Precision Calculation of the *x*-dependence of PDFs from Lattice QCD

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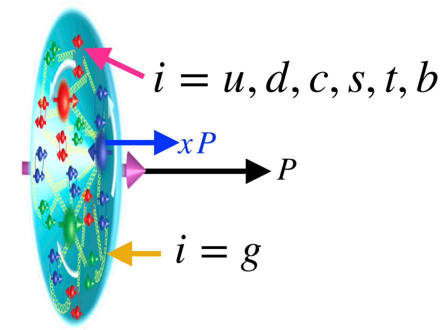
In collaboration with Xiang Gao, Andrew Hanlon, Nikhil Karthik, Swagato Mukherjee, Peter Petreczky, Philipp Scior, Sergey Syritsyn, in preparation.

Large-momentum effective theory (LaMET)

- The quasi-PDF: X. Ji, PRL 110 (2013); SCPMA57 (2014).

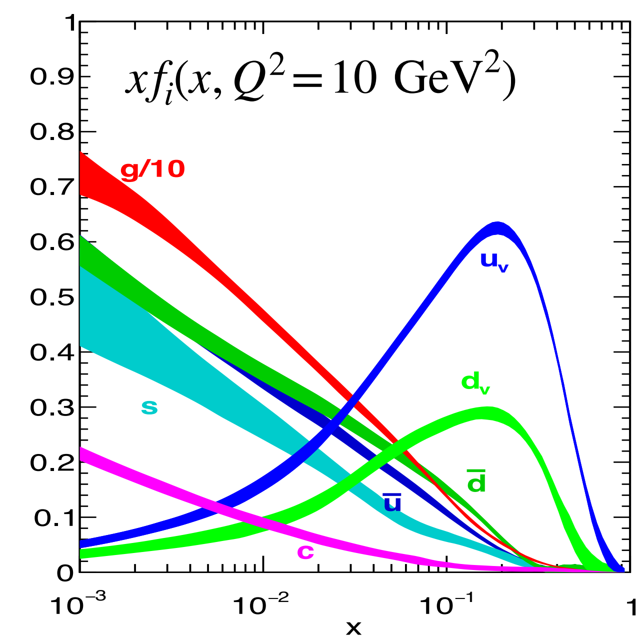
$$\tilde{f}(y, P^z) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{iz(yP^z)} \langle P | \bar{\psi}(z) \Gamma W[z,0] \psi(0) | P \rangle$$



- Direct calculation of x-dependence through large-momentum expansion:

$$f(x, \mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) \tilde{f}(y, P^z, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{(xP^z)^2}, \frac{\Lambda^2_{\rm QCD}}{((1-x)P^z)^2}\right)$$

- X. Xiong, X. Ji, J.-H. Zhang and YZ, PRD 90 (2014);
- Y.-Q. Ma and J. Qiu, PRD98 (2018), PRL 120 (2018);
- T. Izubuchi, X. Ji, L. Jin, I. Stewart, and YZ, PRD98 (2018).



NNPDF Collaboration, EPJ C77 (2017)

Precision calculation = controlled systematics.

- Lattice: excited states, $a \to 0$, physical m_π, $L \to \infty$, etc.;
- Perturbative matching (currently at NNLO) and resummation;
- Power corrections, controllable within $[x_{min}, x_{max}]$.

Renormalization and factorization

- Coordinate space:
- Momentum space/x-space:

$\overline{\text{MS}}$ matrix elements —FT→ $\overline{\text{MS}}$ quasi-PDF:

No IR logs of z or higher-twist effects

Perturbative conversion

Hybrid scheme

No extra higher-twist effects

IR logs of z and higher-twist effects

Nonperturbative conversion

- Ratio schemes
- RI/MOM

Extra higher-twist effects

Rigorously proven factorization

$\overline{\text{MS}}$ PDF:

Hybrid scheme ✔

- Ratio schemes
- RI/MOM

?

Other schemes: $Z_X(z, \mu_R, a)$ —FT→ Quasi-PDF in other schemes

Hybrid renormalization scheme

See X. Ji, YZ, et al., NPB 964 (2021) and references therein.

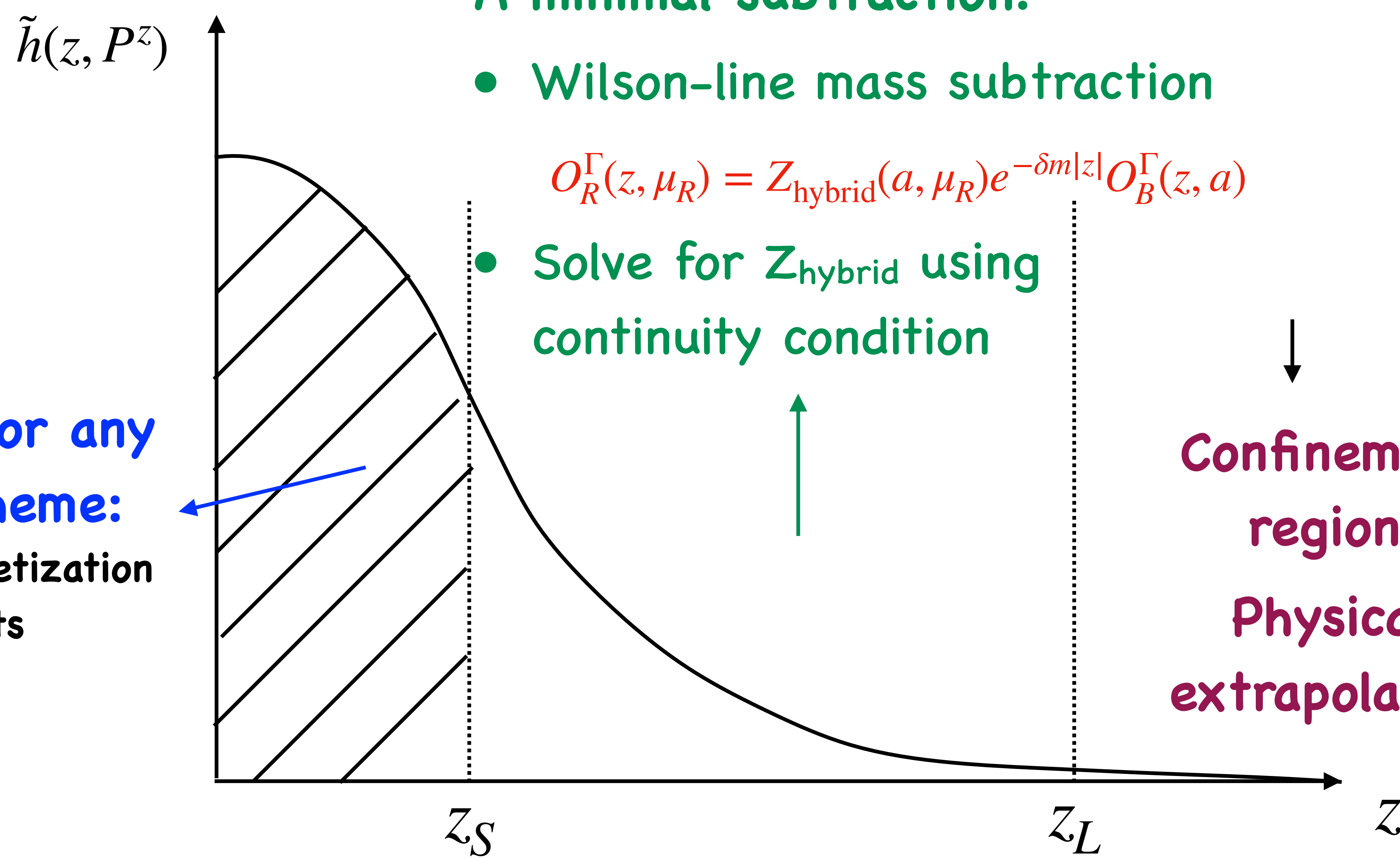
$$O_B^\Gamma(z,a) = e^{-\delta m(a)|z|}\, Z_O(a,\mu) O_R^\Gamma(z,\mu)$$

A minimal subtraction:

- Wilson-line mass subtraction

$$O_R^\Gamma(z,\mu_R) = Z_{\rm hybrid}(a,\mu_R) e^{-\delta m|z|} O_B^\Gamma(z,a)$$

- Solve for $Z_{\rm hybrid}$ using continuity condition



RI/MOM or any ratio scheme:

Cancel discretization effects

Confinement region, Physical extrapolation

$a \ll z_S \ll \Lambda_{\rm QCD}^{-1}$

$z_L \sim \Lambda_{\rm QCD}^{-1}$

Lattice calculation

- Wilson-clover fermion on 2+1 flavor HISQ configurations.

