Toward Simulations of Scalar Quantum Electrodynamics on Quantum Computers

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Outline

Why $1+1$ Scalar QED

- Theory has continuous symmetries

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- Theory has continuous symmetries
- Current hardware best supports 1-d models
- Straight forward model to test qudit based hardware on

Why Qudits?

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- Some Hilbert spaces for physics don't map nicely (wasted space)
- Qudits *generally* have better two-qudit gate scalings than native qubits (arXiv:1905.10481)

Euclidean Time Action

Formalism developed in: arXiv:1512.01737, arXiv:1803.11166 , arXiv:1807.09186

Model

$$\mathcal{S} = -\frac{1}{g^2 a_s a_\tau} \sum_x \sum_{\nu<\mu} Re(U_{x,\mu\nu}) - \frac{a_\tau}{a_s} \sum_x \left(\phi^\dagger U_{x,s} \phi_{x+\hat{s}} + h.c.\right)$$
$$- \frac{a_s}{a_\tau} \sum_x \left(\phi_x^\dagger U_{x,\tau} \phi_{x+\tau} + h.c.\right) \tag{1}$$

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Compact Variables

$$U_{x,\mu\nu} = U_{x,\mu} U_{x+\hat{\mu},\nu} U^\dagger_{x+\hat{\nu},\mu} U^\dagger_{x,\nu}, \;\; U_{x,\nu} = e^{i a_\mu A_{x,\nu}}, \text{ and } \phi_x = e^{i\theta_x} \tag{2}$$

Continuous Time

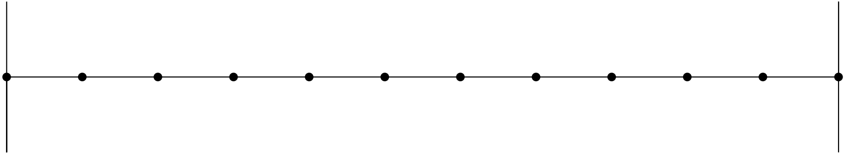
Hamiltonian

$$\hat{H} = \frac{U}{2}\sum_{i=1}^{N_s}(\hat{L}_i^z)^2 + \frac{Y}{2}\sum_{i=1}^{N_s-1}(\hat{L}_i^z - \hat{L}_{i+1}^z)^2 + \frac{Y}{2}((\hat{L}_1^z)^2 + (\hat{L}_{N_s}^z)^2) - X\sum_{i=1}^{N_s}\hat{U}^x \quad (3)$$

Operators

$\hat{L}^z|m\rangle = m|m\rangle$ and $\hat{U}^x = (\hat{U}^- + \hat{U}^+)/2$; $\hat{U}^\pm|m\rangle = |m \pm 1\rangle$
$U = g^2 a_s$, $Y = 1/2a_s$, and $X = 2/a_s$.

Model: A picture



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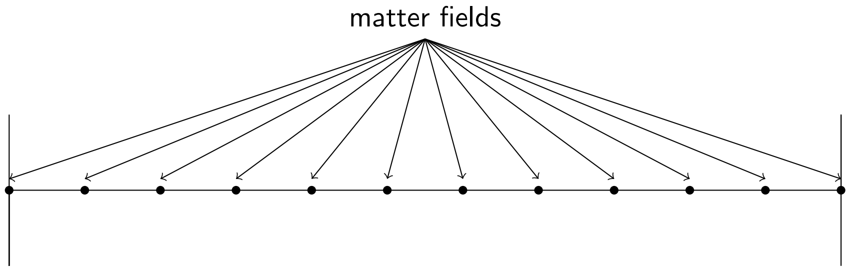

