LATTICE 21
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Massachusetts Institute of Technology

Investigating $N \to N\pi$ matrix elements

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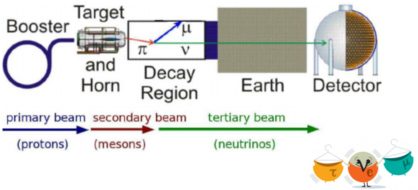
RQCD

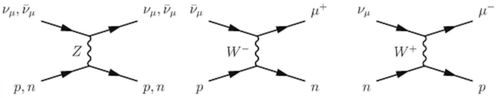
EuroPLEx

H2020

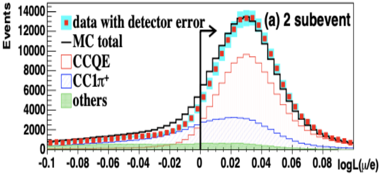
Motivation: neutrino oscillation experiments



[MiniBooNE 2010, 2013, 2018, 2020]



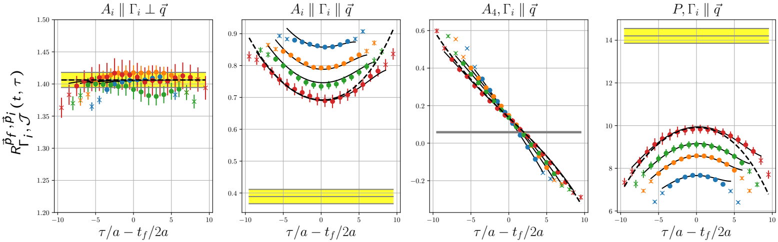
neutrino process	abbreviation	reaction	fraction (%)
CC quasielastic	CCQE	$\nu_\mu + n \to \mu^- + p$	39
NC elastic	NCE	$\nu_\mu + p(n) \to \nu_\mu + p(n)$	16
CC $1\pi^+$ production	$\mathrm{CC}1\pi^+$	$\nu_\mu + p(n) \to \mu^- + \pi^+ + p(n)$	25
CC $1\pi^0$ production	$\mathrm{CC}1\pi^0$	$\nu_\mu + n \to \mu^- + \pi^0 + p$	4
NC $1\pi^\pm$ production	$\mathrm{NC}1\pi^\pm$	$\nu_\mu + p(n) \to \nu_\mu + \pi^+(\pi^-) + n(p)$	4
NC $1\pi^0$ production	$\mathrm{NC}1\pi^0$	$\nu_\mu + p(n) \to \nu_\mu + \pi^0 + p(n)$	8
multi pion production, DIS, etc.	other	$\nu_\mu + p(n) \to \mu^- + N\pi^\pm + X$, etc.	4



Takeaway message: π production had to be taken into account!

[E. Hernandez et al. 2007] - "Weak pion production off the Nucleon"

Motivation: Evidence of $N\pi$ contamination on the lattice



$R_{A_i \parallel \Gamma_i \perp \vec{q}} \propto G_A(Q^2)$ $\quad R_{A_i \parallel \Gamma_i \parallel \vec{q}} \propto (m_N + E_{\vec{q}}) G_A(Q^2) - \frac{q_i}{2m_N} G_{\tilde{P}}(Q^2)$

[G. Bali et al.; arXiv:1911.13150]

Careful analysis with EFT ansatz helps us take into account unwanted contamination.

[O. Bär; arXiv:1812.09191, ...]

$$R^{\vec{p}_f,\vec{p}_i}_{\Gamma_i,\mathcal{J}}(t,\tau) \equiv \frac{C^{\vec{p}_i,\vec{p}_f}_{3pt}(t,\tau;\mathcal{J};\Gamma_i)}{C^{\vec{p}_f}_{2pt}(t)} \sqrt{\frac{C^{\vec{p}_f}_{2pt}(\tau)\, C^{\vec{p}_f}_{2pt}(t)\, C^{\vec{p}_i}_{2pt}(t-\tau)}{C^{\vec{p}_i}_{2pt}(\tau)\, C^{\vec{p}_i}_{2pt}(t)\, C^{\vec{p}_f}_{2pt}(t-\tau)}}$$

