$N\pi$-state contamination in lattice calculations of nucleon electromagnetic form factors

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Physical point simulations

pion mass at (or close to) the physical value

- Advantage

  No chiral extrapolation needed, i.e. one systematic error eliminated

- Problems
  - Numerically demanding

    need large volumes too
  - Signal-to-noise problem
  - Multi-particle-state contamination in correlation functions

    Example: $N\pi$ states in the nucleon sector

    Light pions ⇒ small gap to the single nucleon ground-state

$N\pi$ excited states

Theoretical tool: Chiral perturbation theory (ChPT) Gasser, Saino, Svarc, NPB 307 (1988) 779

Provides understanding for the $N\pi$-excited state contamination in many observables

- Nucleon mass — OB, PRD 92 (2015) 074504
- Nucleon charges g_A g_S g_T — OB, PRD 94 (2016) 054505
- Pdf moments $\langle x \rangle_{u-d}$ $\langle x \rangle_{\Delta u - \Delta d}$ $\langle x \rangle_{\delta u - \delta d}$ — OB, PRD 95 (2017) 034505
- Axial and pseudoscalar form factors $G_A(Q^2)$, $\tilde{G}_P(Q^2)$, $G_P(Q^2)$
  “PCAC puzzle”, projection method
  OB, PRD 99 (2019) 054506
  OB, PRD 100 (2019) 054507
  OB, PRD 101 (2020) 034515

This talk:
Impact on *electromagnetic form factors* $G_E(Q^2)$, $G_M(Q^2)$ — OB and H. Čolić, PRD 103 (2021) 114514

Electromagnetic form factors

Matrix elements of local isovector vector current V_μ

isospin symmetry assumed

$$\langle N(p',s')|V_\mu(0)|N(p,s)\rangle = \bar{u}(p',s')\left(\gamma_\mu \underset{\uparrow}{F_1(q^2)} + i\frac{\sigma_{\mu\nu}q_\nu}{2M_N}\underset{\uparrow}{F_2(q^2)}\right)u(p,s)$$

Dirac and Pauli form factors

Alternatively:

$$G_\mathrm{E}(q^2) = F_1(q^2) + \frac{q^2}{4M_N^2}F_2(q^2)$$

electric

$$G_\mathrm{M}(q^2) = F_1(q^2) + F_2(q^2)$$

magnetic

(Sachs) form factors

Momentum transfer $q^2 = -Q^2 < 0$

Calculating the form factors

Sketch

- Compute the nucleon 2-pt and 3-pt functions involving current V_μ
- Form appropriate ratio $R_\mu(\vec{q}, t, t')$
- Extract the *effective form factors*

$$G_{\mathrm{X}}^{\mathrm{eff}}(Q^2, t, t') = G_{\mathrm{X}}(Q^2)\left[1 + \epsilon_X(Q^2, t, t')\right] \qquad \mathrm{X} = \mathrm{E}, \mathrm{M}$$

$$\longrightarrow 0 \quad \text{for} \quad t, t' \longrightarrow \infty$$

➥ contain *excited-state contribution* ε_{X} and depend on t, t'

- For given t consider *estimators* for the form factors, e.g.
  - *Midpoint estimate*: $G_{\mathrm{E}}^{\mathrm{mid}}(Q^2, t) = G_{\mathrm{E}}^{\mathrm{eff}}(Q^2, t, t' = t/2)$
  - *Plateau estimate*: $G_{\mathrm{E}}^{\mathrm{plat}}(Q^2, t) = \min\limits_{0<t'<t} G_{\mathrm{E}}^{\mathrm{eff}}(Q^2, t, t')$