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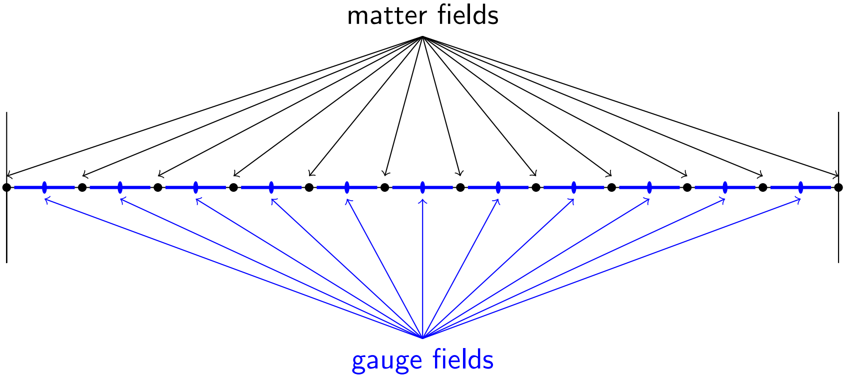
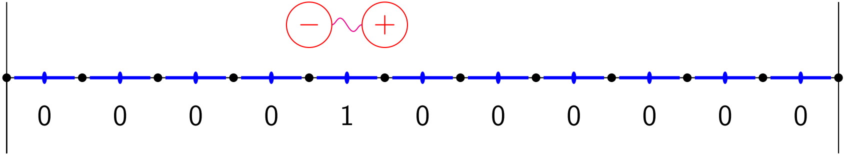


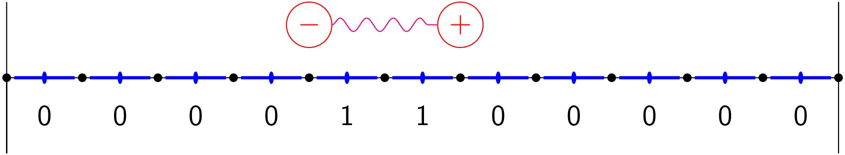
Model: A picture

$$(\hat{L}^z_i - \hat{L}^z_{i+1}) \rightarrow \text{particle number}$$



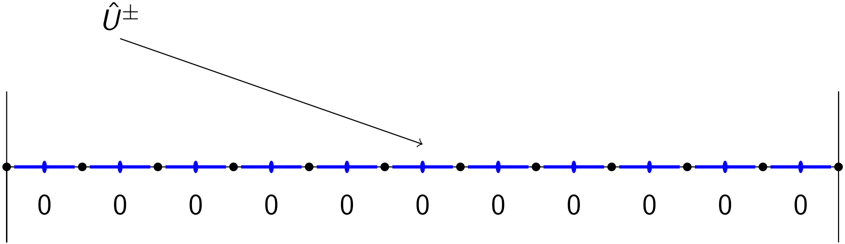
Model: A picture

$(\hat{L}_i^z - \hat{L}_{i+1}^z) \rightarrow$ particle number



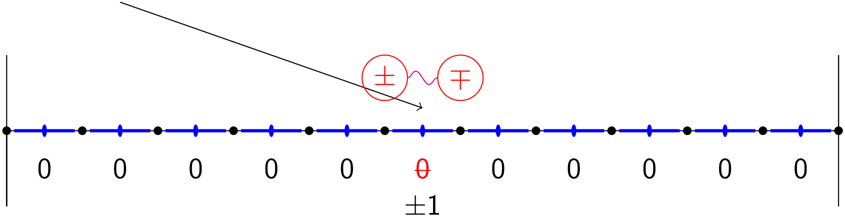
$(\hat{L}^z)^2 \rightarrow$ photons (ish)

Model: A picture



Model: A picture

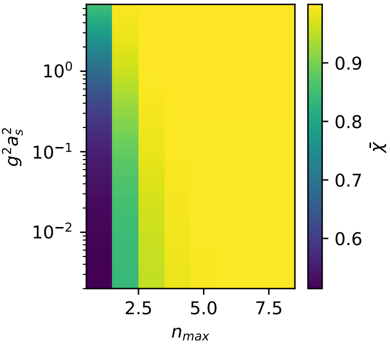
$\hat{U}^{\pm} \quad \rightarrow \quad$ Meson creation / annihilation operator