$$\xrightarrow{t \gg \tau \gg 0} \sqrt{\frac{E_{\vec{p}_f} E_{\vec{p}_i}}{(E_{\vec{p}_f} + m_N)(E_{\vec{p}_i} + m_N)}}\, B^{\vec{p}_f \vec{p}_i}_{\Gamma_i,\mathcal{J}} \xrightarrow[\mathcal{J}=\mathcal{A}_i,\Gamma_i]{\vec{p}_f=\vec{p}_i=\vec{0}} G_A(Q^2=0) \equiv g_A \quad \text{"axial charge"}$$

$$B^{\vec{p}_f \vec{p}_i}_{\Gamma_i,\mathcal{J}} = Tr\left[\, \Gamma_i\, (\not{p}_f + m_N) FF[\mathcal{J}] (\not{p}_i + m_N)\, \right] \qquad \bar{u}_N(\mathbf{p}_f) FF[\mathcal{J}] u_N(\mathbf{p}_i) \equiv \langle N' | \mathcal{J} | N \rangle$$

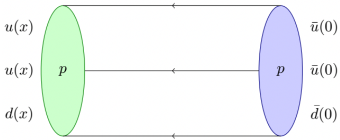
Nucleon Form Factor determination on the lattice

Choosing the appropriate interpolating operators:

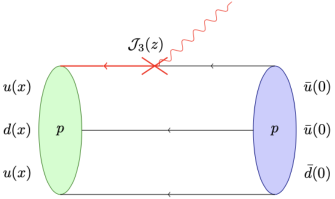
$$\bar{\mathcal{O}}_{P_{\bar{\gamma}}}(\vec{0},0) = \epsilon^{\bar{a}\bar{b}\bar{c}}\bar{u}^{\bar{c}}_{\bar{\gamma}}(0)\Big[\ \bar{u}^{\bar{b}}_{\bar{\beta}}(0)(C\gamma_5)^{\bar{\beta}\bar{\alpha}}\bar{d}^{\bar{a}}_{\bar{\alpha}}(0)\ \Big]$$

$$\mathcal{O}_{P_{\gamma}}(\vec{x},t) = \epsilon^{abc}\Big[\ d^a_\alpha(x)\ (C\gamma_5)_{\alpha\beta}\ u^b_\beta(x)\ \Big]\ u^c_\gamma(x)$$

$$\mathcal{J}_3(\mathbf{z},\tau) = \bar{u}^e_{\bar{\epsilon}}(z)\ \Gamma^{\bar{\epsilon}\epsilon}u^e_\epsilon(z) - \bar{d}^e_{\bar{\epsilon}}(z)\ \Gamma^{\bar{\epsilon}\epsilon}\ d^e_\epsilon(z),$$



C_{2pt}



C_{3pt}

$$\Gamma_\pm = \frac{1}{2}(1 \pm \gamma_4)$$

$$C^{\vec{p}_f}_{2pt}(t) = \sum_{\vec{x}} e^{i\vec{p}_f\cdot\vec{x}}\ \Gamma^{\gamma\bar{\gamma}}_{+}\langle\ \mathcal{O}_{P_\gamma}(\vec{x},t)\ \bar{\mathcal{O}}_{P_{\bar{\gamma}}}(\vec{0},0)\ \rangle$$

$$\Gamma_{i,\pm} = \Gamma_\pm i\gamma_i\gamma_5$$

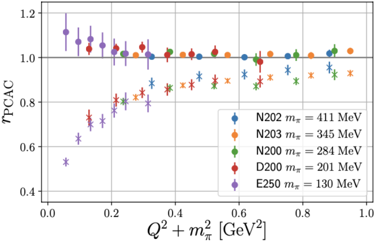
$$C^{\vec{p}_i,\vec{p}_f}_{3pt}(t,\tau) = \sum_{\vec{z},\ \vec{x}} e^{i\vec{q}\cdot\vec{z}}\ e^{-i\vec{p}_f\cdot\vec{x}}\ \Gamma^{\gamma\bar{\gamma}}_{i,+}\langle\ \mathcal{O}_{P_\gamma}(\vec{x},t)\ \mathcal{J}_3(\vec{z},\tau)\ \bar{\mathcal{O}}_{P_{\bar{\gamma}}}(\vec{0},0)\ \rangle$$