Practically the same for small Q^2

Calculating the form factors

Sketch

- Note: You follow the same steps in ChPT …

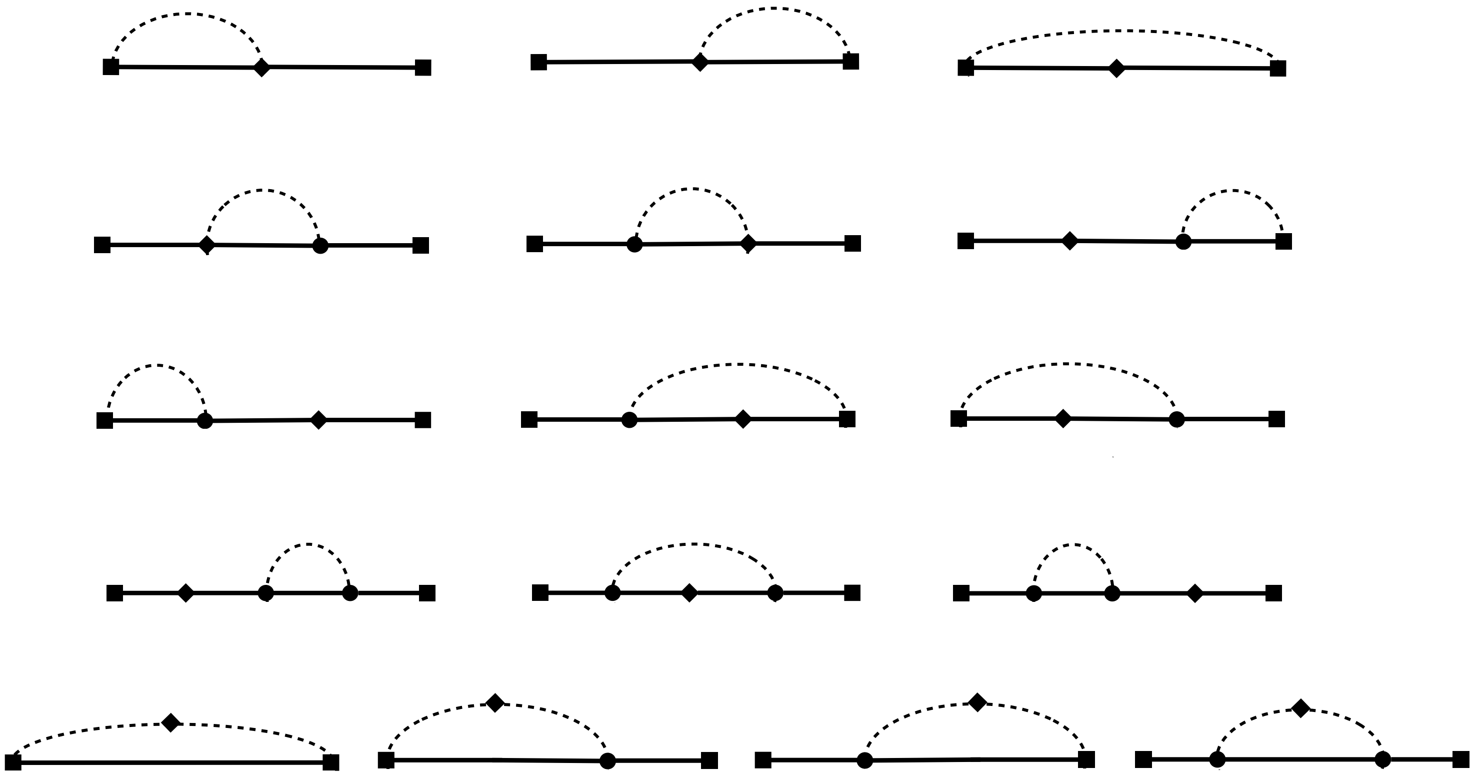
… and obtain the $N\pi$-state contribution $\epsilon_X^{N\pi}(Q^2, t, t')$

dominant one for large time separations

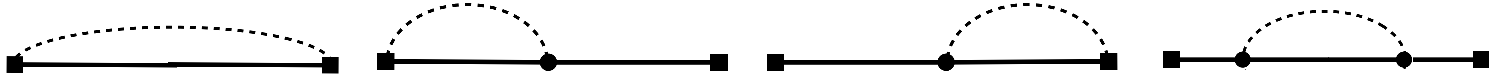
$N\pi$ contribution in ChPT

Perturbative calculation → Feynman diagrams

3-pt function



2-pt function



$N\pi$ contribution in ChPT

Diagrams $\Rightarrow$ $\epsilon_X^{N\pi}(Q^2, t, t')$ X = E, M

- analytic but cumbersome results (not shown here …)
- Fix five input parameters by (approximate) exp. values:

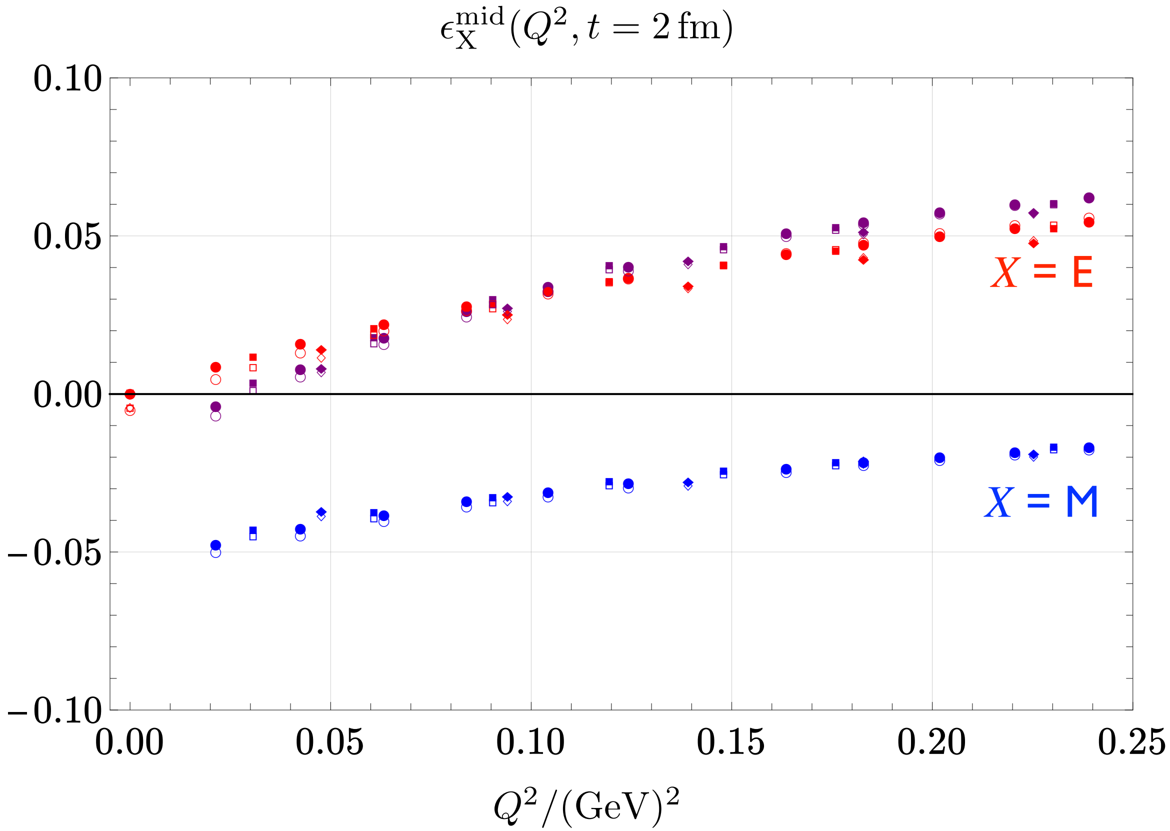
M_π	M_N	f_π	g_A	μ_p - μ_n
140 MeV	940 MeV	93 MeV	1.27	4.71

- finite spatial volume, extent L, periodic BC → assume value $M_\pi L$
  ➥ discrete momentum transfer Q_n^2
- Note:
  LO results do not depend on LECs associated with nucleon interpolating fields!
  Drop out in the ratios (at LO, not true beyond)

Results: Electric and magnetic form factors

for t = 2 fm

ChPT is expected to work well …



$M_\pi L$

●	●	6
■	■	5
◆	◆	4

small FV effect

- $G_{\rm E}^{\rm mid}$ overestimates, increasingly with Q^2, up to 5%
  $\varepsilon_{\rm E} = 0$ for $Q^2 = 0$, vector current conservation*
- $G_{\rm M}^{\rm mid}$ underestimates, decreasingly with Q^2, 2 to 5%

