Sigma Terms and Singlet Charges

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Nucleon Charges

Nucleon charges are matrix elements of the form $\langle H|\overline{q}\Gamma q|H\rangle$, where Γ is a spinor structure that defines the charge.

Focus on the sigma terms $m_q \langle H|\overline{q}q|H\rangle$, however we do perform a joint analysis with additional matrix elements.

Γ	Charge
$\mathbb{I}$	g_S^q
$\gamma^5\gamma^\mu$	g_A^q
γ^μ	g_V^q
$\gamma^\mu\gamma^\nu$	g_T^q

The sigma terms are of physical interest in various areas, notably the decomposition of hadron masses [arXiv: 1603.00827]

$$m_H = -\langle T_{\mu\mu}\rangle_N = \sum_q m_q \langle\overline{q}q\rangle_H - \gamma_m(\alpha)\sum_q m_q \langle\overline{q}q\rangle_H - \frac{\beta(\alpha)}{4\alpha}\langle F_{\mu\nu}F_{\mu\nu}\rangle_N$$

and in spin-independent WIMP-nucleon scattering (where λ_q is the coupling of a scalar particle and the quark q) the cross section is

$$\sigma^{SI} \propto [Zf_p + (A-Z)f_n]^2 \qquad f_{n,p} = \sum_q \frac{\sigma_q^{n,p}\lambda_q}{m_q}.$$

CLS Action

The action used by CLS[arXiv: 1411.3982] consists of the Lüscher-Weisz gluonic action, and the Sheikholeslami-Wohlert fermionic action

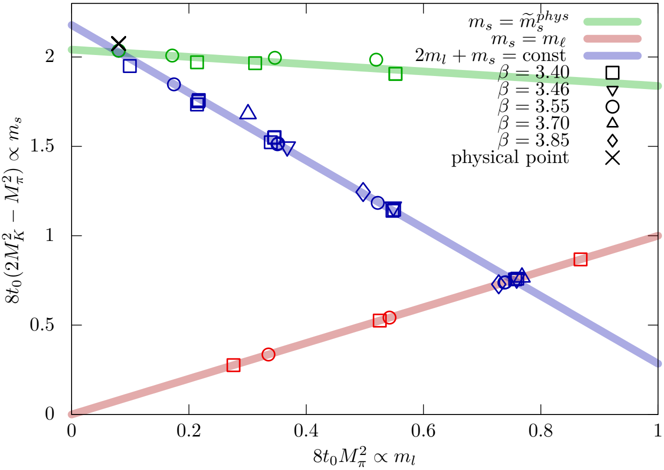
$$S = \frac{\beta}{6}\left(\frac{5}{3}\sum_{p}\mathrm{Tr}\{1-U(p)\} - \frac{1}{12}\sum_{r}\mathrm{Tr}\{1-U(r)\}\right) + \left(a^4\sum_{f=1}^{3}\sum_{x}\overline{\psi}_f(x)D_W(m_{0,f})\psi_f(x)\right)$$

$$D_W(m_{0,f}) = \frac{1}{2}\sum_{\mu=0}^{3}[\gamma_\mu(\nabla_\mu^* + \nabla_\mu) - a\nabla_\mu^*\nabla_\mu] + ac_{SW}\sum_{\mu,\nu=0}^{3}\frac{i}{4}\sigma_{\mu\nu}\widehat{F}_{\mu\nu} + m_{0,f}$$

Aspects of note:

- Chiral symmetry breaking by the Wilson Fermion Term $a\nabla_\mu^*\nabla_\mu$
- $\mathbb{O}(a)$ improvement via the Clover term $\widehat{F}_{\mu\nu}$, where the c_{SW} is determined non-perturbatively [arXiv: 1304.7093].
- Some ensembles have open boundary conditions in time, periodic in space

CLS Ensembles



Typically between 1000 and 2000 configs each. Six lattice spacings $0.039\,\text{fm} < a < 0.098\,\text{fm}$
$Lm_\pi \gtrsim 4$, with some smaller L for volume studies
11 geometries, ranging between $(24^3, 48)$ and $(96^3, 192)$