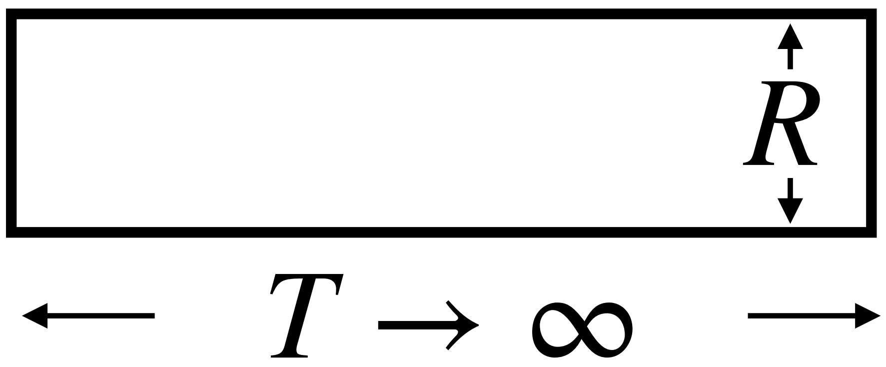
n_z	P_z (GeV)		ζ
	$a = 0.06$ fm	$a = 0.04$ fm	
0	0	0	0
1	0.43	0.48	0
2	0.86	0.97	1
3	1.29	1.45	2/3
4	1.72	1.93	3/4
5	2.15	2.42	3/5
	$48^3 \times 64$	$64^3 \times 64$	

$m_\pi = 300$ MeV

- X. Gao, YZ, et al., PRD102 (2020).
- X. Gao, YZ, et al., 2102.01101.

Wilson-line mass renormalization

- Polyakov loop

$\langle \Omega |$  $| \Omega \rangle \propto \exp[-V(R)T]$

- Renormalization condition:

$$V^{\rm lat}(r,a)\Big|_{r=r_0} + 2\delta m(a) = 0.95/r_0$$

$$\delta m(a) = \frac{1}{a}\sum_n c_n \alpha_s^n(1/a) + \delta m_0^{\rm lat}$$

$$\delta m_0^{\rm lat} \sim \Lambda_{\rm QCD}$$

C. Bauer, G. Bali and A. Pineda, PRL108 (2012).

$$a\delta m(a = 0.04\ \text{fm}) = 0.1508(12)$$

$$a\delta m(a = 0.06\ \text{fm}) = 0.1586(8)$$

$$a\delta m(a = 0.076\ \text{fm}) = 0.1597(16)$$

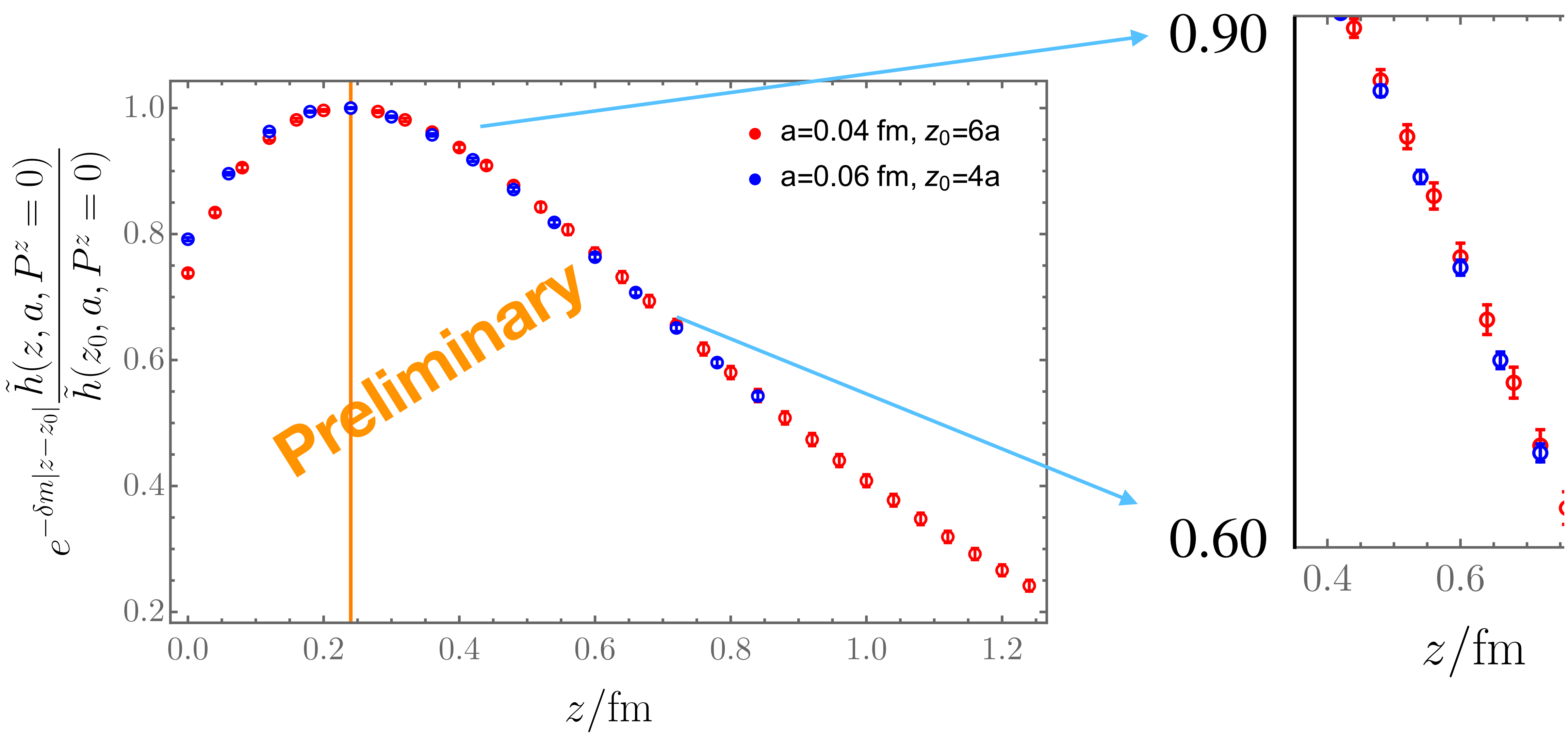
A. Bazavov et al., TUMQCD, PRD98 (2018).

Wilson-line mass renormalization

- Check of continuum limit:

$$O_B^\Gamma(z,a) = e^{-\delta m|z|}\, Z_O(a) O_R^\Gamma(z)$$

$$\lim_{a\to 0} e^{-\delta m(z-z_0)} \frac{\tilde{h}(z,a,P^z=0)}{\tilde{h}(z_0,a,P^z=0)} = \frac{\tilde{h}(z,P^z=0,\ \mu)}{\tilde{h}(z_0,P^z=0,\ \mu)} \qquad z, z_0 \gg a$$