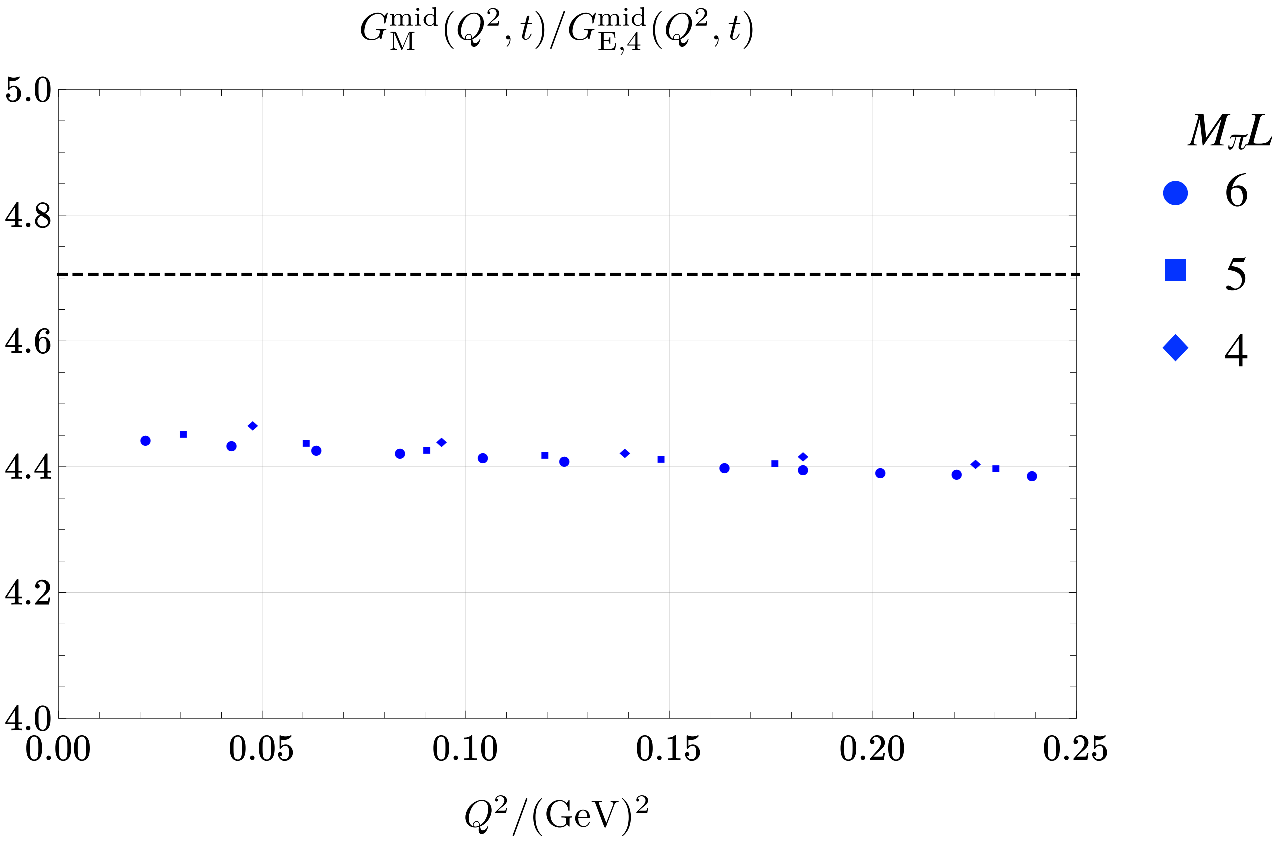
for t = 2 fm

about times 2 for t = 1.6 fm

* $G_{\rm E}$ computed using V_4

Electric and magnetic form factors

for t = 2 fm



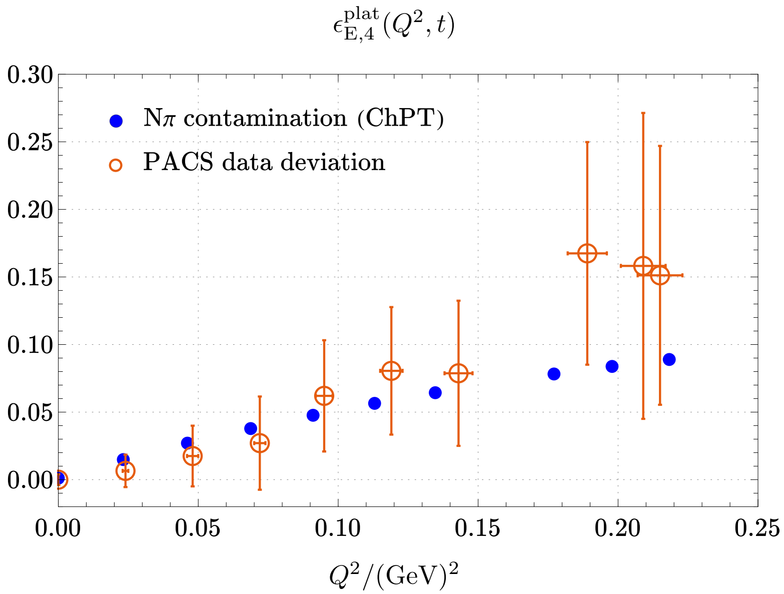
- $G_{\rm M}^{\rm mid}/G_{\rm E}^{\rm mid}$ shows flat Q^2 dependence
- Underestimates by about 6% for $t = 2$ fm