Correlation Functions

The Correlation Functions used are:

$$C_{2pt}(t_f, t_i) = \left\langle \mathcal{H}(t_f)\overline{\mathcal{H}}(t_i) \right\rangle$$
$$C_{3pt}(t_f, t, t_i) = \left\langle \mathcal{H}(t_f)S(t)\overline{\mathcal{H}}(t_i) \right\rangle - \langle S(t) \rangle \left\langle \mathcal{H}(t_f)\overline{\mathcal{H}}(t_i) \right\rangle$$

- $\mathcal{H}$ (resp. $\overline{\mathcal{H}}$) is the interpolation operator that destroys (creates) a nucleon
- $S(t)$ is an insertion operator that defines the matrix element we can extract

$$S(t) = \overline{q}(t)\Gamma q(t) \qquad \text{with } \Gamma = \mathbb{I} \text{ for the scalar current.}$$

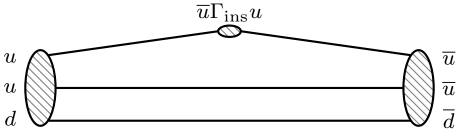
Related to the matrix elements via the spectral decomposition (with the first excitation).

$$C_{2pt}(t_f, 0) \sim Z_1^2 \mathrm{e}^{-t_f m} \left[1 + \frac{Z_2^2}{Z_1^2} \mathrm{e}^{-\Delta m t_f} \right]$$

$$C_{3pt}(t_f, t, 0) \sim Z_1^2 \mathrm{e}^{-t_f m} \left[\langle 1|S|1 \rangle + \frac{Z_2 Z_1}{Z_1^2} \langle 2|S|1 \rangle \left(\mathrm{e}^{-\Delta m (t_f - t)} + \mathrm{e}^{-\Delta m t} \right) \right]$$

where $Z_j = \langle 0|\overline{\mathcal{H}}(0)|j\rangle = Z_j^*$, and $|0\rangle$ and $|1\rangle$ are the vacuum and ground state respectively. As $m_\pi \to m_\pi^{phys}$ we expect the first excited state to be either $N(0)\pi(0)\pi(0)$ or $N(\vec{p})\pi(-\vec{p})$.

Connected Contribution



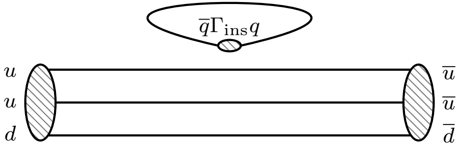
Connected Quark Line Diagram

- Sequential Source Method

- Coherent Source Method [arXiv: 1001.3620]
  - Typically 4 source-sink separations (typically 10 measurements) $0.7\,\text{fm}\ (1), 0.9\,\text{fm}\ (2), 1.0\,\text{fm}\ (3), 1.2\,\text{fm}\ (4)$.

- Wuppertal Smearing for the source and sink
  - Quark rms radius between 0.6 and 0.85fm

Disconnected Contribution



Disconnected Quark Line Diagram

- Stochastic Loop Estimation
  - Truncated Solver Method[arXiv:0910.3970], Hopping Parameter Expansion[arXiv:hep-lat/9707001], Time Partitioning[S. Bernardson 1993]
- Solvers
  - IDFLS [arxiv:0710.5417] or DD-αAMG[arxiv:1303.1377]
- Two-point Functions
  - Typically 20 measurements, lead into the 20 measurements of the disconnected three-point function

Fit Form

- The expected excited state is either N$\pi\pi$ or a Nπ p-wave state, which is difficult to resolve in two-point functions, so we use a ratio

$$R(t_f, t, 0) = \frac{C_{3pt}(t_f, t, 0)}{C_{2pt}(t_f, 0)} = \langle 1|S|1\rangle + A\,\langle 2|S|1\rangle \left(\mathrm{e}^{-\Delta m(t_f - t)} + \mathrm{e}^{-\Delta m t}\right) + \ldots$$

  where A is the fraction of overlap factors. While (normally) small, the resulting excited state contribution can be enhanced by the $\langle 2|S|1\rangle$ matrix element.

