Finite volume corrections to forward Compton scattering off the nucleon

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In collaboration with: Agadjanov, Gegelia, Meißner and Rusetsky

July 27, 2021
Spin-averaged Compton tensor:

\[
T^{\mu\nu} = (q^\mu q^\nu - g^{\mu\nu} q^2) T_1(\nu, q^2) \\
+ \frac{1}{m^2} \left\{ (p^\mu q^\nu + p^\nu q^\mu) p \cdot q - g^{\mu\nu} (p \cdot q)^2 - p^\mu p^\nu q^2 \right\} T_2(\nu, q^2)
\]

Two scalar amplitudes involved: \( T_1(\nu, q^2) \) and \( T_2(\nu, q^2) \), \( \nu \equiv p \cdot q / m \)
Nucleon Forward Compton Scattering

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- the proton-neutron mass difference
Nucleon Forward Compton Scattering

Spin-averaged Compton tensor:

\[ T^{\mu\nu} = (q^{\mu} q^{\nu} - g^{\mu\nu} q^2) T_1(\nu, q^2) \]
\[ + \frac{1}{m^2} \left\{ (p^{\mu} q^{\nu} + p^{\nu} q^{\mu}) p \cdot q - g^{\mu\nu} (p \cdot q)^2 - p^{\mu} p^{\nu} q^2 \right\} T_2(\nu, q^2) \]

Two scalar amplitudes involved: \( T_1(\nu, q^2) \) and \( T_2(\nu, q^2) \), \( \nu \equiv p \cdot q / m \)
- the proton-neutron mass difference
- Lamb shift in the muonic hydrogen
Lamb Shift: Polarizability Contribution

Two-photon exchange of lepton-nucleon scattering.

- Higher-order proton structure corrections to the Lamb shift in muonic hydrogen: Carlson, Vanderhaeghen 2011

\[
\Delta E_{nS} = \frac{\alpha_{em} \phi_n^2}{4\pi^3 m_l} \frac{1}{i} \int d^3 q \int_0^\infty d\nu \frac{(q^2 - 2\nu^2) T_1(\nu, q^2) - (q^2 + \nu^2) T_2(\nu, q^2)}{q^4[(q^2/4m_l^2) - \nu^2]}. 
\]

- Muonic Lamb-shift → \( T_1, T_2 \) (\( q^2 < 0 \))
Lamb Shift: Polarizability Contribution

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- Muonic Lamb-shift \( \rightarrow T_1, T_2 (q^2 < 0) \)
- The \( T_1(\nu, q^2) \) can be evaluated using the once-subtracted dispersion integral

\[ q^4[(q^2/4m_l^2) - \nu^2] \]
Lamb Shift: Polarizability Contribution

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\[ \Delta E_{nS} = \frac{\alpha_{em} \phi_n^2}{4\pi^3 m_e} \frac{1}{i} \int d^3 q \int_0^{\infty} d\nu \frac{(q^2 - 2\nu^2) T_1(\nu, q^2) - (q^2 + \nu^2) T_2(\nu, q^2)}{q^4[(q^2/4m_e^2) - \nu^2]} \]

- Muonic Lamb-shift \( \rightarrow T_1, T_2 \ (q^2 < 0) \)
- The \( T_1(\nu, q^2) \) can be evaluated using the once-subtracted dispersion integral
- A problem: \( S_1(q^2) \equiv T_1(0, q^2) \) is not fixed by experiments.
Reggeon dominance hypothesis. Gasser et al. 2015.

- Chiral effective field theories J. Alarcón et al. 2014
- Reggeon dominance Gasser et al. 2015
Determination of $S_1$

Reggeon dominance hypothesis. Gasser et al. 2015.

- Chiral effective field theories J. Alarcón et al. 2014
- Reggeon dominance Gasser et al. 2015
- Lattice QCD: model-independent
Using **lattice QCD**, the **Compton tensor** can be studied.

Study the two-point function in an **external** em. field.

- Uniform electromagnetic field $\rightarrow$ polarizabilities  

  Detmold et al. 2006
Nucleon in a Periodic Magnetic Field

- **Static** magnetic field $\rightarrow$ stable energy levels.
- The **energy shift** of a nucleon on the lattice, using the external field method, is
  
  $\delta E = \frac{e^2 B^2}{4m} S_1(-\omega^2) + O(B^3)$.

- $B = (0, 0, -eB \cos(\omega nx))$, $n = (0, 1, 0)$ and $\omega = \frac{2\pi n}{L}$

- Recently, this result was rederived in a finite volume
  
  $\delta E = -\frac{1}{4m} \left( eB \omega^2 \right) T_{11}(p, q) + O(B^3)$.

- Kinematics: $p = (m, \vec{0})$, $q = (0, 0, \omega, 0)$.

- Work left to do: Subtract finite-volume correction to $T_{11} \rightarrow S_1$. 