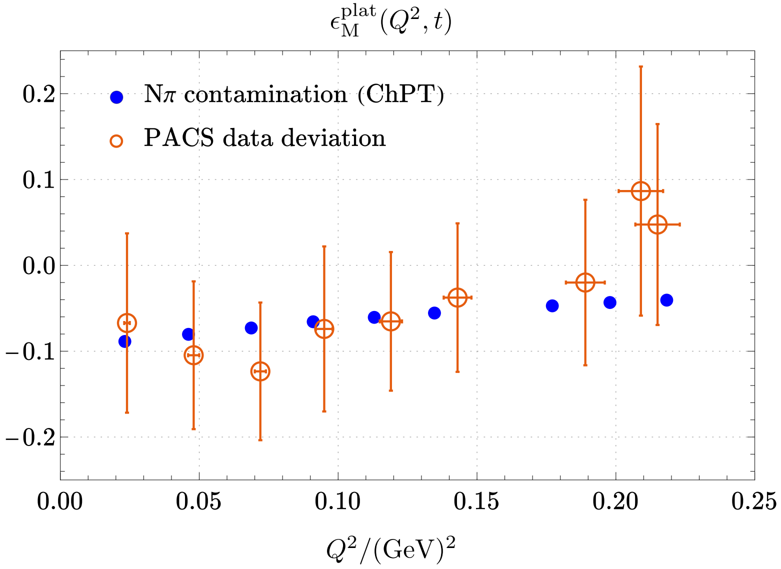
Comparison with lattice data

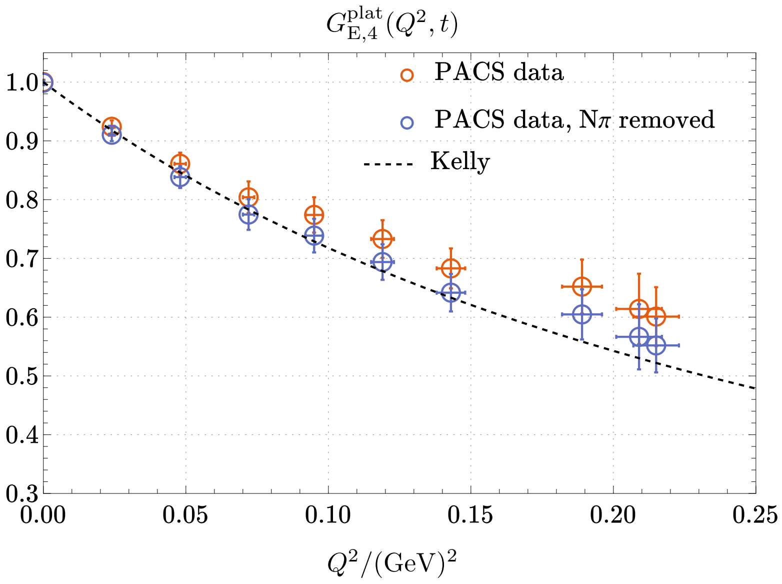
Compare with PACS data in Ishikawa *et al*, PRD 98 (2018) :

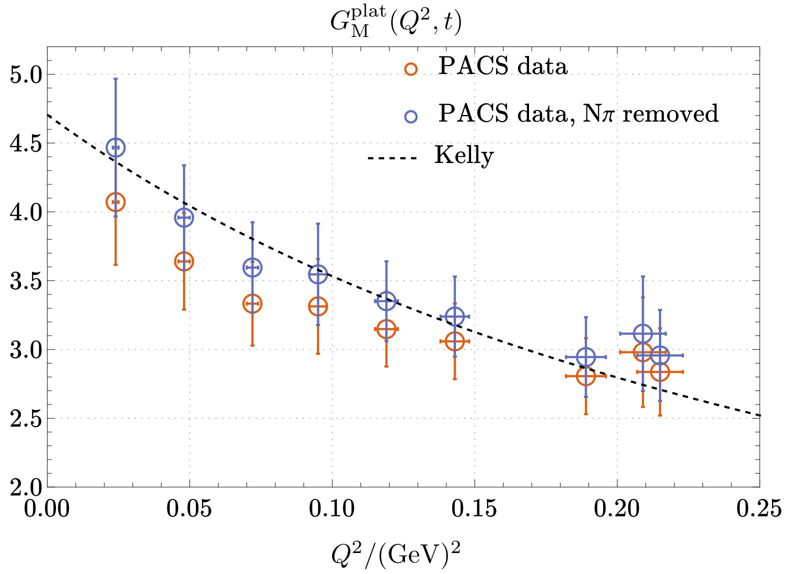
- ✔ simple plateau estimates
- ✔ almost physical pion mass: $M_\pi \approx 146\ \mathrm{MeV}$
- ✔ large volume: $M_\pi L \approx 6.0$ → many small Q^2
- ✘ one small source-sink separation: $t \approx 1.3\ \mathrm{fm}$
- ✘ large statistical errors
- ✘ one lattice spacing $a \approx 0.085\ \mathrm{fm}$