- Fitting ratios alone can lead to unstable fits. To better resolve excited states, we fit several matrix elements with different currents simultaneously with the same excited state mass.

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$\mathbb{I}$	g_S^q
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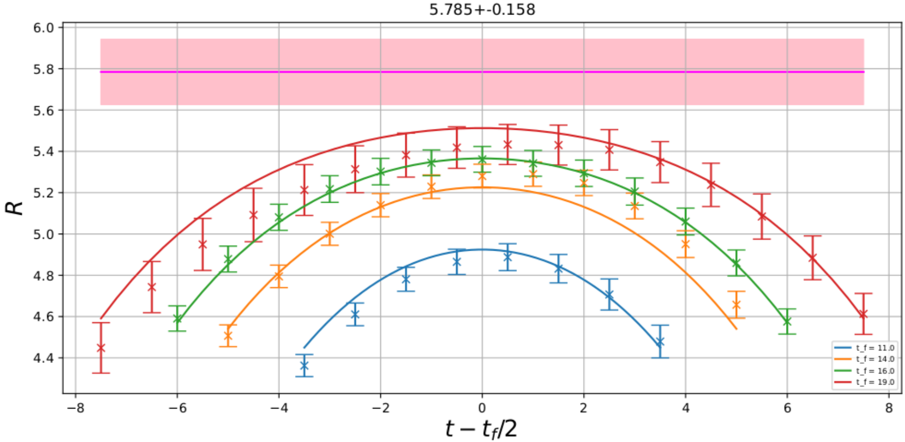
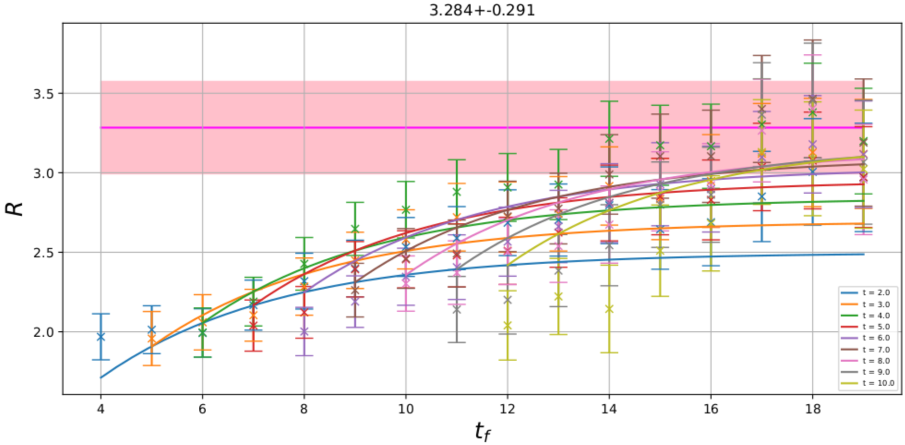
Connected Fit (Preliminary Results)



Connected Data

The fit above is to ensemble N203,

- Volume: $(48^3, 128) \rightarrow (3.08^3, 8.22)$ fm
- m_π: 345 MeV, m_K: 442 MeV
- Beta: 3.55, Lattice spacing: 0.0642 fm



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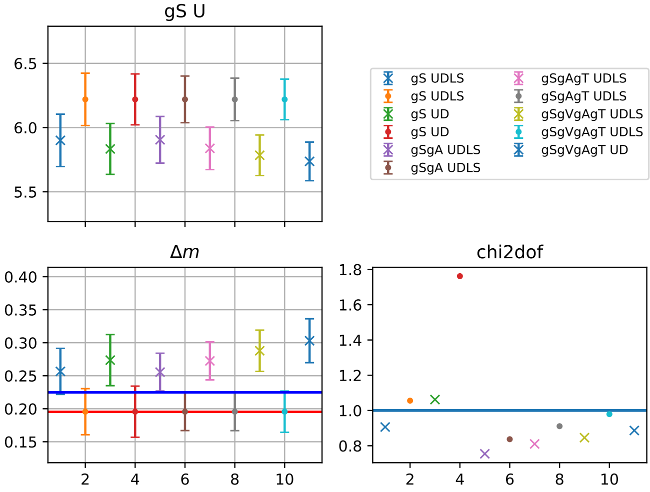
Fit Method Variation (Preliminary Results)



