Finite volume corrections to forward Compton scattering off the nucleon

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In collaboration with: Agadjanov, Gegelia, Meißner and Rusetsky

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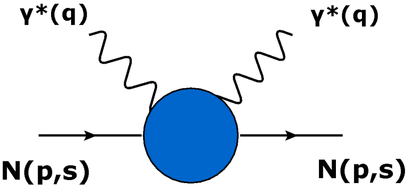
Nucleon Forward Compton Scattering



The doubly-virtual forward Compton scattering.

- Spin-averaged Compton tensor:

$$T^{\mu\nu} = (q^\mu q^\nu - g^{\mu\nu} q^2)\, T_1(\nu, q^2) + \frac{1}{m^2}\left\{(p^\mu q^\nu + p^\nu q^\mu) p\cdot q - g^{\mu\nu}(p\cdot q)^2 - p^\mu p^\nu q^2\right\} T_2(\nu, q^2)$$

- Two scalar amplitudes involved: $T_1(\nu, q^2)$ and $T_2(\nu, q^2)$, $\nu \equiv p\cdot q/m$

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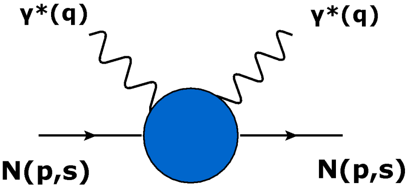

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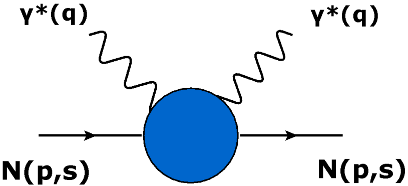

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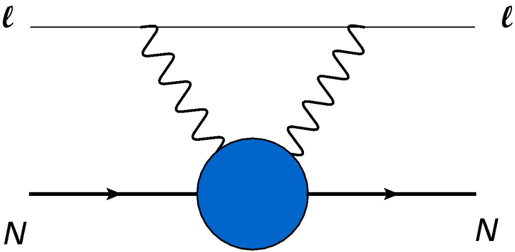
  - the proton-neutron mass difference
  - Lamb shift in the muonic hydrogen

Lamb Shift: Polarizability Contribution



Two-photon exchange of lepton-nucleon scattering.

- Higher-order proton structure corrections to the Lamb shift in muonic hydrogen: Carlson, Vanderhaeghen 2011

$$\Delta E_{nS} = \frac{\alpha_{em}\phi_n^2}{4\pi^3 m_l}\frac{1}{i}\int d^3q\int_0^\infty d\nu\,\frac{(q^2-2\nu^2)\,T_1(\nu,q^2)-(q^2+\nu^2)\,T_2(\nu,q^2)}{q^4[(q^2/4m_l^2)-\nu^2]}.$$

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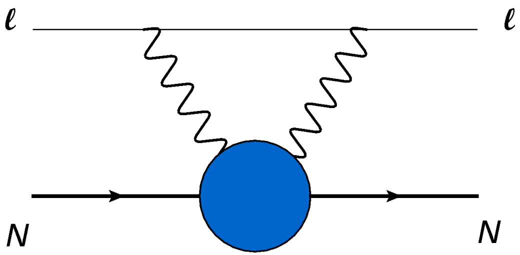

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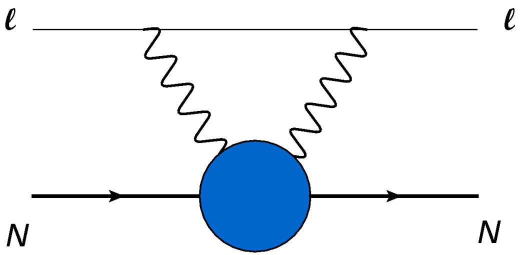

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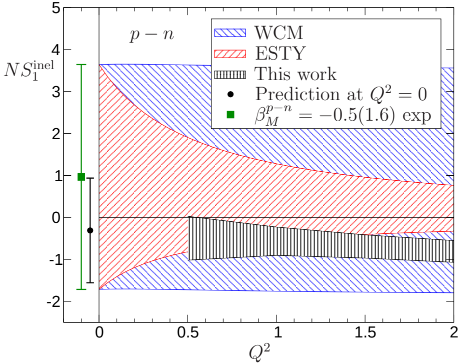
- Muonic Lamb-shift $\rightarrow$ T_1, T_2 $(q^2<0)$
- The $T_1(\nu, q^2)$ can be evaluated using the once-subtracted dispersion integral
- A problem: $S_1(q^2) \equiv T_1(0,q^2)$ is not fixed by experiments.

Determination of S_1



Reggeon dominance hypothesis. Gasser et al. 2015.