Truncation systematics

- Measure Accuracy with

$$\chi = \frac{1}{N_s}\sum_{i=1}^{N_s}\sum_{j=1}^{N_s}\langle\Omega|(\hat{L}_i^z - \hat{L}_j^z)^2|\Omega\rangle$$



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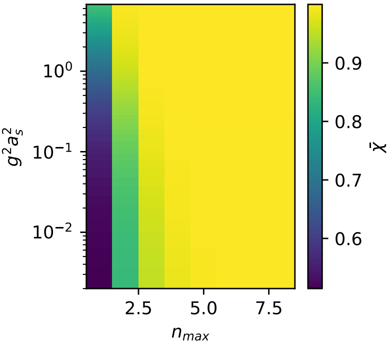
- Strong coupling (Low truncation is good)



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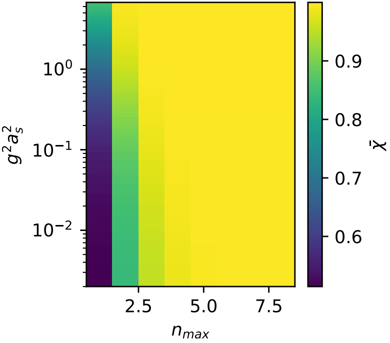
- Strong coupling (Low truncation is good)
- Weak coupling (Higher truncation is necessary)



$exp(-i\theta(\hat{L}_i^z)^2)$ rotations

- $\hat{\sigma}^x_{a,b}|c\rangle = \begin{cases} |b\rangle & c = a \\ |a\rangle & c = b \\ 0 & c \neq a, b \end{cases}$ $\hat{\sigma}^y_{a,b}|c\rangle = \begin{cases} i|b\rangle & c = a \\ -i|a\rangle & c = b \\ 0 & c \neq a, b \end{cases}$

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- $(\hat{L}^z)^2 = \alpha_0 \mathbf{1} + \sum_{j=0}^{2n_{max}-1} \alpha_{j,j+1}\hat{\sigma}^z_{j,j+1}$

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- $\alpha_{j,j+1}(n) = \frac{2j+2}{3}n^2 - \frac{3j^2-4j-1}{3}n + \frac{2j^3+3j^2+j}{6}$

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- requires $2n_{max} + 1$ single qudit rotations to implement

$exp(-i\theta(\hat{U}_i^x))$ rotations

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- if U^x wraps around can be implemented with a single qudit Hadamard.

$exp(-i\theta(\hat{L}^z \otimes \hat{L}^z))$ rotations

- $e^{-i\theta\hat{L}^z\otimes\hat{L}^z} = C_{sum}\prod_{i=0}^{2n_{max}-1}\left(\prod_{j=0}^{2n_{max}-1} R^z_{j,j+1}(\theta\beta_{i,j})C_{sum}\right)$

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- $C_{sum} = \sum_{j=0}^{2n_{max}+1} |j\rangle\langle j| \otimes \mathcal{X}^j$
- $\mathcal{X} = \sum_{j=0}^{2n_{max}+1} |j\rangle\langle mod_{2n_{max}+1}(j+1)|$

Trotter operators: $exp(-i\delta t(U/2+Y)(\hat{L}^z)^2)$

Plaquette Operator

$$R^{Lz}(\delta t) = e^{-i\delta t\frac{U+2Y}{2}(\hat{L}^z)^2} = R^z_{0,1}\Big(-\frac{\delta t(U+2Y)}{6}\Big)R^z_{1,2}\Big(\frac{\delta t(U+2Y)}{6}\Big)$$