Lattice results

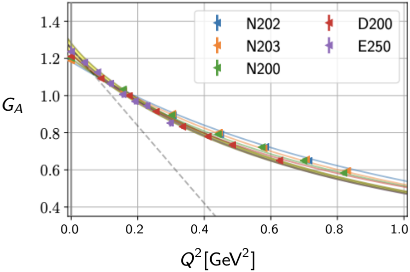
$G_A(Q^2),\ G_P(Q^2),\ \tilde{G}_P(Q^2) \quad \rightarrow \quad$ check PCAC: $\partial^\mu \mathcal{A}_\mu = 2im_l\ \mathcal{P}$

$$\frac{m_l}{M_N} G_P(Q^2) = G_A(Q^2) - \frac{Q^2}{4M_N^2} G_{\tilde{P}}(Q^2) + \mathcal{O}(a^2)$$

$$r_{PCAC} = \frac{\frac{m_l}{M_N} G_P(Q^2) + \frac{Q^2}{4M_N^2} G_{\tilde{P}}(Q^2)}{G_A(Q^2)} \quad \Longrightarrow$$



[G. Bali et al.; arXiv:1911.13150]



$$G_A(Q^2) = \frac{g_A}{(1+Q^2/M_A^2)^2} \qquad M_A,\ g_A,\ < r_A^2 >$$

$$G_A(Q^2) = \sum_n^N a_n\ z^n(Q^2)$$

$$z(Q^2) = \frac{\sqrt{t_{cut}+Q^2} - \sqrt{t_{cut}-t_0}}{\sqrt{t_{cut}+Q^2} + \sqrt{t_{cut}-t_0}}$$

z-expansion

Multiparticle interpolator approach

$$C_{3pt}^{\Gamma_i,\mathcal{J}}(t,\tau|\,\mathbf{p}',\mathbf{p}) = \sum_{\mathbf{x},\mathbf{z}} e^{-i\mathbf{p}'\cdot\mathbf{x}}\; e^{+i\mathbf{q}\cdot\mathbf{z}}\; \Gamma_i^{\alpha\beta} \langle\, \mathcal{O}_P^{\beta}(\mathbf{x},t)\; \mathcal{J}^{\mu}(\mathbf{z},\tau)\; \bar{\mathcal{O}}_P^{\alpha}(\mathbf{0},0)\, \rangle$$

$$\approx \sum_{\sigma,\sigma'} \Gamma_i^{\alpha\beta} \langle 0|\,\mathcal{O}_P^{\beta}\,|P_{\mathbf{p}'}^{\sigma'}\rangle \boxed{\langle P_{\mathbf{p}'}^{\sigma'}|\,\mathcal{J}^{\mu}\,|P_{\mathbf{p}}^{\sigma}\rangle} \langle P_{\mathbf{p}}^{\sigma}|\,\bar{\mathcal{O}}_P^{\alpha}\,|0\rangle\; \frac{e^{-E'(t-\tau)}e^{-E\tau}}{2E2E'}$$

Each state with the same quantum numbers as the proton can contribute!

$$|p\rangle = |1/2;+1/2\rangle$$

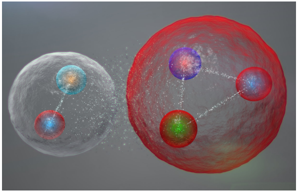
$$|I=1/2\rangle = |I_1=1\rangle \oplus |I_2=1/2\rangle$$

$$|\tfrac{1}{2};-\tfrac{1}{2}\rangle = -\sqrt{\tfrac{1}{3}}\,|\pi^0\rangle\,|n\rangle - \sqrt{\tfrac{2}{3}}\,|\pi^-\rangle\,|p\rangle$$

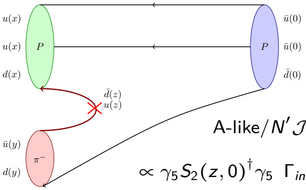
$$\langle n|\,\mathcal{J}^-\,|p\rangle = \langle p|\,\mathcal{J}_3\,|p\rangle$$