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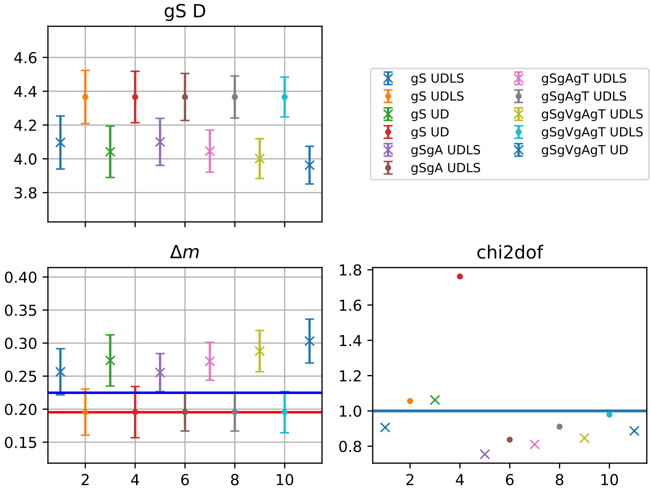

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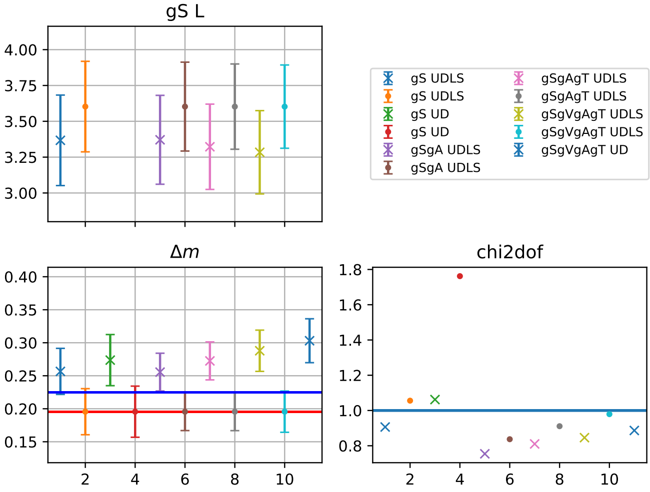
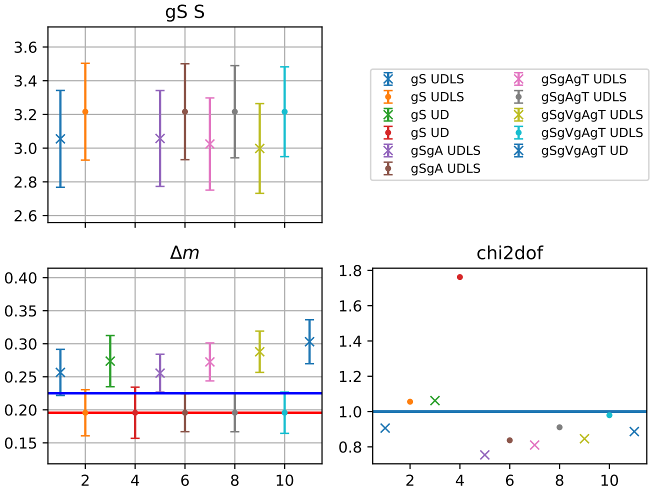


Fit Method Variation (Preliminary Results)



All plots going forward are from the fit which uses all currents, connected and disconnected, and no priors.