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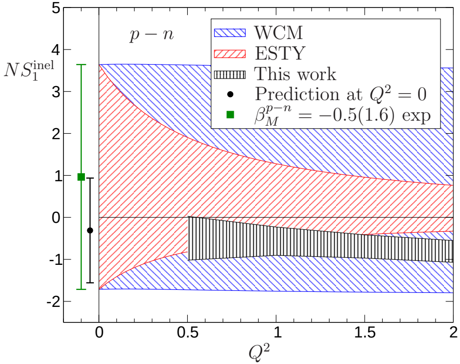

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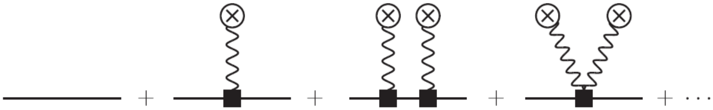
- Lattice QCD: model-independent

External Field Method

- Using lattice QCD, the Compton tensor can be studied
- Study the two-point function in an external em. field



Detmold et al. 2006

- Uniform electromagnetic field $\to$ polarizabilities   Detmold et al. 2006

Nucleon in a Periodic Magnetic Field

- Static magnetic field $\rightarrow$ stable energy levels.
- The energy shift of a nucleon on the lattice, using the external field method, is

  Agadjanov, Rusetsky and Meißner 2017, Schierholz et al. 2017

$$\delta E = \frac{e^2 B^2}{4m} S_1(-\omega^2) + O(B^3).$$

- $\mathbf{B} = (0, 0, -eB\cos(\omega \mathbf{n}\mathbf{x}))$, $\mathbf{n} = (0, 1, 0)$ and $\omega = \frac{2\pi n}{L}$

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- Kinematics: $p = (m, \vec{0})$, $q = (0, 0, \omega, 0)$.
- Work left to do: Subtract finite-volume correction to $T^{11} \rightarrow S_1$.

Finite Volume Artifacts

- Lattice simulations are done in a finite-volume $\rightarrow$ finite-volume effects.
- Two kinds of FVE:
  - Type 1: Polarization Effects: exponential
  - Type 2: Multi-hadron intermediate states: power law

In this work, we deal with FVE of the first type.

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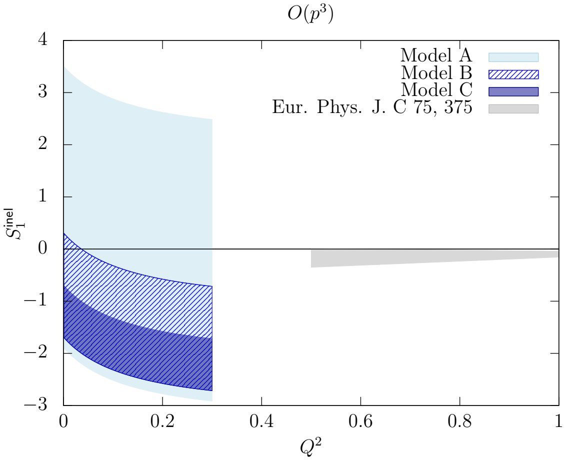
- How to estimate them? $\rightarrow$ baryon ChPT in a finite-volume
- The Lagrangians are the same in the infinite and in a finite volume
- The 3-momentum integrals changed by sums:

$$\int \frac{d^3\mathbf{k}}{(2\pi)^3} \rightarrow \frac{1}{L^3}\sum_{\mathbf{k}}, \qquad \mathbf{k} = \frac{2\pi}{L}\mathbf{n}, \qquad \mathbf{n} \in \mathbb{Z}^3$$

Results in the infinite-volume



- Model A: $\mathtt{S}^{\mathtt{inel}}(0) = (0.8 \pm 2.7)\mathtt{GeV}^{-2} \longrightarrow$ Purely experimental
- Model B: $\mathtt{S}^{\mathtt{inel}}(0) = (-0.7 \pm 1.0)\mathtt{GeV}^{-2} \longrightarrow$ Experimental + Reggeon
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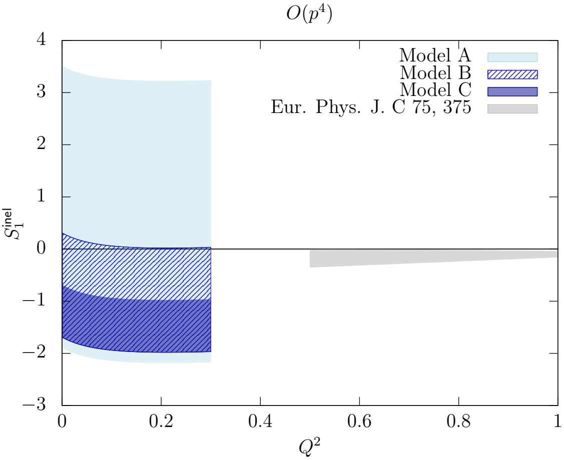