Spoiler: This and more on next talk by R. Horsley

- $p_{\bar\gamma} \xrightarrow{\mathcal{J}_\mu^-} n_\gamma$ $\quad\langle\, \mathcal{O}_{n_\gamma}(x)\; \mathcal{J}_\mu^-(z)\; \bar{\mathcal{O}}_{P_{\bar\gamma}}(0)\,\rangle$
- $p_{\bar\gamma} \xrightarrow{\mathcal{J}_\mu^-} n_\gamma\;\pi^0$ $\quad\langle\, \mathcal{O}_{n_\gamma}(x)\; \mathcal{O}_{\pi^0}(y)\; \mathcal{J}_\mu^-(z)\; \bar{\mathcal{O}}_{P_{\bar\gamma}}(0)\,\rangle$
- $p_{\bar\gamma} \xrightarrow{\mathcal{J}_\mu^-} p_\gamma\;\pi^-$ $\quad\langle\, \mathcal{O}_{P_\gamma}(x)\; \mathcal{O}_{\pi^-}(y)\; \mathcal{J}_\mu^-(z)\; \bar{\mathcal{O}}_{P_{\bar\gamma}}0)\,\rangle$

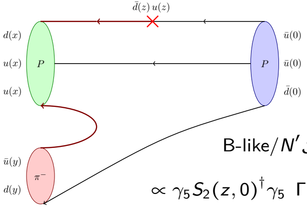


$P_{\bar{\gamma}} \xrightarrow{\mathcal{J}_\mu^-} P_\gamma\ \pi^-$



A-like/$N'\mathcal{J}\pi$

$\propto \gamma_5 S_2(z,0)^\dagger \gamma_5\ \ \Gamma_{ins}\ \ S_1(z,0)$

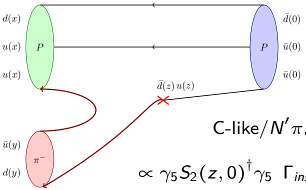


B-like/$N'\mathcal{J}N$

$\propto \gamma_5 S_2(z,0)^\dagger \gamma_5\ \ \Gamma_{ins}\ \ D(z,0)$

$S_2(z,0) \propto D(z,x) S_1(x,0)$

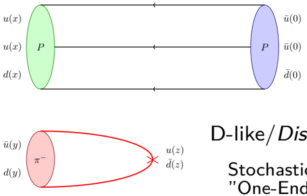
Sequential method



C-like/$N'\pi\mathcal{J}$

$\propto \gamma_5 S_2(z,0)^\dagger \gamma_5\ \ \Gamma_{ins}\ \ U(z,0)$

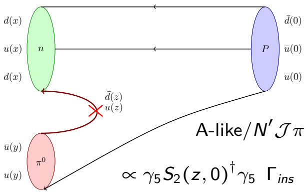
$S_2(z,0) \propto D(z,y) S_1(y,0)$



D-like/*Disconnected*

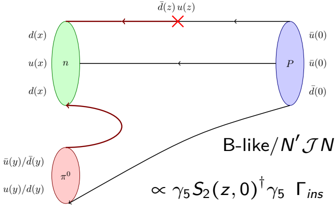
Stochastic method:
"One-End trick"

$P_{\bar{\gamma}} \xrightarrow{\mathcal{J}_\mu^-} n_\gamma \ \pi^0$



A-like/$N'\mathcal{J}\pi$

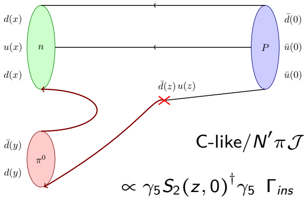
$$\propto \gamma_5 S_2(z,0)^\dagger \gamma_5 \ \ \Gamma_{ins} \ \ S_1(z,0)$$



