Implementing noise reduction techniques into the OpenQ*D package

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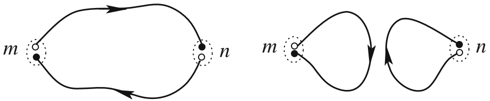
Vector correlator

$$\Pi(q)_{\mu\nu} = \int d^4 x e^{iq\cdot x} \langle V_\mu(x) V_\nu(0) \rangle$$



[Gattringer and Lang, 2010]

$$\begin{aligned} \langle V_{\mu,s}(n) V_{\nu,s}(m) \rangle_F &= \langle \overline{\psi}_s(n) \gamma_\mu \psi_s(n) \overline{\psi}_s(m) \gamma_\nu \psi_s(m) \rangle_F \\ &= -\mathrm{tr}[\gamma_\mu D^{-1}(n|m) \gamma_\nu D^{-1}(m|n)] \\ &\quad - \mathrm{tr}[\gamma_\mu D^{-1}(n|n)] \mathrm{tr}[\gamma_\nu D^{-1}(m|m)] \end{aligned} \tag{1}$$

$$\begin{aligned} &\sum_z D(n,z)\phi(z) = \eta^{(r)}(n), \qquad r = 1, ..., N_r \\ &\langle\langle \eta(n) \eta^\dagger(m) \rangle\rangle := \lim_{N_r \to \infty} \frac{1}{N_r} \sum_r \eta(n)^{(r)} \eta^{(r)}(m)^\dagger = \delta_{n,m} \end{aligned} \tag{2}$$

Noise and Low mode averaging

- High Gauge noise at long distance, scaling $\propto e^{2(m_\rho - m_\pi)t}$
- Stochastic noise from random sources
- Stochastic noise scaling improvements: One-end trick, dilution schemes, low-mode averaging

Low mode averaging:

Calculate N lowest eigenvectors of the Dirac operator

Split quark propagators into low and high eigenmode contributions

$$\psi = \psi_{low} + \psi_{high}$$

Calculate ψ_{low} using the eigenvectors, ψ_{high} stochastically

[DeGrand and Schaefer, 2004]

Distillation

Idea: Low-mode averaging, but cheaper

Instead of Dirac operator, use the gauge-inavariant spatial Laplacian

$$\bigtriangledown^2_{mn}(t) = -6\delta_{mn} + \sum_{j=1}^{3}\Big(U_j(m,t)\delta_{m+\hat{j},n} + U_j^\dagger(m-\hat{j},t)\delta_{m-\hat{j},n}\Big) \tag{3}$$

Use lowest N eigenmodes to form distillation projector

$$\Box(t) = V(t)V^\dagger(t) = \sum_{k=1}^{N} v^{(k)}(t)v^{(k)\dagger}(t) \tag{4}$$

[Peardon et al., 2009]

Insert $1 - \Box(t) + \Box(t)$ into correlator C at source and sink. Then:

$$C(t) = C_{dist}(t) + C_{rest}(t) + C_{cross}(t) \tag{5}$$

Number of required Dirac solves is now $N_{dist} + 2N_s$

Distillation

Implemented for the connected contribution in OpenQ*D, Laplacian solved with Primme(Preconditioned Iterative MultiMethod Eigensolver) [Wu et al., 2016]

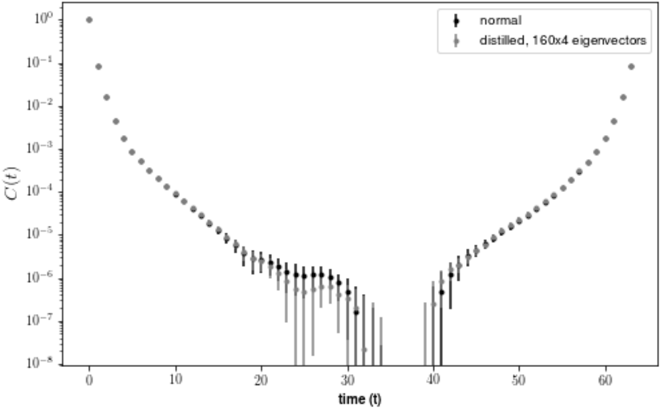
Tests in OpenQ*D on 30 configurations from two CLS data sets with two flavours of dynamical $O(a)$ improved Wilson fermions. Computed with periodic boundary conditions and stochastic wall sources

[Campos et al., 2020]

Config	V	β	κ	$m_\pi L$	a [fm]	m_π [MeV]
E5	$64x32^3$	5.30	0.13625	4.7	0.0658(7)(7)	437
A5	$64x32^3$	5.20	0.13594	4.0	0.0755(9)(7)	331