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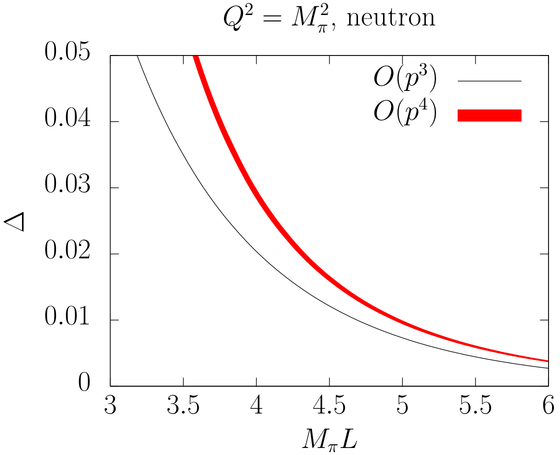

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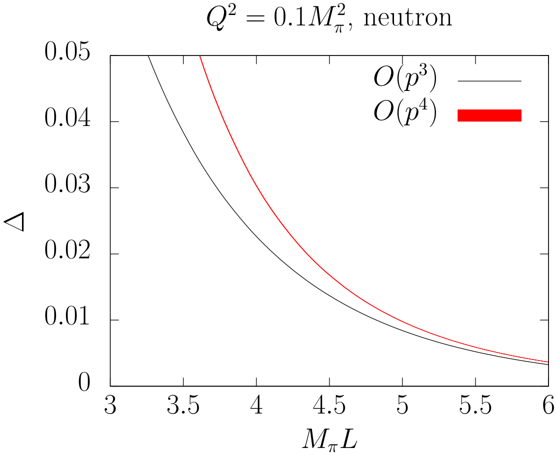

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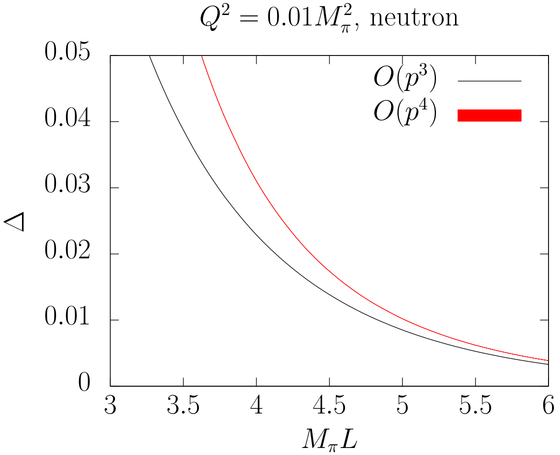

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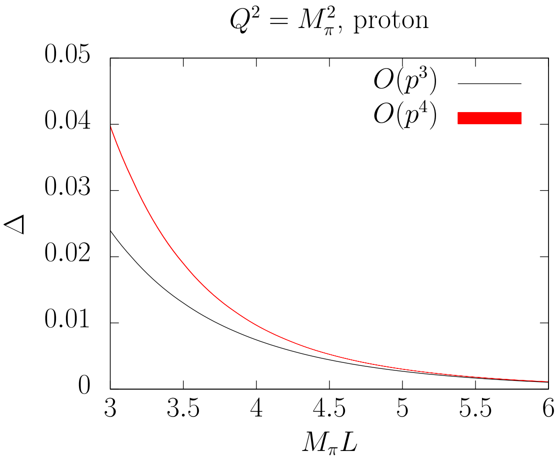

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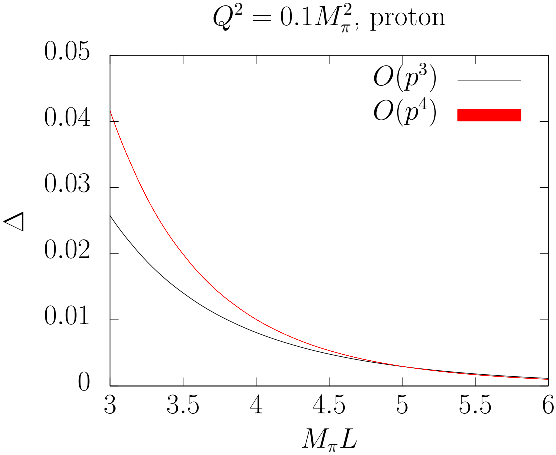

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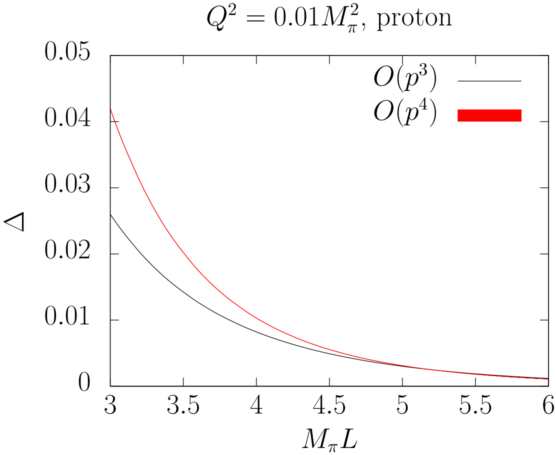

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- Low energy constants at $O(p^4)$, although not accurately known, do not pose a problem in the convergence of our results.
- The extraction of the infinite-volume S_1 with good accuracy is possible for reasonable large lattices.