Comparison with lattice data


$\epsilon^{\mathrm{plat}}_{\mathrm{E},4}(Q^2,t)$


$\epsilon^{\mathrm{plat}}_{\mathrm{M}}(Q^2,t)$


$G^{\mathrm{plat}}_{\mathrm{E},4}(Q^2,t)$

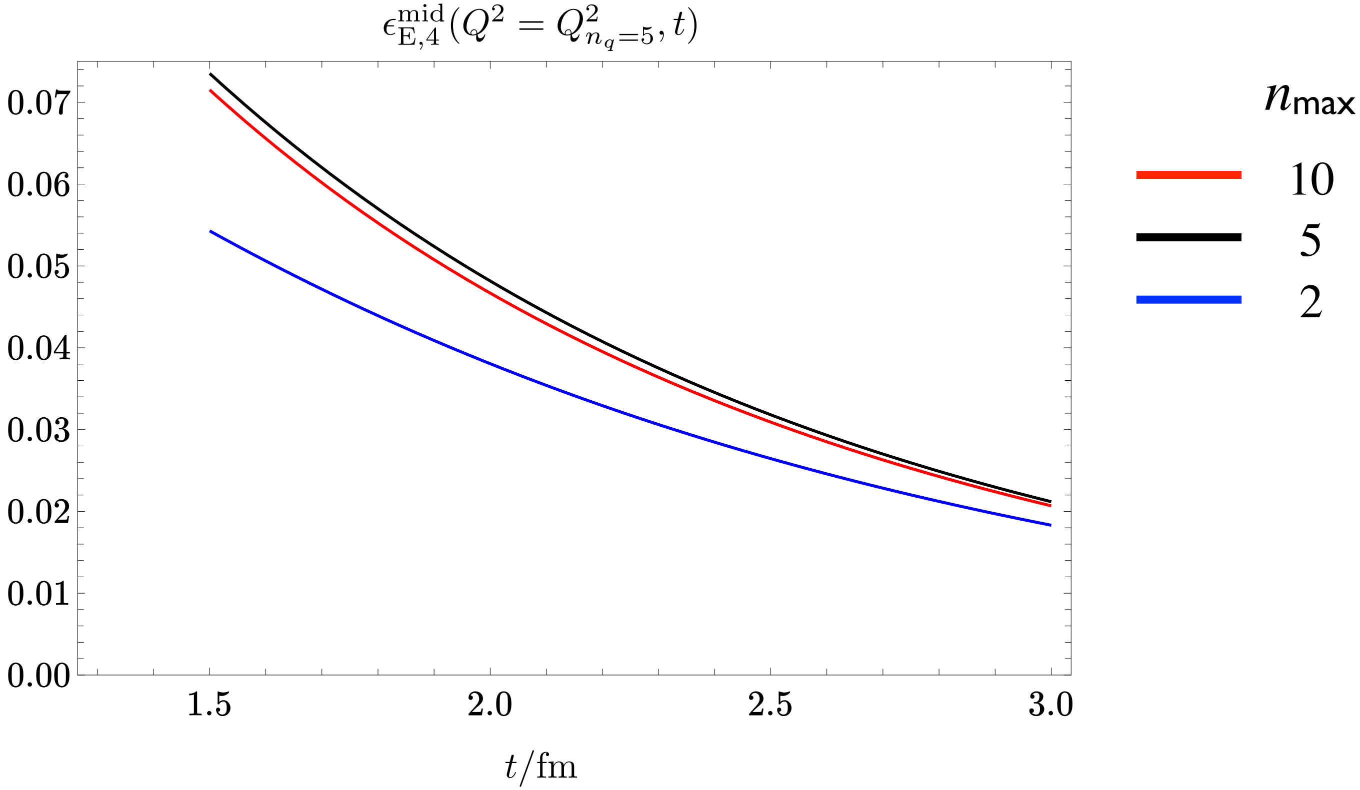

$G^{\mathrm{plat}}_{\mathrm{M}}(Q^2,t)$

- LO ChPT works surprisingly well (large statistical errors in the data …)

Comment I

The total $N\pi$ contamination is the *cumulative contribution* of many $N\pi$-states:

Example:

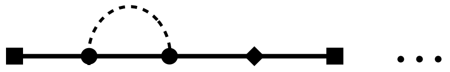


➥ total contamination ≈ sum of lowest 5 non-zero momentum states for $M_\pi L = 4$

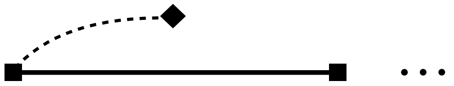
Comment 2

Comparing the $N\pi$ contribution in various form factors:

$\epsilon_{\rm X}^{\rm mid}$ $\quad {\rm X} = E,\ M,\ A$: dominated by loop diagrams