Renormalisation & Mixing

Due to chiral symmetry breaking, non-singlet quark mass combinations renormalise with Z_m^{ns} and singlet quark mass combinations with Z_m^s and $Z_m^s/Z_m^{ns} = r_m = 1 + \alpha_z$, which gives mixing of the form[arXiv: 1111.1600]

$$\begin{pmatrix} m_u(\mu) \\ m_d(\mu) \\ m_s(\mu) \end{pmatrix}^{\text{ren}} = Z_m^{ns}(\mu, a) \begin{pmatrix} 1 + \frac{\alpha_z}{3} & \frac{\alpha_z}{3} & \frac{\alpha_z}{3} \\ \frac{\alpha_z}{3} & 1 + \frac{\alpha_z}{3} & \frac{\alpha_z}{3} \\ \frac{\alpha_z}{3} & \frac{\alpha_z}{3} & 1 + \frac{\alpha_z}{3} \end{pmatrix} \begin{pmatrix} m_u \\ m_d \\ m_s \end{pmatrix}^{\text{lat}}$$

Defining $\operatorname{Tr} M = \sum_q m_q$, $\operatorname{Tr} g_S = \sum_q g_{q,S}$, and $\widehat{\mathbb{O}}$ to be the renormalised $\mathbb{O}$ we can write

$$\widehat{m}_q = Z_m \left(m_q + \frac{r_m - 1}{3} \operatorname{Tr} M \right), \qquad \widehat{g}_{q,S} = Z_m^{-1} \left(g_{q,S} + \frac{r_m^{-1} - 1}{3} \operatorname{Tr} g_S \right).$$

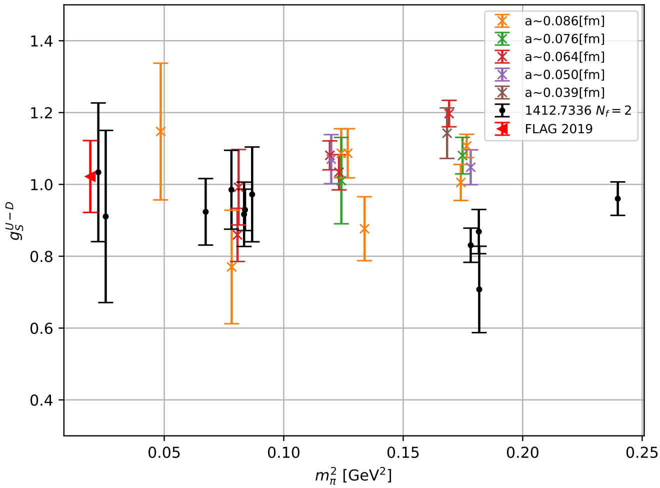
Combining these gives

$$\sigma_q = \left(m_q + \frac{r_m - 1}{3} \operatorname{Tr} M \right) \left(g_{q,S} + \frac{r_m^{-1} - 1}{3} \operatorname{Tr} g_S \right).$$

Combinations of note include the pion-nucleon sigma-term and the singlet combination, with the latter being RG invariant

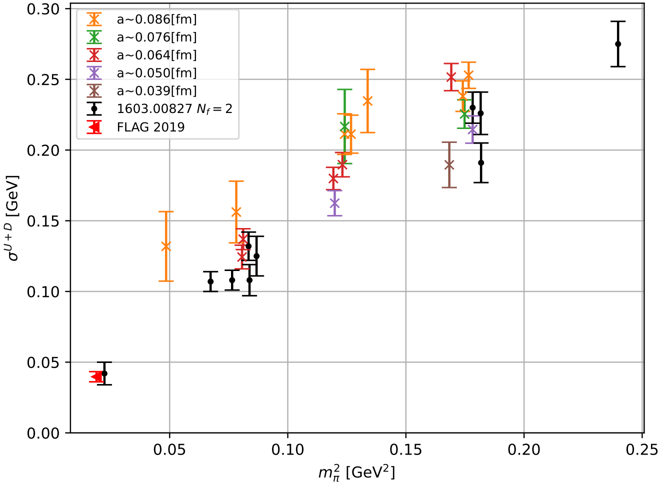
$$\sigma_{N\pi} = \sigma_u + \sigma_d, \qquad \sum_q \sigma_q$$

Isovector Scalar Charge Comparison (Preliminary Results)