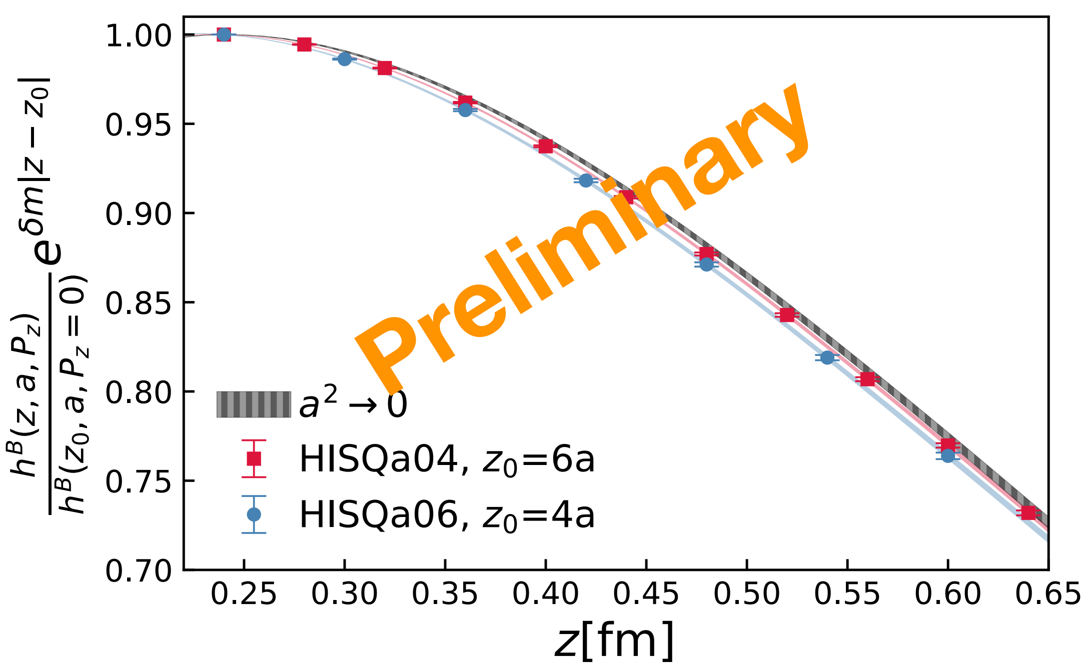


Matching to the MSbar scheme

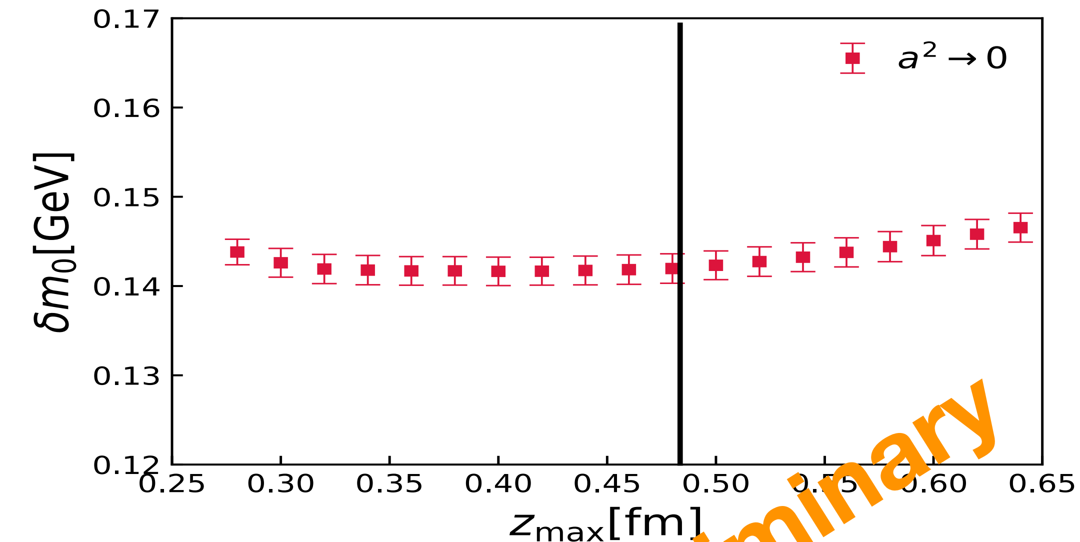
$$a \ll z,\ z_0 \ll \Lambda_{\rm QCD}^{-1}$$

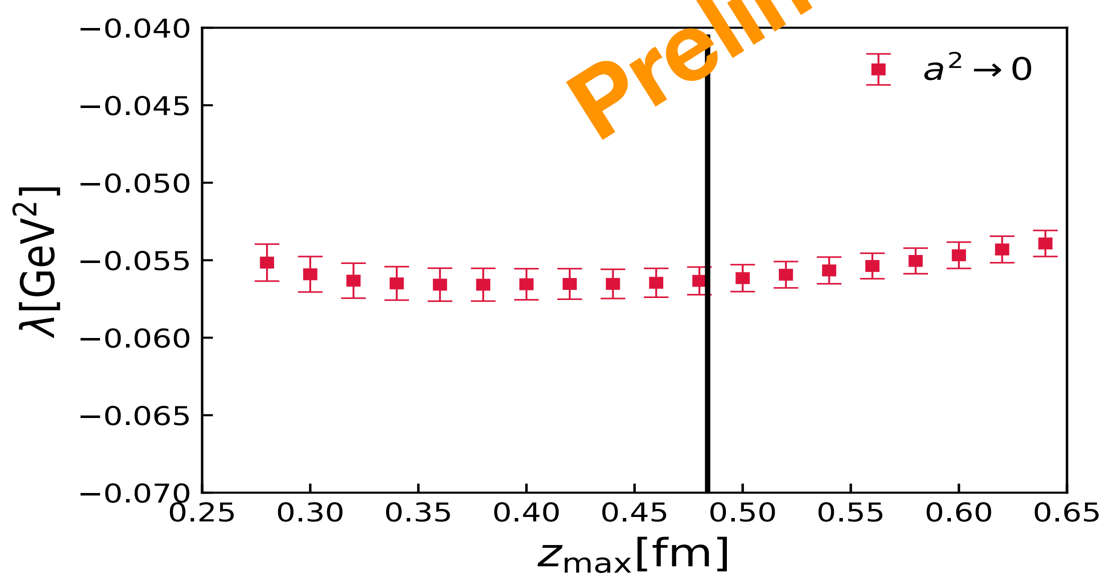
$$e^{-m_0(z-z_0)} \lim_{a\to 0} e^{\delta m(a)(z-z_0)} \frac{\tilde{Q}(z,a,P^z=0)}{\tilde{Q}(z_0,a,P^z=0)} = e^{-m_0(z-z_0)} \frac{C_0(\alpha_s(\mu), z^2\mu^2) + \lambda z^2}{C_0(\alpha_s(\mu), z_0^2\mu^2) + \lambda z_0^2}$$