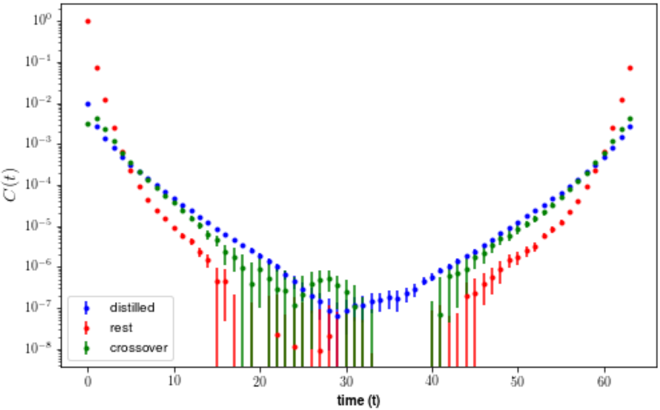
[Della Morte et al., 2017] [Fritzsch et al., 2012]

Vector-Vector correlator with distillation



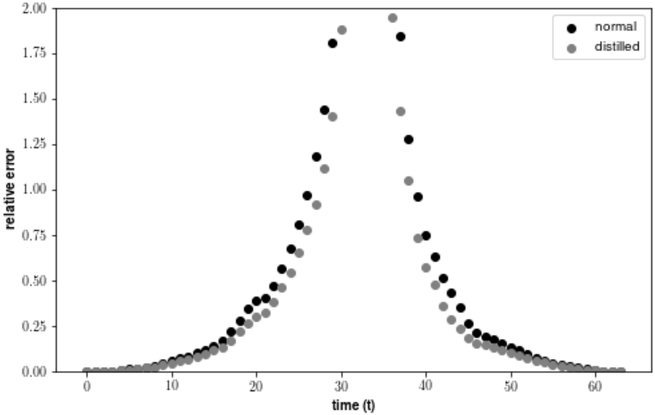
E5 vector correlator, 24 sources

Vector-Vector correlator with distillation



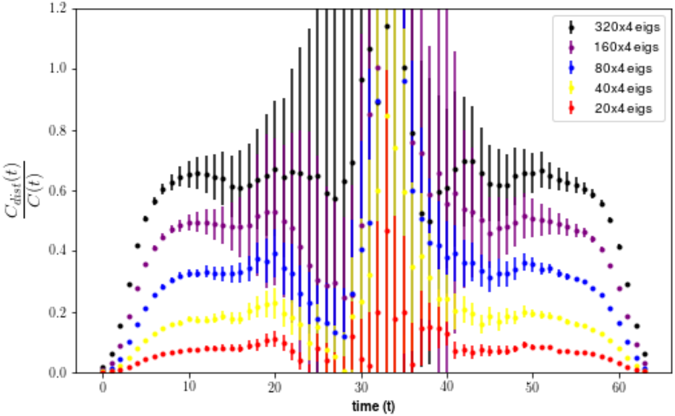
E5 vector correlator by parts, 24 sources

Vector-Vector correlator with distillation



E5 vector correlator precision, 24 sources

Distilled sub-space contribution

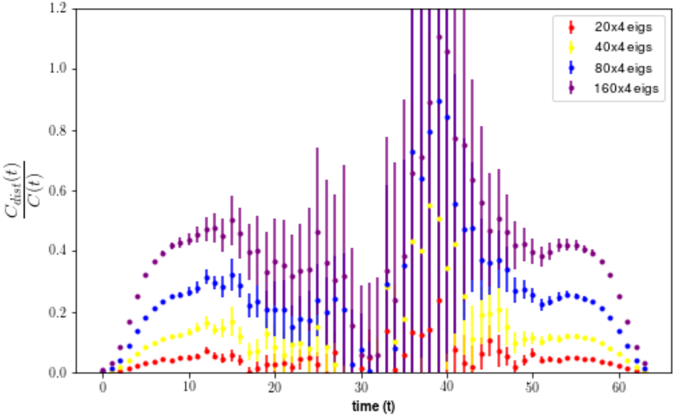


E5 vector correlator C_{dist} subspace, 24 sources

Laplacian Solver performance scaling

N_{dist}	C_{dist} Dirac solver	Lap. solver	Lap. solver Orthogonalisation	Lap. solver MatVec
20x4	1.18e+02 s	4.63e+01 s	3.51e+00 s	3.11e+01 s
40x4	2.53e+02 s	1.04e+02 s	1.08e+01 s	6.87e+01 s
80x4	4.84e+02 s	2.43e+02 s	4.18e+01 s	1.52e+02 s
160x4	1.12e+03 s	7.77e+02 s	2.51e+02 s	3.46e+02 s
320x4	2.82e+03 s	2.14e+03 s	1.14e+03 s	7.40e+02 s