2 1-qutrit gates

Trotter operators: $exp(-i\delta tX\hat{U}^x)$

U^x rotation

$$R^U(\delta t) = e^{-i\delta tX\hat{U}^x} = R^y_{0,1}\left(-\frac{\pi}{4}\right)R^y_{0,2}\left(\frac{\pi}{4}\right)R^z\left(\frac{\delta tX}{\sqrt{2}}\right)R^y_{0,2}\left(-\frac{\pi}{4}\right)R^y_{0,1}\left(\frac{\pi}{4}\right)$$

5 1-qutrit gates

Trotter operators: $exp(i\delta tY\hat{L}^z \otimes \hat{L}^z)$

Interaction

$$R^{LzLz}(\delta t) = e^{i\delta tY\hat{L}^z\hat{L}^z} = C_{sum}^{c\to t} R_{0,1}^{z;t}\left(\frac{2\delta tY}{3}\right) R_{1,2}^{z;t}\left(\frac{\delta tY}{3}\right) C_{sum}^{c\to t} R_{1,2}^{z,1}\left(\frac{2\delta tY}{3}\right) R_{0,1}^{z,1}\left(\frac{\delta tY}{3}\right) C_{sum}^{c\to t}$$

4 1-qutrit gates, 3 2-qutrit gates

Trotter operators: $exp(-i\delta t(U/2+Y)(\hat{L}^z)^2)$

Plaquette Operator

$$\hat{L}^z = (\hat{Z}^2 + \hat{Z}^1\hat{Z}^2)/2 \rightarrow (\hat{L}^z)^2 = (1+\hat{Z}^1)/2$$

1 1-qubit gate

Trotter operators: $exp(-i\delta t X \hat{U}^x)$

$\hat{U}^x$ rotation

$$\hat{U}^x = \hat{X}^1(\mathbf{1} + \hat{X}^2 + \hat{Z}^2)/2 + \hat{Y}^1\hat{Y}^2/2$$

¡15 1-qubit gates; 3 CNOTs

Trotter operators: $exp(i\delta t Y \hat{L}^z \otimes \hat{L}^z)$

Interaction

$$\hat{L}_1^z \hat{L}_2^z = (\hat{Z}_2^1 + \hat{Z}_1^1 \hat{Z}_1^2) \otimes (\hat{Z}_2^2 + \hat{Z}_1^2 \hat{Z}_2^2)/4$$

4 1-qubit gates, 8 CNOTs

Summary

Term	Qubits 1-q	Qubits 2-q	Qutrits 1-q	Qutrits 2-q
$(L^z)^2$	1	0	2	0
$L^z L^z$	4	8	4	3
U^x	15	3	5	0

Parameters and Observable

- $U = 5,\ Y = 1/2$, and $X = 2$; 4 site lattice
- $\langle \Gamma | e^{it\hat{H}} \hat{U}_0^- e^{-it\hat{H}} \hat{U}^+ | \Gamma \rangle$

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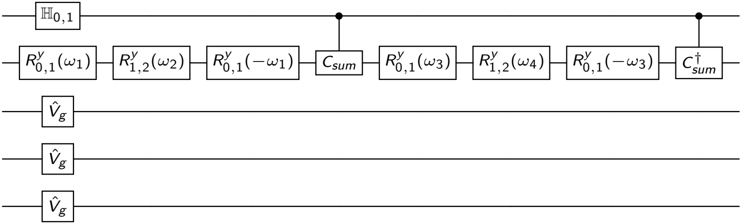
- $|\Gamma\rangle = \prod_{i=1}^{4}(|\Psi_0\rangle)^{\otimes}$
- $|\Psi_0\rangle = \frac{1}{\mathcal{N}}(|0\rangle + b|1\rangle + |2\rangle)$
- $$\mathcal{A} = \frac{1}{2}\begin{pmatrix} U+Y & -X & 0 \\ -X & 0 & -X \\ 0 & -X & U+Y \end{pmatrix}$$