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FV corrections to VVCS off the nucleon
July 27, 2021
Nucleon in a Periodic Magnetic Field

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- The *energy shift* of a nucleon on the lattice, using the external field method, is

  \[ \delta E = \frac{e^2 B^2}{4m} S_1(-\omega^2) + O(B^3). \]

  \[ B = (0, 0, -eB \cos(\omega nx)), \quad n = (0, 1, 0) \text{ and } \omega = \frac{2\pi n}{L} \]

  More recently, this result was rederived in a finite volume

  \[ \delta E = -\frac{1}{4m} \left( \frac{eB}{\omega} \right)^2 T^{11}(p, q) + O(B^3). \]

- Kinematics: \( p = (m, \vec{0}), \quad q = (0, 0, \omega, 0). \)
Nucleon in a Periodic Magnetic Field

- **Static** magnetic field $\rightarrow$ stable energy levels.
- The **energy shift** of a nucleon on the lattice, using the external field method, is

  Agadjanov, Rusetsky and Meißner 2017, Schierholz et al. 2017

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- Kinematics: \( p = (m, \vec{0}) \), \( q = (0, 0, \omega, 0) \).
- Work left to do: Subtract **finite-volume** correction to \( T^{11} \rightarrow S_1 \).
Lattice simulations are done in a **finite-volume** effects.

Two kinds of **FVE**:
- Type 1: Polarization Effects: exponential
- Type 2: Multi-hadron intermediate states: power law

In this work, we deal with FVE of the first type.
Lattice simulations are done in a finite-volume effects.

Two kinds of FVE:
- Type 1: Polarization Effects: exponential
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In this work, we deal with FVE of the first type.

How to estimate them? → baryon ChPT in a finite-volume

The Lagrangians are the same in the infinite and in a finite volume.

The 3-momentum integrals changed by sums:

\[
\int \frac{d^3k}{(2\pi)^3} \rightarrow \frac{1}{L^3} \sum_k, \quad k = \frac{2\pi}{L} n, \quad n \in \mathbb{Z}^3
\]
Results in the infinite-volume

\[ O(p^3) \]

- **Model A**: \( S_{\text{inel}}(0) = (0.8 \pm 2.7) \text{GeV}^{-2} \rightarrow \text{Purely experimental} \)
- **Model B**: \( S_{\text{inel}}(0) = (-0.7 \pm 1.0) \text{GeV}^{-2} \rightarrow \text{Experimental + Reggeon} \)
- **Model C**: \( S_{\text{inel}}(0) = (-1.2 \pm 0.5) \text{GeV}^{-2} \rightarrow \text{Experimental + Lattice} \)
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Model C: $S_{\text{inel}}(0) = (-1.2 \pm 0.5)\text{GeV}^{-2} \rightarrow$ Experimental + Lattice
\[ Q^2 = M^2_\pi, \text{ neutron} \]

\[ \Delta \equiv \frac{T^{11}_L(p,q) - T^{11}(p,q)}{T^{11}(p,q)} \]
$Q^2 = 0.1M_{\pi}^2, \ \text{neutron}$

$$\Delta \equiv \frac{T_{L}^{11}(p,q) - T_{11}^{11}(p,q)}{T_{11}^{11}(p,q)}$$
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\[ Q^2 = 0.01M_{\pi}^2, \text{ neutron} \]
$Q^2 = M_{\pi}^2$, proton

$\Delta \equiv \frac{T_{11}^{11}(p,q) - T_{11}^{11}(p,q)}{T_{11}^{11}(p,q)}$
FV Correction: Proton

\[ Q^2 = 0.1 M^2_{\pi}, \text{ proton} \]

\[ \Delta \equiv \frac{T_{L}^{11}(p, q) - T^{11}(p, q)}{T^{11}(p, q)} \]
$Q^2 = 0.01M_{\pi}^2$, proton

$\Delta \equiv \frac{T_{L}^{11}(p,q) - T_{11}^{11}(p,q)}{T_{11}^{11}(p,q)}$
A finite-volume calculation of the forward Compton scattering tensor was performed in the framework of ChPT to order $O(p^4)$. For the physical pion mass, and for $M_\pi L \simeq 4$, $\Delta$ is not bigger than 3% for both proton and neutron. Low energy constants at $O(p^4)$, although not accurately known, do not pose a problem in the convergence of our results. The extraction of the infinite-volume $S_1$ with good accuracy is possible for reasonable large lattices.
Summary and Conclusion

- A finite-volume calculation of the forward Compton scattering tensor was performed in the framework of ChPT to order $O(p^4)$.
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Summary and Conclusion

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- For the physical pion mass, and for $M_\pi L \approx 4$, $\Delta$ is not bigger than 3% for both proton and neutron.
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- The extraction of the infinite-volume $S_1$ with good accuracy is possible for reasonable large lattices.