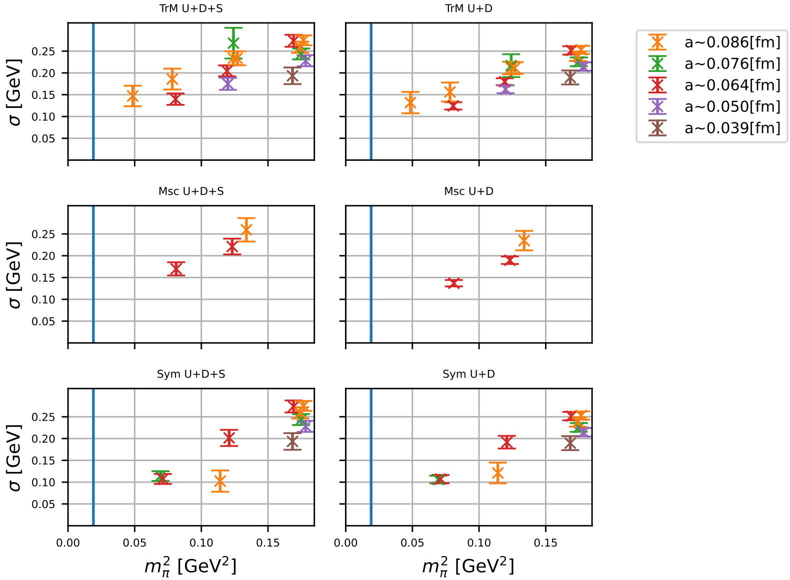
The FLAG point is for $N_f = 2 + 1 + 1$ and is formed from only one result [arXiv:1806.09006]
Renormalisation scale $\mu = 2$GeV, with Z_S taken from [arXiv:2012.06284]

Pion-Nucleon Sigma Term Comparison (Preliminary Results)



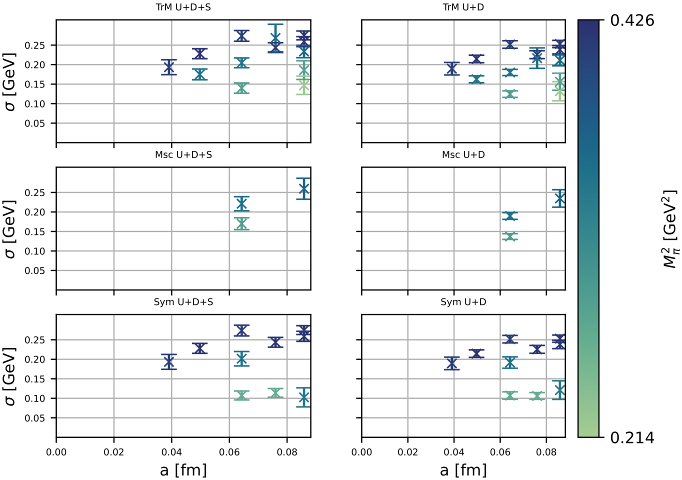
The FLAG point is for $N_f = 2 + 1$ and is formed from [arxiv:1109.4265], [arxiv:1510.08013], and [arxiv:1511.09089]

Sigma Term Overview Plots: M_π (Preliminary Results)



Sigma Term Overview Plots: Lattice Spacing a (Preliminary Results)

While the action is $\mathcal{O}(a)$-improved, the iso-singlet current is subject to $\mathcal{O}(a)$ lattice effects. Their removal would require adding an improvement term $\propto aFF$, whose coefficient is unknown.



Conclusions and Outlook

- Analysis is ongoing, with results being generated on ensembles with a wide range of lattice spacings
- Simultaneous fits to two- and three-point function ratios with different currents seem to be stable
- Compatible results when compared with fits where the excited state is set to $N\pi$ using priors, but there is a systematic difference
- For a similar analysis see the poster: Towards the Determination of Sigma Terms for the Baryon Octet on $N_f = 2 + 1$ Ensembles by Pia Leonie Jones Petrak (Wednesday 8am EST)
- Next steps include:
  - Further exploration of the excited states is needed
  - Push analysis closer to physical pion mass
  - Quark mass, lattice spacing, and volume extrapolations
  - Data is also available for g_A and g_T

Thank you for your attention!