Lattice data



Continuum extrapolation



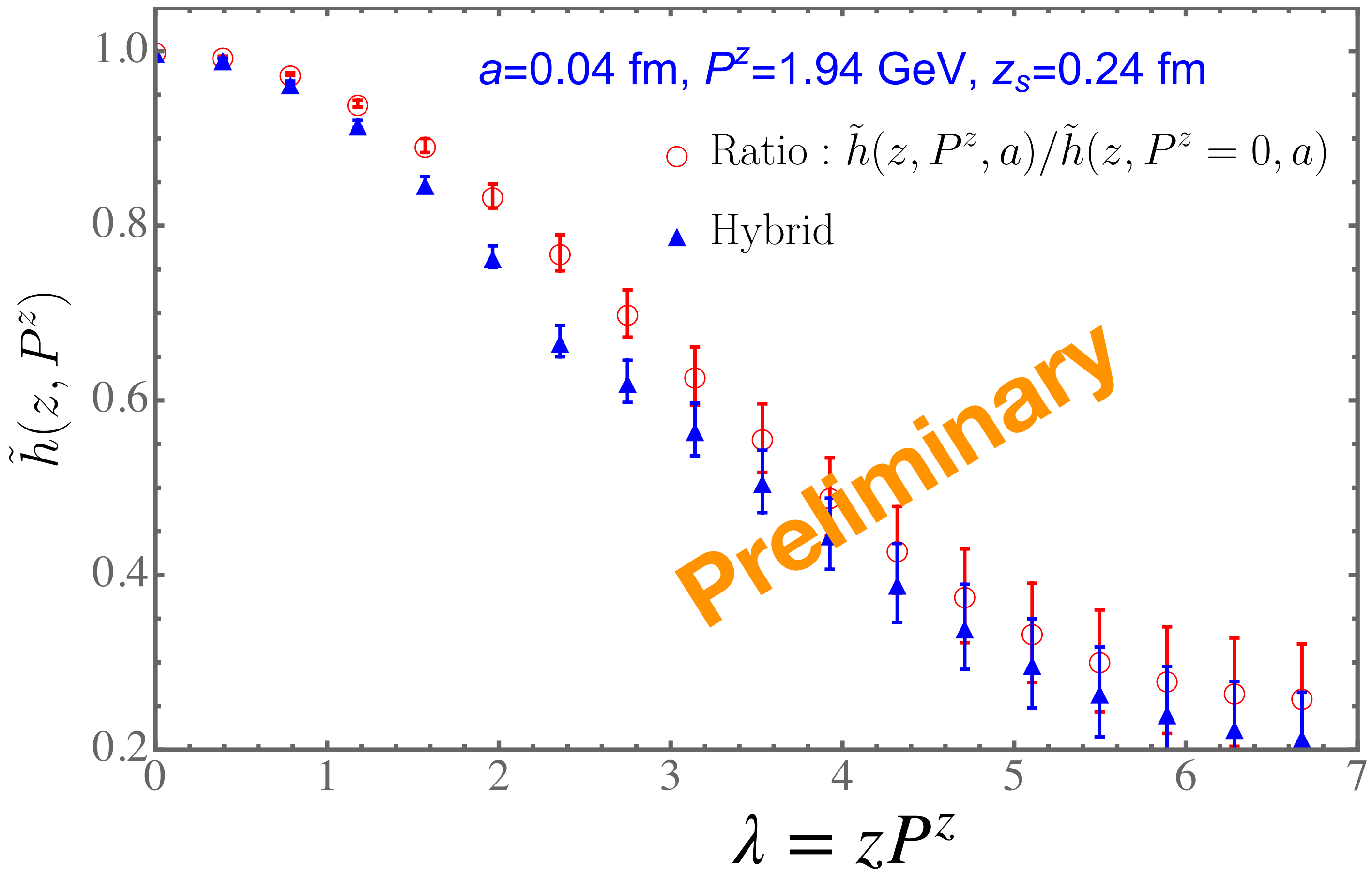


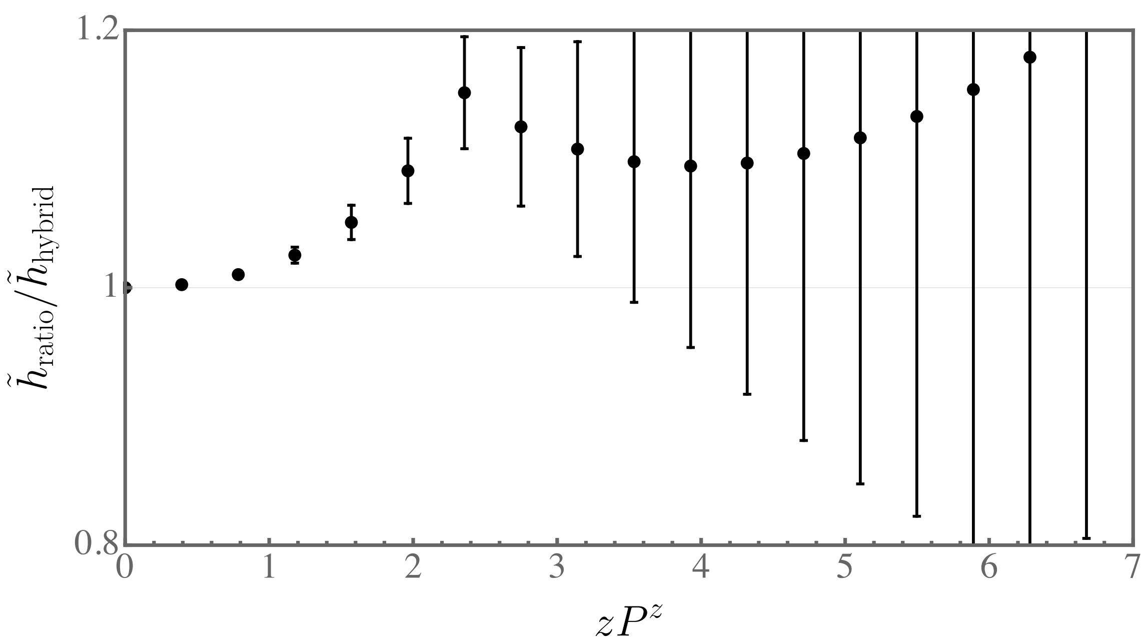
Renormalized matrix element

- Power-correction corrected ratio at short distance:

$$\lim_{a\to 0}\frac{\tilde{h}(z,a,P^z)}{\tilde{h}(z,a,P^z=0)}\frac{C_0(\alpha_s(\mu),z^2\mu^2)+\lambda z^2}{C_0(\alpha_s(\mu),z^2\mu^2)}=\frac{\tilde{h}(z,P^z,\mu)}{C_0(\alpha_s(\mu),z^2\mu^2)} \qquad 0\le z\le z_S$$

- Hybrid scheme matrix element:

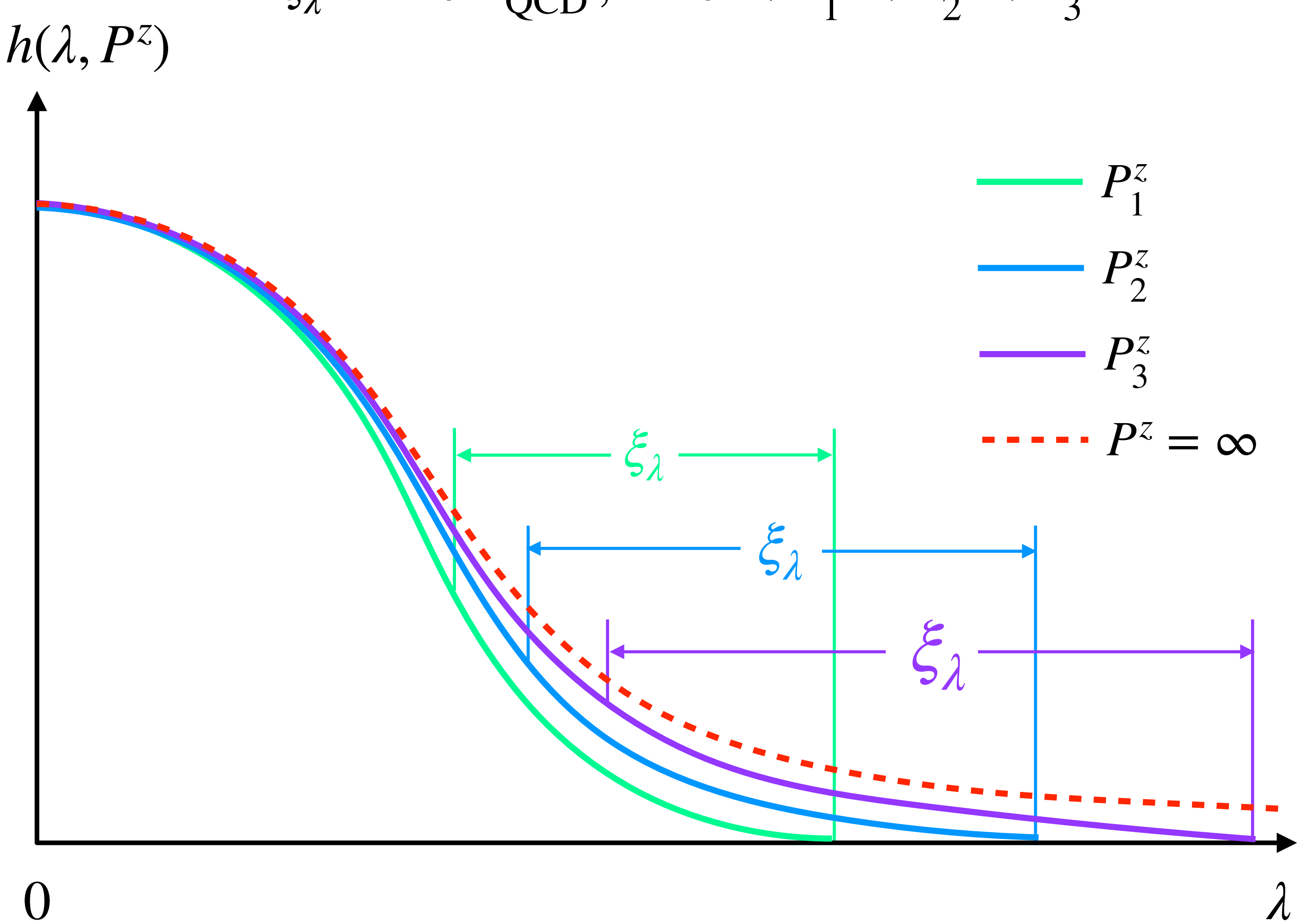




Physical extrapolation beyond z_L

- When z is larger than the hadron size, $\tilde{h}(z, P^z) \propto e^{-z/\xi_z}$ with $\xi_z \sim 1/\Lambda_{\rm QCD}$; in $\lambda = zP^z$ space, the correlation length $\xi_\lambda = P^z\xi_z \sim P^z/\Lambda_{\rm QCD}$;
- As $P^z \to \infty$, $\xi_\lambda \to \infty$, only the twist-2 contribution survives, $\tilde{h}(\lambda, P^z) \sim 1/\lambda^d$, which is determined by the small-x Regge behavior of the PDF.

$$\xi_\lambda \sim P^z/\Lambda_{\rm QCD}\,, \qquad 0 < P_1^z < P_2^z < P_3^z$$



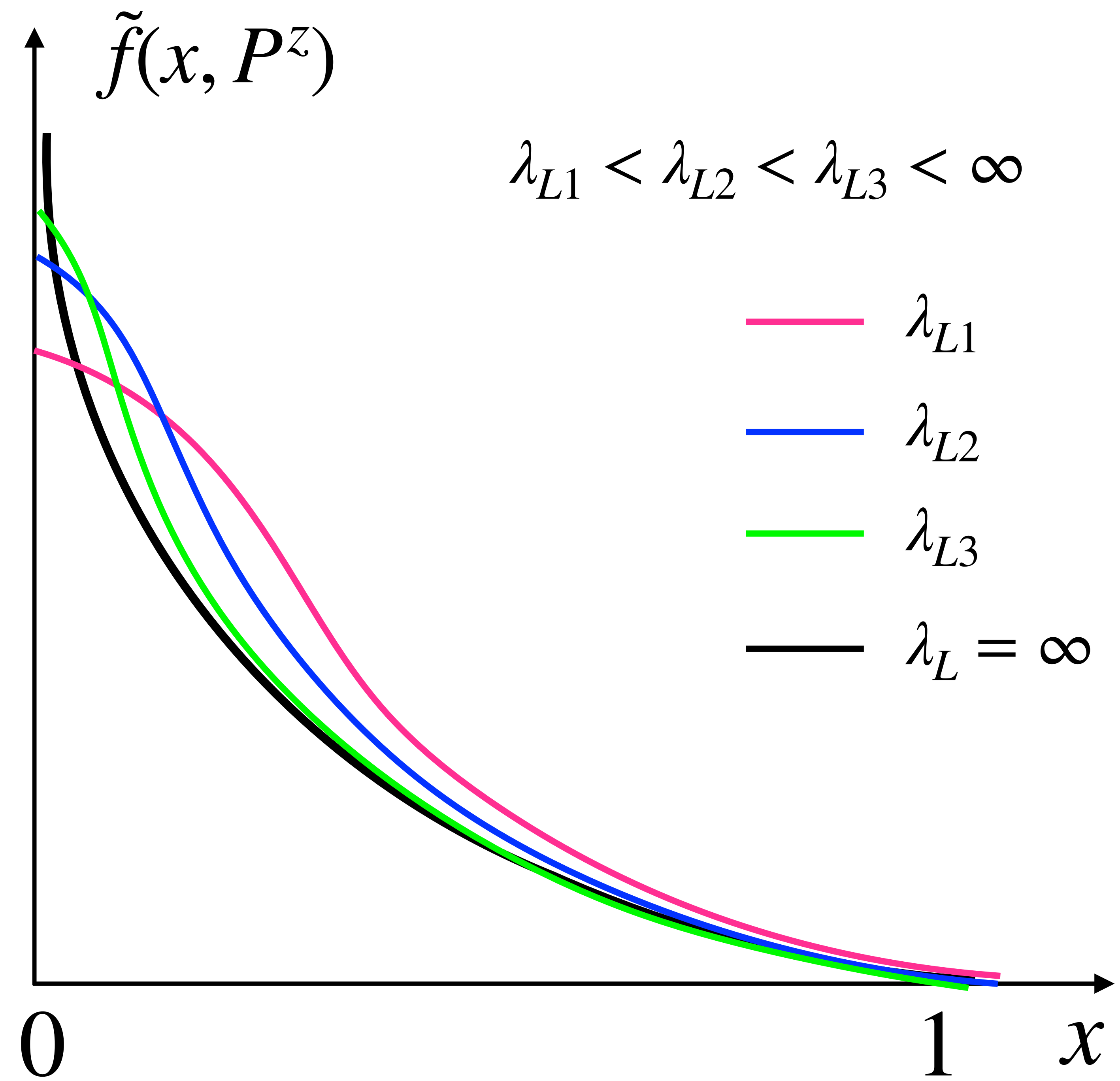
Therefore, if

- P^z **is not very large, e.g. 2-5 GeV, use an exponential form;**
- P^z **is very large, use an algebraic form.**