B-like/$N'\mathcal{J}N$

$$\propto \gamma_5 S_2(z,0)^\dagger \gamma_5 \ \ \Gamma_{ins} \ \ D(z,0)$$

$$S_2(z,0) \propto D(z,x) S_1(x,0)$$



C-like/$N'\pi\mathcal{J}$

$$\propto \gamma_5 S_2(z,0)^\dagger \gamma_5 \ \ \Gamma_{ins} \ \ D(z,0)$$

$$S_2(z,0) \propto D(z,y) S_1(y,0)$$

$$C_{2pt}(t)^{p\pi^- \to n\pi^0} \ = \ C_{3pt}(t, \tau = 0)^{p \xrightarrow{\mathcal{P}} n\pi^0}$$

3pt		2pt
$p_{\bar{\gamma}} \xrightarrow{\mathcal{P}} n_\gamma \ \pi^0$	$\Rightarrow$	$p_{\bar{\gamma}} \ \pi^- \ \to \ n_\gamma \ \pi^0$
$p_{\bar{\gamma}} \xrightarrow{\mathcal{P}} p_{\bar{\gamma}} \ \pi^-$	$\Rightarrow$	$p_{\bar{\gamma}} \ \pi^- \ \to \ p_{\bar{\gamma}} \ \pi^-$

Spectral decomposition of C_{3pt}

$$C^{\mathcal{J},G_1}_{3pt,\Gamma_i}(t_{sink},\tau,|\,\mathbf{p}'_N,\mathbf{p}'_\pi,\mathbf{q}) \equiv \sum_{\mathbf{x},\mathbf{y},\mathbf{z}} e^{-i\mathbf{p}'_N\cdot\mathbf{x}}\ e^{-i\mathbf{p}'_\pi\cdot\mathbf{y}}\ e^{+i\mathbf{q}\cdot\mathbf{z}}\ \Gamma_i^{\alpha\beta}\langle\mathcal{O}^{\beta,G_1}_{N\pi}(x,y)\ \mathcal{J}^-_\mu(z)\ \bar{\mathcal{O}}^\alpha_N(0)\rangle$$

where $\mathcal{O}^{\beta,G_1}_{N\pi}(x,y) = Proj_{G_1}\left[\mathcal{O}^\beta_N(x)\ \mathcal{O}_\pi(y)\right]$.

Inserting 2 complete set of states and defining $\mathbf{k}\equiv\mathbf{p}'_N+\mathbf{p}'_\pi$:

$$C^{G_1}_{3pt} \approx \sum_{\sigma'\sigma}\ \Gamma_i^{\alpha\beta}\langle 0|\,\mathcal{O}^{\beta,G_1}_{N\pi}\,|N_{\sigma'},\pi;\ \mathbf{k}\rangle\ \langle N_{\sigma'},\pi;\ \mathbf{k}|\,\mathcal{J}^\mu\,|N^{\mathbf{p}}_\sigma\rangle\ \langle N^{\mathbf{p}}_\sigma\ |\,\bar{\mathcal{O}}^\alpha_N\,|0\rangle\ \frac{e^{-E'_{N\pi}(t_{sink}-\tau)}}{4EE'_{N\pi}}e^{-E\tau}$$

If so...

$$C^{N\pi,G_1}_{2pt}(t|\,\mathbf{k}) = \sum_\sigma\ \Gamma_+^{\alpha\beta}\ \langle 0|\,\mathcal{O}^{\beta,G_1}_{N\pi}\,|\ N_\sigma,\pi;\mathbf{k}\rangle\,\langle\mathbf{k};\pi,N_\sigma|\,\bar{\mathcal{O}}^{\alpha,G_1}_{N\pi}\,|0\rangle\ \frac{e^{-E'_{N\pi}t}}{2E'_{N\pi}} + \ldots$$

$$C^N_{2pt}(t|\,\mathbf{p}) = \sum_\sigma\ \Gamma_+^{\alpha\beta}\ \langle 0|\,\mathcal{O}^\beta_N\,|N^{\mathbf{p}}_\sigma\rangle\,\langle N^{\mathbf{p}}_\sigma|\,\bar{\mathcal{O}}^\alpha_N\,|0\rangle\ \frac{e^{-Et}}{2E} + \ldots$$

Nucleon effective mass

Ensemble used for the simulations:

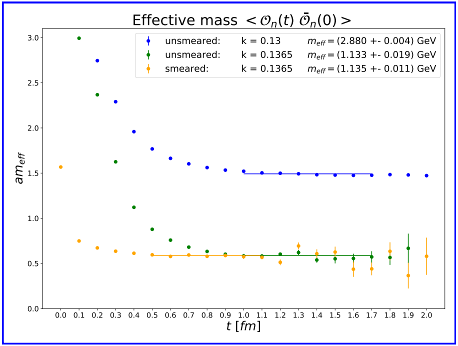
Ensemble	N_s	N_t	β	a [fm]	m_π [MeV]
A653	24	48	3.34	≈ 0.1	≈ 400

Nucleon 2-point function: $\langle \mathcal{O}_n(t)\ \bar{\mathcal{O}}_n(0) \rangle$

2 different kappas:

- very heavy quark mass: $k = 0.13$
- lighter quark mass: $k = 0.1365$

$k = 0.1365$:
with the smearing the plateau starts earlier

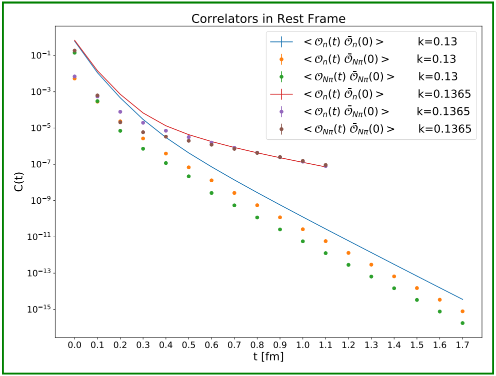


Solving the GEVP

$$\mathcal{M}(t) = \begin{pmatrix} < \mathcal{O}_n(t) \;\; \bar{\mathcal{O}}_n(0) > & < \mathcal{O}_n(t) \;\; \bar{\mathcal{O}}^{G_1}_{N\pi}(0) > \\ < \mathcal{O}^{G_1}_{N\pi}(t) \;\; \bar{\mathcal{O}}_n(0) > & < \mathcal{O}^{G_1}_{N\pi}(t) \;\; \bar{\mathcal{O}}^{G_1}_{N\pi}(0) > \end{pmatrix}$$

$$|N\pi\rangle^{I=1/2}_{I_3=-1/2} = -\frac{1}{\sqrt{3}}|n\rangle|\pi^0\rangle - \sqrt{\frac{2}{3}}|p\rangle|\pi^-\rangle$$

$$\mathcal{O}^{G_1}_{N\pi} \; : \; \mathcal{O}_{N\pi} \xrightarrow[\text{onto irrep}]{\text{Projected}} G_1 \xrightarrow{\text{continuum}} J = 1/2\{n, p, \dots\}$$



$$\mathcal{M}(t) \;\; V(t-t_0) = \mathcal{M}(t_0) \;\; V(t-t_0) \;\; \Lambda(t-t_0)$$

$$\Lambda(\tau) = diag(\lambda_1(\tau), \lambda_2(\tau))$$

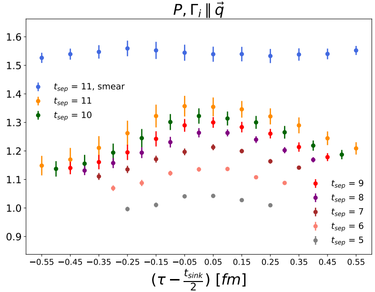
$$V(\tau) = (v_1(\tau), v_2(\tau)); \qquad \lambda_n(\tau) = d_n^2 e^{-E_n\tau}$$

Preliminary results for $C_{3pt}^{p\to p\pi^-}$, $C_{3pt}^{p\to n\pi^0}$

Analysis is still undergoing:

- Find λ_n, v_n
- $C_{3pt} = -\frac{1}{\sqrt{3}} C_{3pt}^{p\to n\pi^0} - \sqrt{\frac{2}{3}} C_{3pt}^{p\to p\pi^-}$