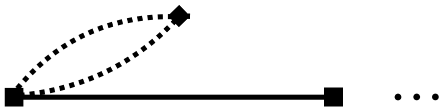
${\rm X} = \tilde{P}$: dominated by tree diagrams



$\Rightarrow$ $\epsilon_{\tilde{\rm P}}^{\rm mid}$ is much larger than the others

OB, PRD 99 (2019) 054506

Note: Analoque of the tree diagrams
for vector current form factors :



$N\pi\pi$ contribution
expected to be small

Conclusions

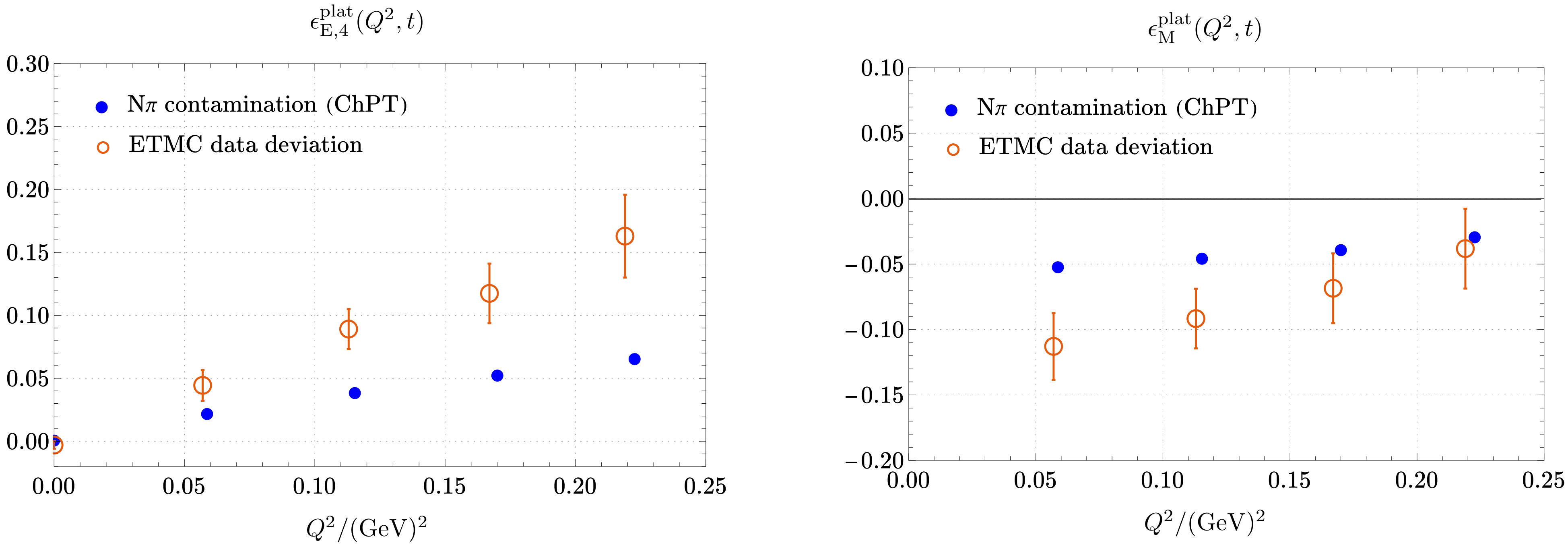
- $N\pi$-states result in a non-negligible contamination in $G_E(Q^2)$, $G_M(Q^2)$
  - O(±5%) for $t \approx 2$ fm
  - twice as large for $t \approx 1.5$ fm
- For $t \gtrsim 2$ fm: Use ChPT results to analytically remove the $N\pi$-contamination

  (similarly to removing FV effects)
- For $t \lesssim 2$ fm: Exercise caution when analyzing lattice data

  ??? usage of two-state-fits even though many more $N\pi$-states contribute ???

Backup slides

Comparison with ETMC data



- LO ChPT describes the data reasonably well (within expectation)
- Suggests that $N\pi$-states are at least partially responsible for deviation with experiment

Comparison with data - alternative form

$$G_{\rm X}^{\rm mid}(Q^2,t) = G_{\rm X}(Q^2)\Big[1+\epsilon_{\rm X}^{\rm mid}(Q^2,t)\Big]$$

➥ Remove analytically the anticipated LO $N\pi$ contamination from the data:

$$G_{\rm X}^{\rm corr}(Q^2,t) \equiv \frac{G_{\rm X}^{\rm mid}(Q^2,t)}{1+\epsilon_{\rm X}^{{\rm mid},N\pi}(Q^2,t)}$$