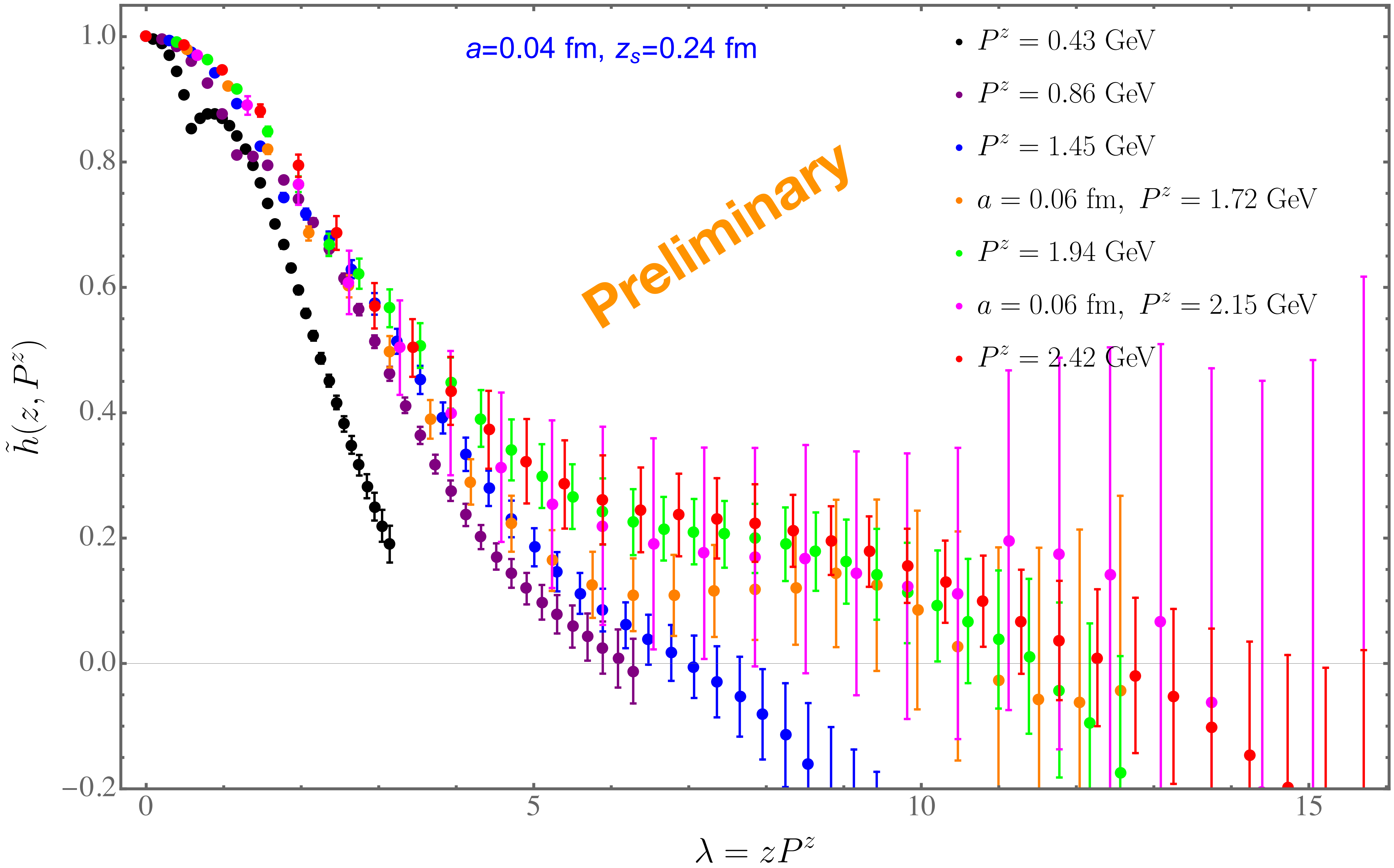
Physical extrapolation beyond z_L

Impact of exponential extrapolation:

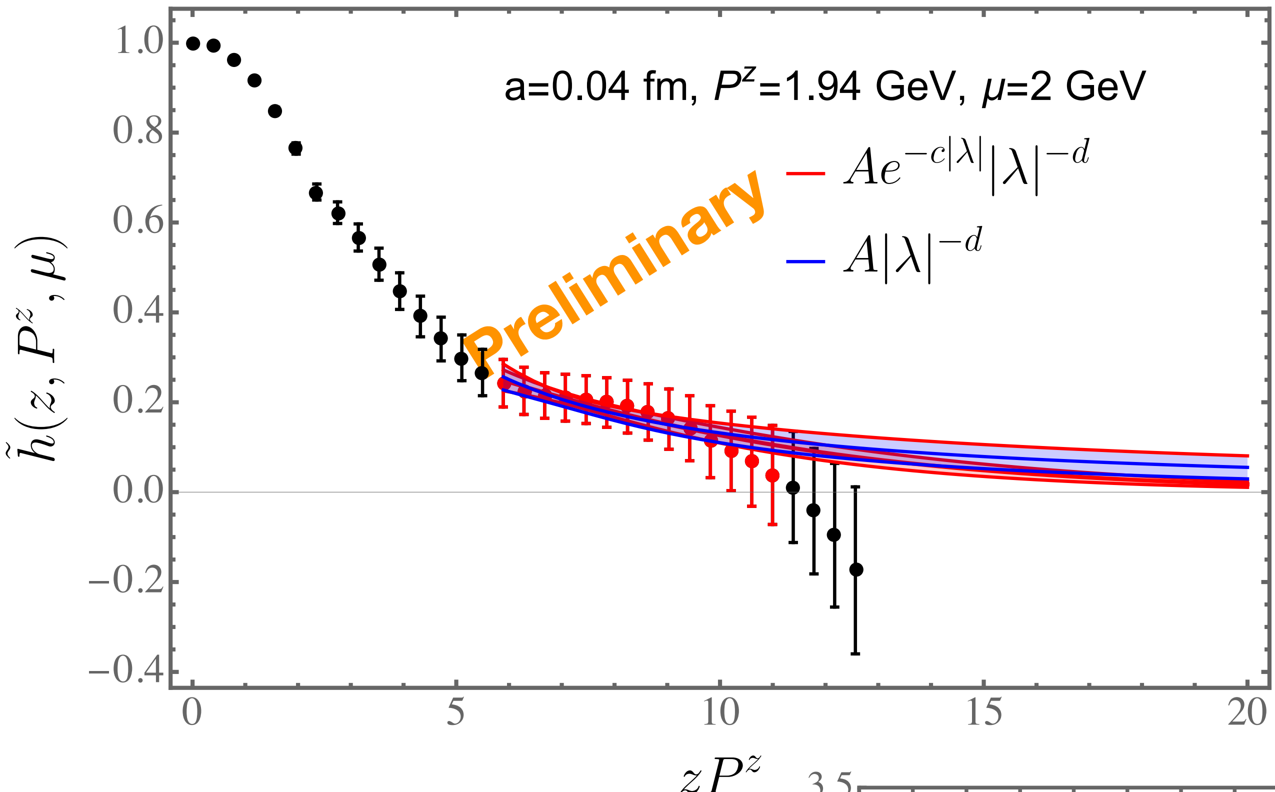
- Remove unphysical oscillation from Fourier transform;
- Affects the $x \to 0$ and $x \to 1$ regions most, but not [x_{min}, x_{max}] where LaMET can have controlled prediction.