Ratio method: plots on the ensemble A653

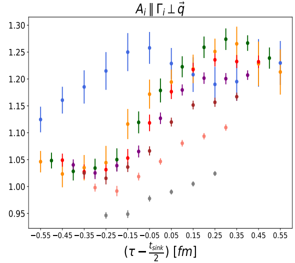


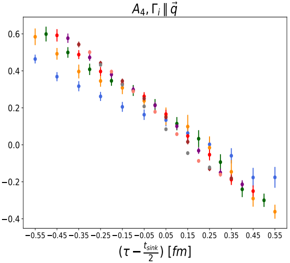
Forward limit

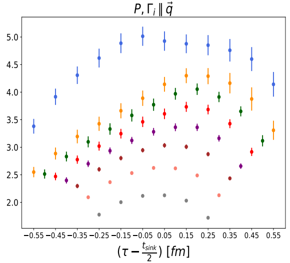
$$R^{\vec{p}_f, \vec{p}_i}_{\Gamma_i, \mathcal{J}}(t, \tau) \xrightarrow[\mathcal{J}=\mathcal{A}_i;\ \Gamma_i]{t \gg \tau \gg 0,\ \ \vec{p}_f = \vec{p}_i = \vec{0}} G_A(Q^2 = 0) \equiv g_A$$

The effect of the smearing is visible.

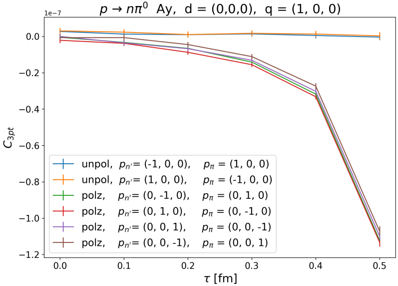
↑ increasing $t_{sep} = t_{sink} - t_{source}$







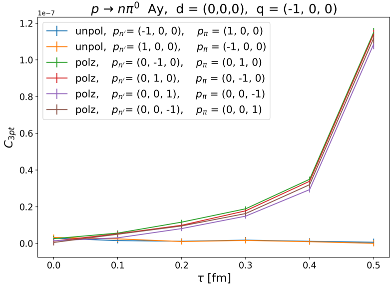
Examples of $C_{3pt}^{p\to N\pi}$ on the ensemble $A653$ in the rest frame ($\mathbf{d} = \mathbf{0}$).



$$C_{3pt}(\tau) \;=\; \Gamma_i \;\; \langle \mathcal{O}_{n\pi^0}^{\vec{d}=\vec{0}}(t_{sink}) \;\; \mathcal{A}_y^{\vec{q}=\hat{e}_x}(\tau) \;\; \bar{\mathcal{O}}_p^{-\hat{e}_x}(0)\rangle$$

(Heavy quark mass!)

$$C_{3pt}(\tau) \;=\; \Gamma_i \;\; \langle \mathcal{O}_{n\pi^0}^{\vec{d}=\vec{0}}(t_{sink}) \;\; \mathcal{A}_y^{\vec{q}=-\hat{e}_x}(\tau) \;\; \bar{\mathcal{O}}_p^{\hat{e}_x}(0)\rangle$$



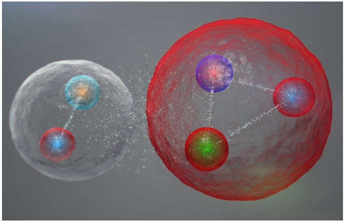
Preliminary

Analysis in progress..

Summary & Conclusions

Next tasks:

1. Extraction of $\langle N\pi|\mathcal{J}|N\rangle$
2. Compare $\langle N\pi|\mathcal{J}|N\rangle$ to $\langle N|\mathcal{J}|N\rangle$
3. Check χPT predictions
4. Apply smearing
5. Lorentz decomposition of $\langle N\pi|\mathcal{J}|N\rangle$
6. $N\pi$ scattering



This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 813942

EuroPLEx

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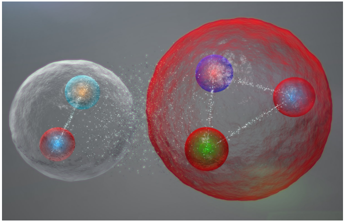

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Thanks everyone for the attention! Questions?

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