Circuit: State Prep



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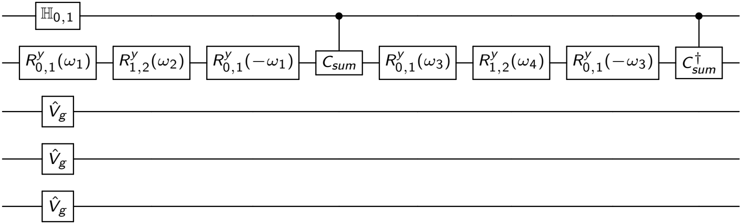
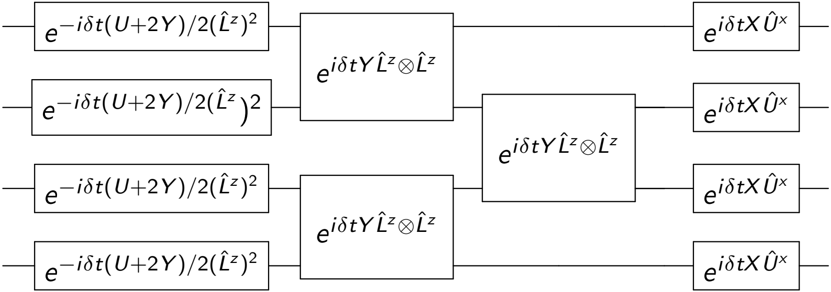


$$\hat{V}^g = \hat{R}^y_{1,2}(-arcsin(1/\sqrt{\mathcal{N}^2 - 1}))\hat{R}^y_{0,1}arccos(1/\mathcal{N}))$$

$$\omega_1 = -0.65273,\ \omega_2 = -1.43696,\ \omega_3 = 1.7837,\ \text{and}\ \omega_4 = 2.65568.$$

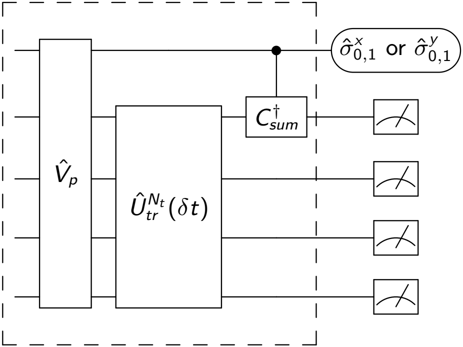
Circuit: time evolution



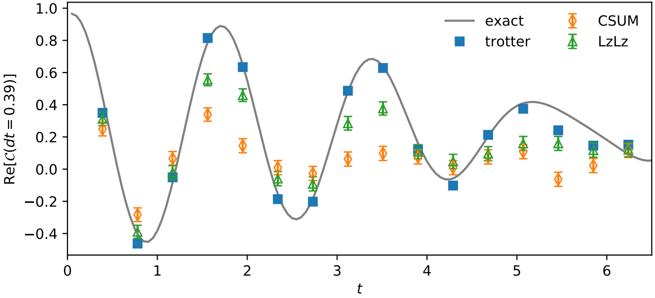
Circuit: Full

$$\mathcal{C} = (\langle|\hat{C}^\dagger(\hat{\sigma}^x_{0,1})_a\hat{C}|\rangle + \langle|\hat{C}^\dagger(\hat{\sigma}^x_{0,1})_a(\hat{Z}_2)\hat{C}|\rangle + i\langle|\hat{C}^\dagger(\hat{\sigma}^y_{0,1})_a\hat{C}|\rangle + \langle|\hat{C}^\dagger(\hat{\sigma}^y_{0,1})_a(\hat{Z}_2)\hat{C}|\rangle)/2,$$

$\hat{C}$ is defined below



Noisy Simulation



- Noise Levels taken from Morvan et al. arxiv:2008.09134
- Noise assumed to apply only to R^y, R^x ($\approx$ 99 per-cent fidelity) , and two-qutrit rotations ($\approx$ 82 per-cent fidelity).

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- Have a way to implement Abelian Higgs in 1+1d onto digital quantum computers
- Appears feasible to simulate on current reported implementations of qutrit based hardware

Acknowledgements

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