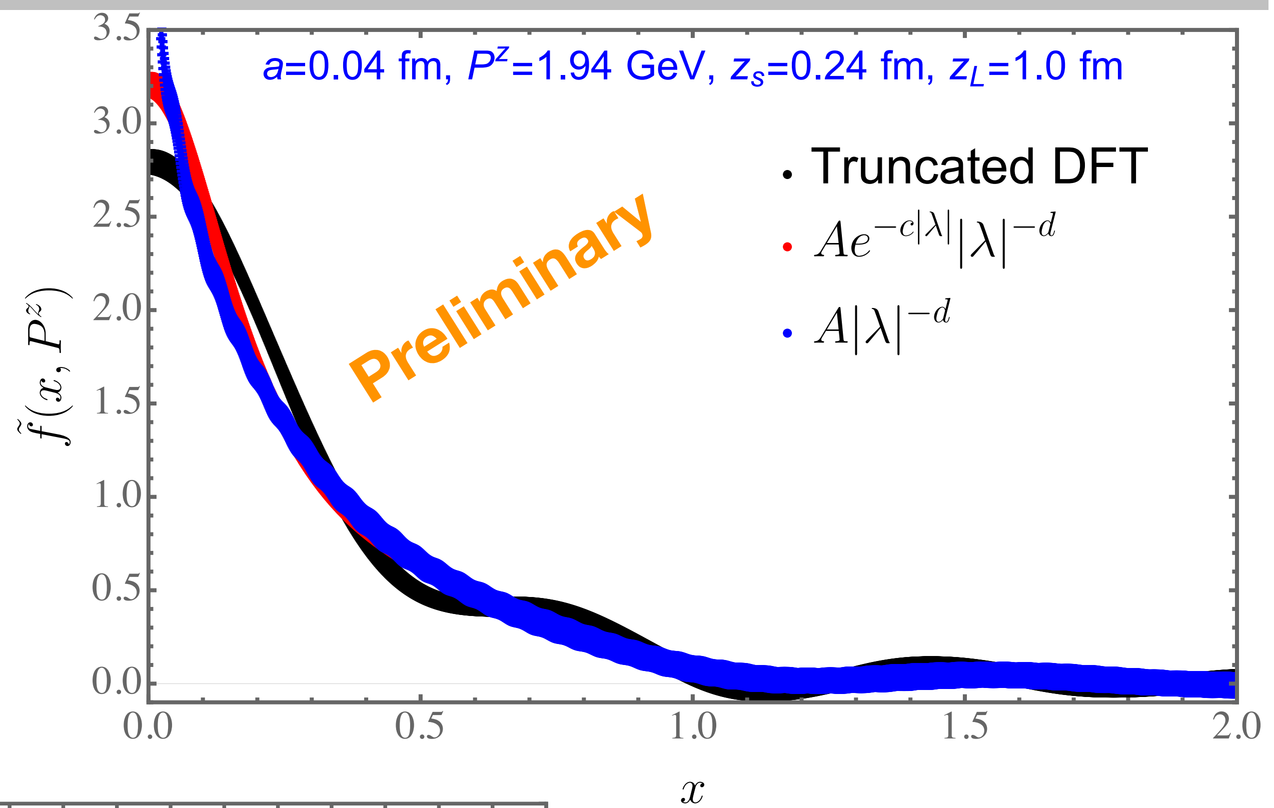


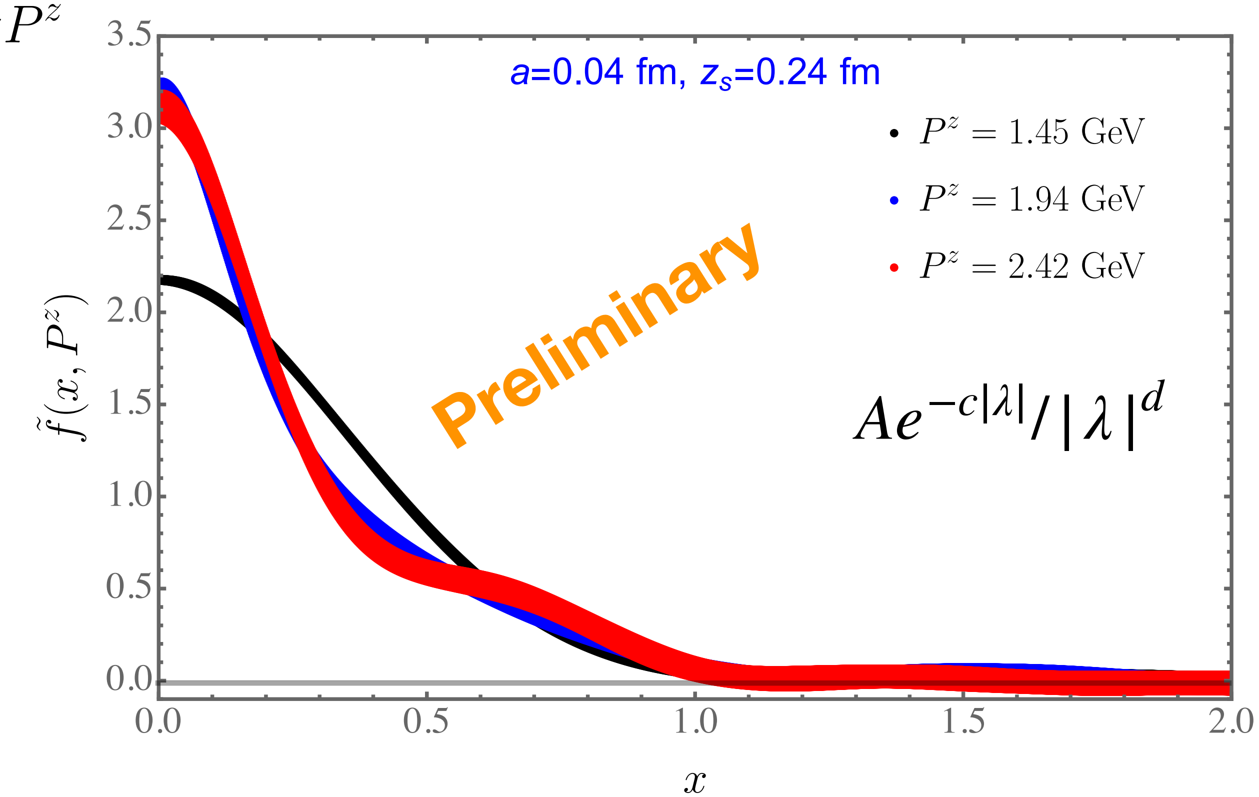
Renormalized matrix elements



Physical extrapolation and Fourier transform

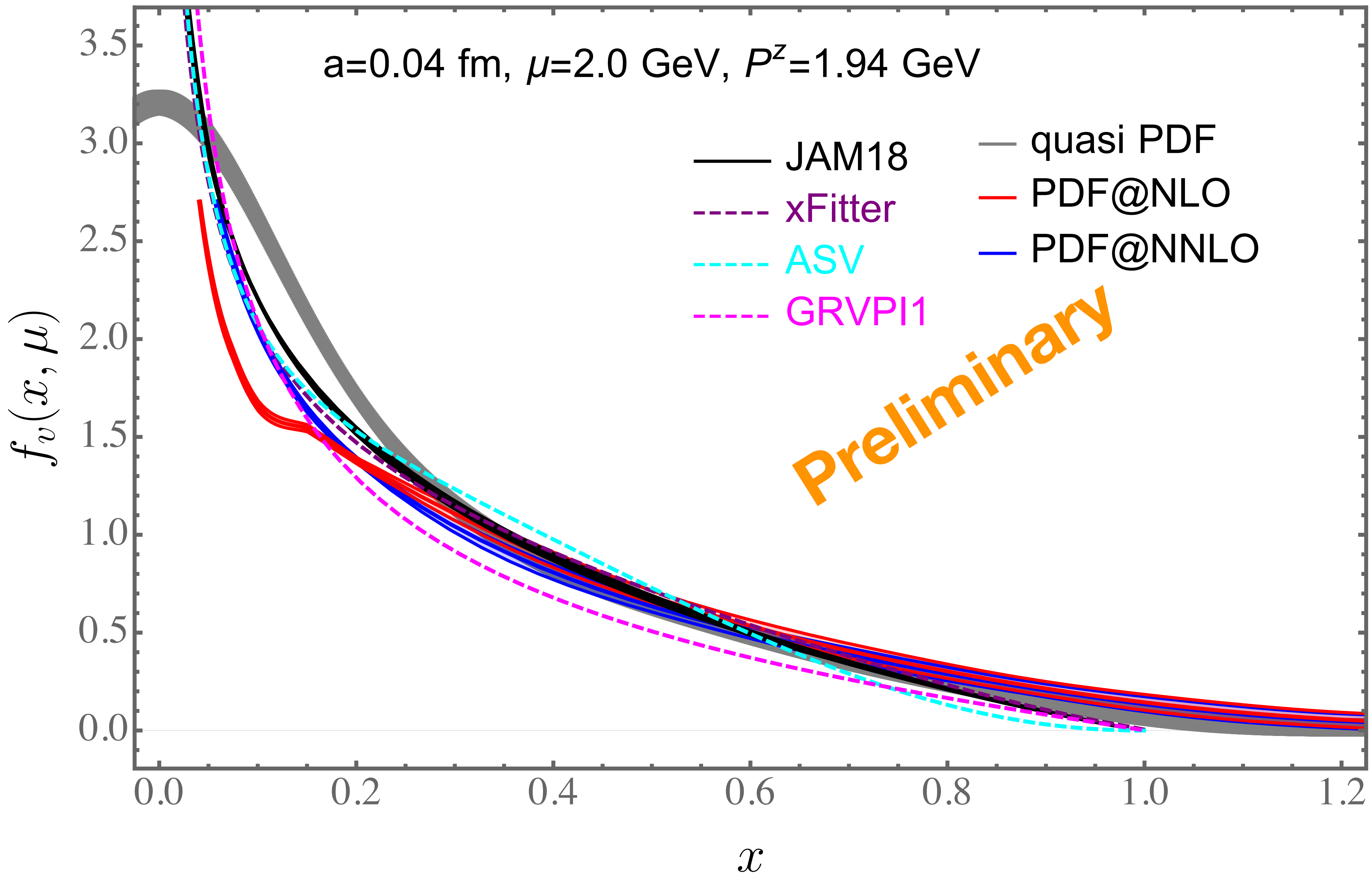






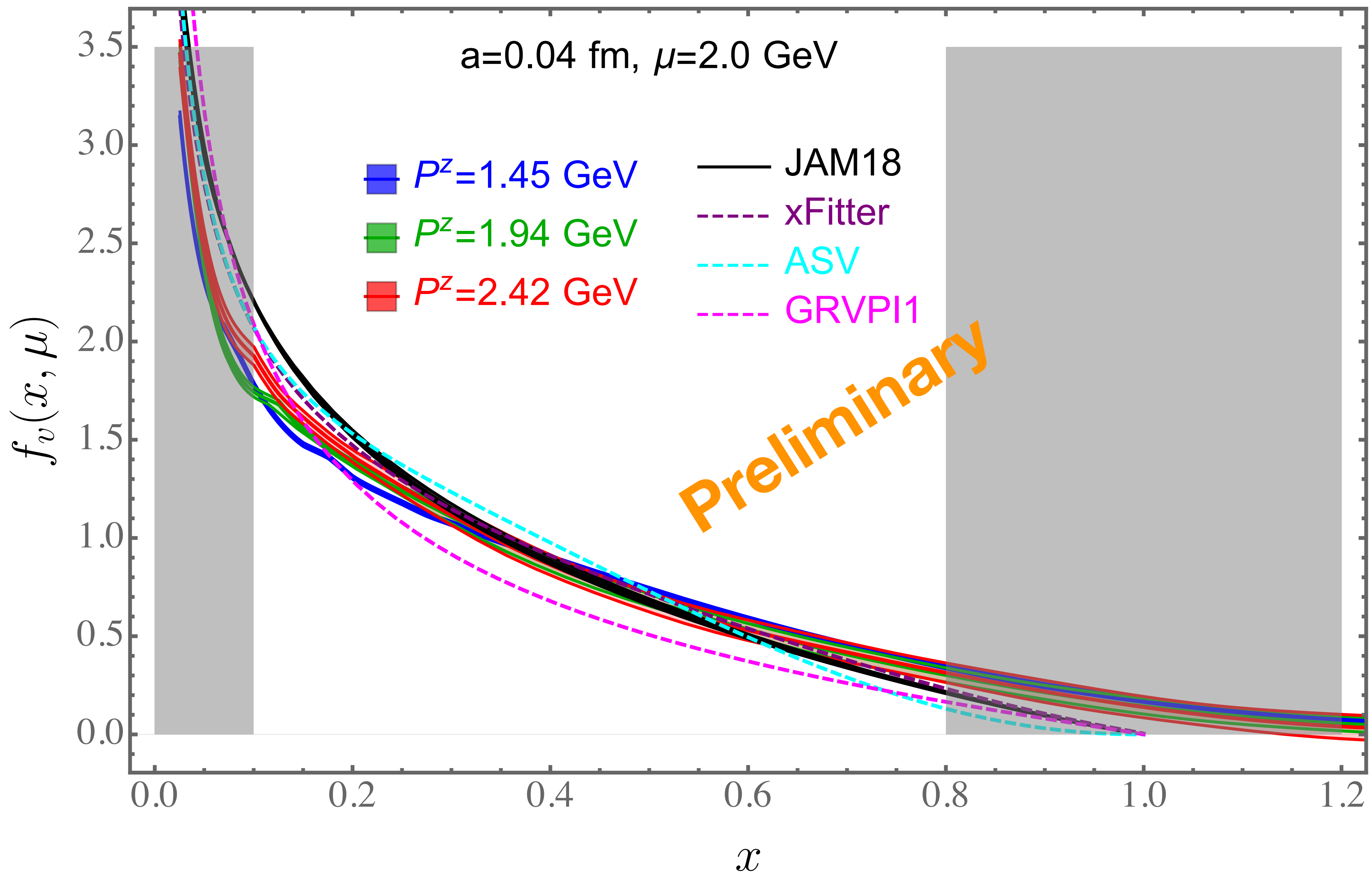
Perturbative matching at NNLO

- Perturbative correction shows good convergence.



Error band assumes 100% correlation of points at all x during the matching.

Comparison with phenomenology



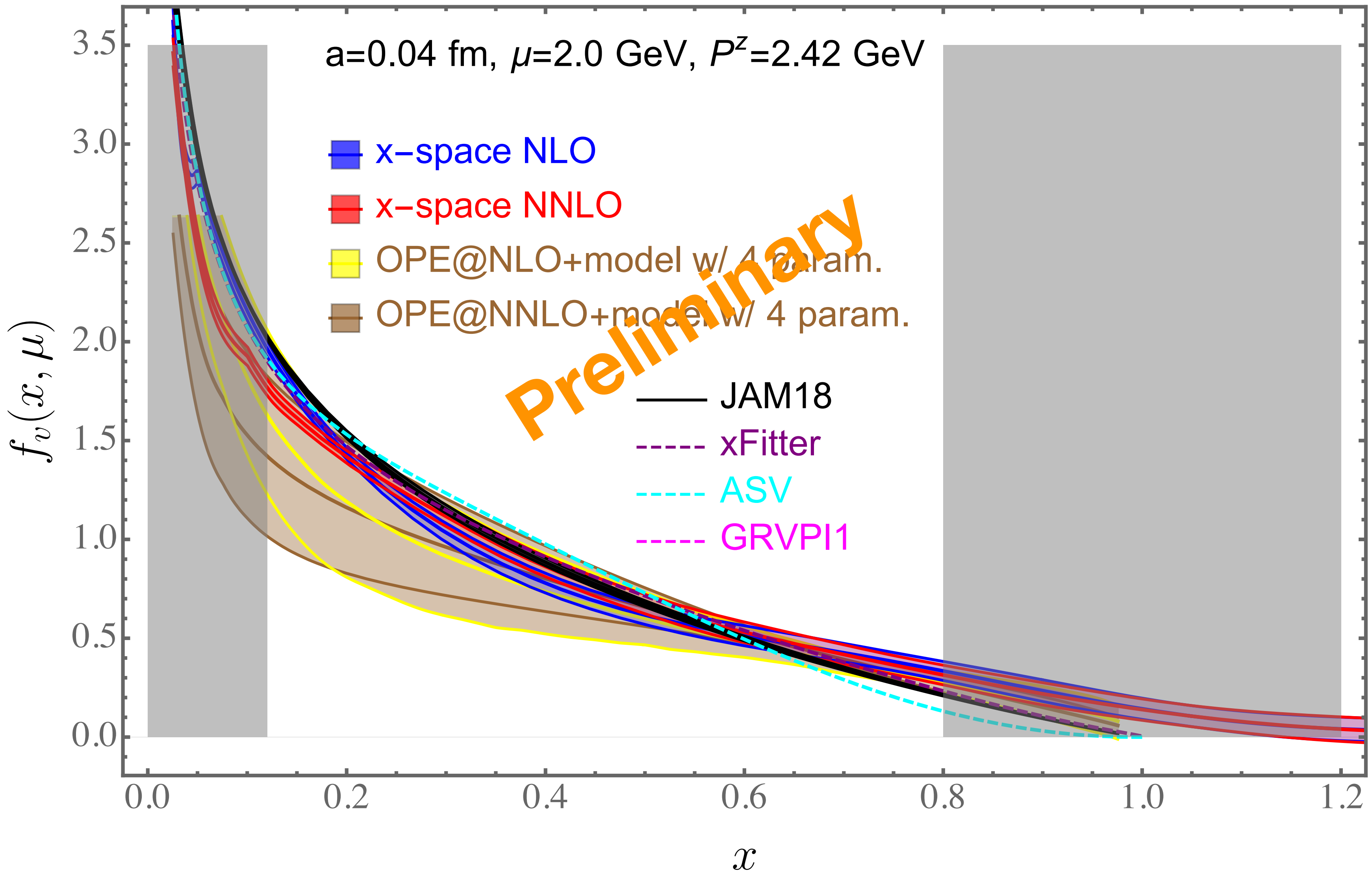
Gray band: x<0.12 or x>0.8. (A very preliminary estimate)

Error at P^z=2.42 GeV > 15% **or** variation in P^z > 1% between P^z=1.94 and 2.42 GeV.

Comparison with ratio scheme analysis in coordinate space

Ratio scheme with nonzero-momentum matrix elements in the dominator.

X. Gao, YZ, et al., PRD102 (2020).



Systematics to be included in x-space analysis: physical pion mass, lattice spacing dependence, variation of factorization scale (α_s), variation of z_L, etc.

Conclusion

- We have carried out lattice calculation of the x-dependence of pion valence PDF with the hybrid renormalization scheme;
- The Wilson-line mass correction can be well determined from lattice and matched to the MSbar scheme by using the NNLO OPE formula with quadratic power correction;
- NNLO matching shows good perturbative convergence;
- We demonstrate that we can predict the x-dependence with controlled systematic uncertainties within a subregion of x;
- Systematics to be analyzed: physical pion mass, lattice spacing dependence, variation of factorization scale (α_s), variation of z_L, etc.