Distilled sub-space A5



A5 vector correlator C_{dist} subspace

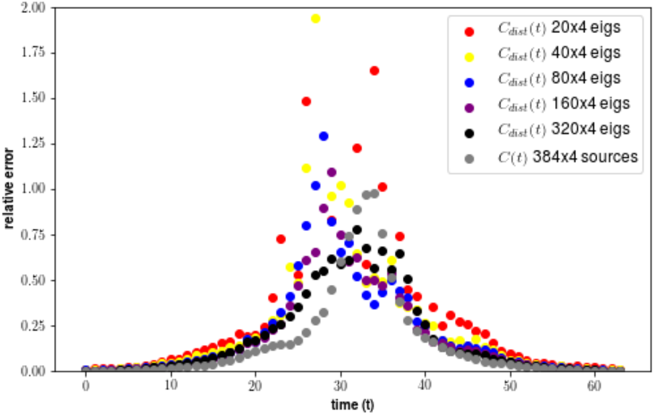
Conclusion

Idea: Use gauge-covariant Laplacian modes instead of Low mode averaging with Dirac modes

- The contribution of the eigenspace to the vector-vector correlator grows too slowly with the number of eigenmodes
- Also tested with the pseudoscalar, similar result

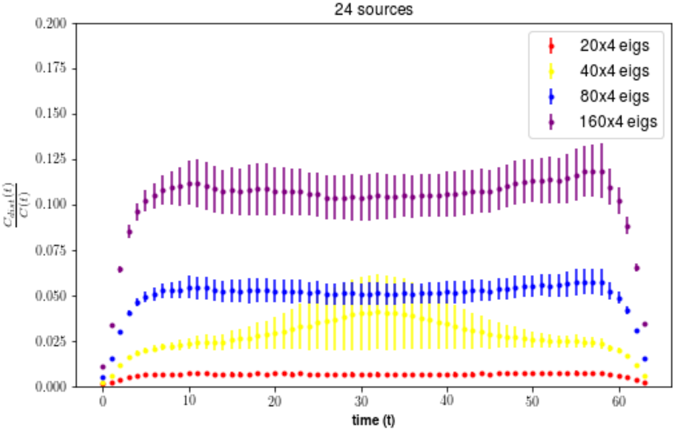
Stick with Low mode averaging

Backup: distillation subspace gauge noise



E5 vector correlator C_{dist} noise scaling

Backup: pseudoscalar



A5 pseudoscalar C_{dist} noise scaling

Backup: Insertion

$$
\begin{aligned}
&\langle \overline{\psi}_s^A(t)\gamma_\mu \psi_s^B(t)\overline{\psi}_s^B(t')\gamma_\nu \psi_s^A(t')\rangle_F \\
&= \langle \overline{\psi}_s^A(t)\gamma_\mu \left(1-\Box(t)+\Box(t)\right)\psi_s^B(t)\overline{\psi}_s^B(t')\gamma_\nu \left(1-\Box(t')+\Box(t')\right)\psi_s^A(t')\rangle_F \\
&= \langle \overline{\psi}_s^A(t)\gamma_\mu \left(1-\Box(t)\right)\psi_s^B(t)\overline{\psi}_s^B(t')\gamma_\nu \left(1-\Box(t')\right)\psi_s^A(t')\rangle_F \\
&+ 2\langle \overline{\psi}_s^A(t)\gamma_\mu \Box(t)\psi_s^B(t)\overline{\psi}_s^B(t')\gamma_\nu \left(1-\Box(t')\right)\psi_s^A(t')\rangle_F \\
&+ \langle \overline{\psi}_s^A(t)\gamma_\mu \Box(t)\psi_s^B(t)\overline{\psi}_s^B(t')\gamma_\nu \Box(t')\psi_s^A(t')\rangle_F \\
&= \frac{1}{N_r}\mathrm{tr}[\gamma_5\gamma_\mu \left(1-\Box(t)\right)D^{-1}(t,t')\eta^{(r)}(t') \\
&\qquad \eta^{\dagger(r)}(t')\gamma_\nu\gamma_5 \left(1-\Box(t')\right)D^{-1\dagger}(t',t)] \\
&+ \frac{2}{N_r}\mathrm{tr}[\gamma_5\gamma_\mu \Box(t)D^{-1}(t,t')\eta^{(r)}(t')\eta^{\dagger(r)}(t')\gamma_\nu\gamma_5 \left(1-\Box(t')\right)D^{-1\dagger}(t',t)] \\
&+ \mathrm{tr}[\gamma_5\gamma_\mu V^\dagger(t)D^{-1}(t,t')V(t')\gamma_\nu\gamma_5 V^\dagger(t')D^{-1\dagger}(t',t)V(t)]
\end{aligned}
\qquad (6)
$$

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