← lattice results

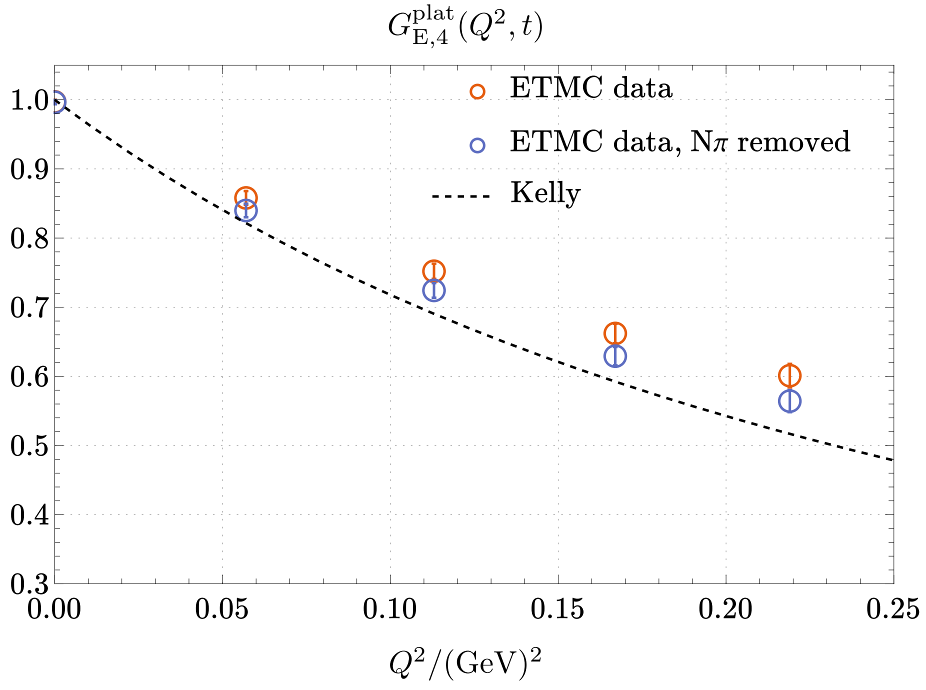
← ChPT prediction

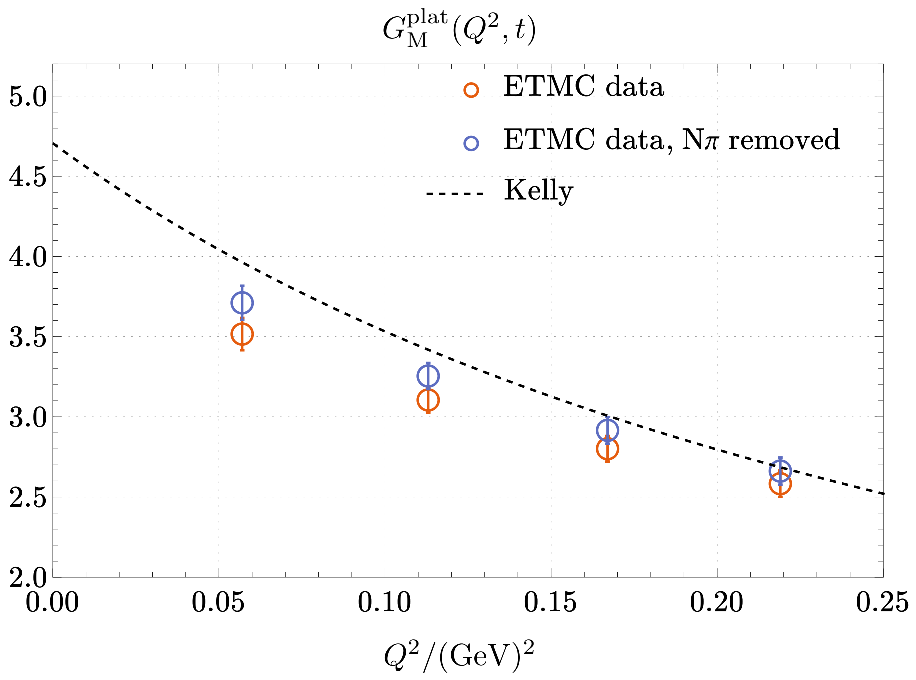
If higher order and other excited-state contributions are negligible:

$$G_{\rm X}^{\rm corr}(Q^2,t) \approx G_{\rm X}(Q^2)$$

← phys form factor

Comparison with ETMC data





- Same conclusion as before
  LO ChPT eases discrepancy with experimental data (here given by Kelly line)