# Electric Polarizability of Hadrons from Lattice QCD

## Lattice 2021

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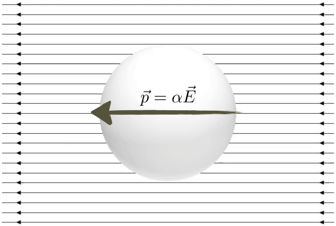
July 29, 2021

# Electric and Magnetic Polarizabilities




$$\boldsymbol{p} = \alpha \boldsymbol{E} \tag{1}$$

$$\begin{aligned} H_{\mathrm{em}} = & -\boldsymbol{\mu} \cdot \boldsymbol{B} - \frac{1}{2}\alpha \boldsymbol{E}^2 - \frac{1}{2}\beta \boldsymbol{B}^2 \\ & - \frac{1}{2}\gamma_{E1}\boldsymbol{\sigma} \cdot \boldsymbol{E} \times \dot{\boldsymbol{E}} - \frac{1}{2}\gamma_{M1}\boldsymbol{\sigma} \cdot \boldsymbol{B} \times \dot{\boldsymbol{B}} \\ & + \gamma_{E2}\sigma_i E_{ij} B_j - \gamma_{M2}\sigma_i B_{ij} E_j \\ & - \frac{1}{12}\alpha_{E2} E_{ij}^2 - \frac{1}{12}\beta_{M2} B_{ij}^2 + \dots \end{aligned} \tag{2}$$

Charged Pion Polarizabilities:

- $\chi$PT

$$\alpha_E^{\pi^+} = -\beta_M^{\pi^+} = (2.8 \pm 0.2) \times 10^{-4}\,\text{fm}^3 \tag{3}$$

- Experiment

| Experiment | $\alpha_E^{\pi^+}$ $(10^{-4}\texttt{fm}^3)$ |
|---|---|
| $\gamma p \to \gamma n \pi^+$ | $20 \pm 12$ |
| $\gamma \pi^+ \to \gamma \pi^+$ | $2.0 \pm 0.6 \pm 0.7$ |
| $\gamma\gamma \to \pi^+\pi^-$ | $2.2 \pm 1.1$ |

## Background Field Method

Multiply the $SU(3)$ links by $U(1)$ links

$$U_\mu(x) = e^{iqA_\mu(x)}$$
$$A_\mu(x) = -\mathcal{E} x_4 \delta_{\mu,3} \tag{4}$$

For charged hadrons

$$\boxed{C(t) \neq A e^{-Et}} \tag{5}$$

Continuum effective correlator:

$$C(t) = \frac{1}{2}\int_0^\infty ds \sqrt{\frac{q\mathcal{E}}{2\pi \sinh(q\mathcal{E}s)}}\; e^{-\frac{1}{2}m^2 s - \frac{1}{2}q\mathcal{E}t^2 \coth(q\mathcal{E}s)} \tag{6}$$

But, it can't be used on the finite lattice ($L \neq \infty, a \neq 0$).

$\mathcal{E}$ produces charged pairs

These are accelerated forever unless they hit a wall!
↓
Dirichlet boundaries in the field direction
&
weak electric fields.

Relativistic particle with spin zero in 2D

Discretize the effective action on a lattice of size $L \times T$

$$S_E = \frac{1}{2} a^2 \sum_{n,m} \phi_n K_{nm} \phi_m \tag{7}$$

with

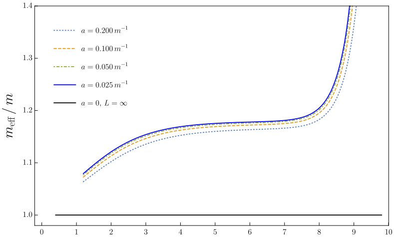
$$K_{nm} = [4 + (am)^2]\delta_{nm} - \sum_{\hat{\mu}>0} \left[\delta_{n+\hat{\mu},m}\, e^{-iqaA_\mu(n)} + \delta_{n-\hat{\mu},m}\, e^{+iqaA_\mu(m)}\right] \tag{8}$$

Effective model is the propagator:

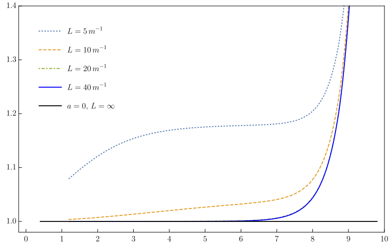
$$G_0(x,t;A_\mu,\tilde{L}) = K^{-1}_{(x,t);(x,t)_s} \tag{9}$$

# Comparison of the effective mass from the model and the continuum function

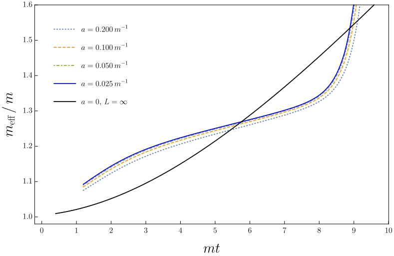
Continuum limit for $E = 0$



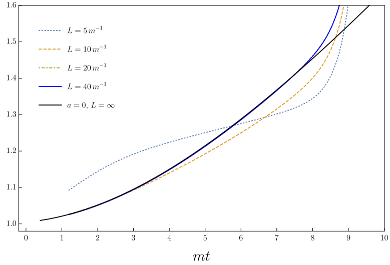

Infinite volume limit in continuum for $E = 0$




Continuum limit for $E \neq 0$




Infinite volume limit in continuum for $E \neq 0$



Lattice QCD propagator for $\pi^+$:

$$G(x,t;A_\mu,L) = \sum_{y,z} \langle 0|\hat{O}(x,y,z,t)\hat{O}^\dagger(x_s,y_s,z_s,t_s)|0\rangle, \qquad \hat{O} = \bar{d}\gamma_5 u \tag{10}$$

Distance between the walls: $\tilde{L}$ for the effective correlator and $L$ for the QCD correlator.

Dirichlet walls at $x=0$ and $x=L$ in the lattice QCD box and at $x=(L-\tilde{L})/2$ and $x=(L+\tilde{L})/2$ in the effective model.
Time boundaries are $t=0$, and $t/a=(N_t+1)$ for both.
Position of the sources are aligned in both.

$$\chi^2 \equiv \delta^\dagger C^{-1}\delta \quad \text{with the } \textit{residues} \quad \delta_i \equiv \langle y_i\rangle - f_i \tag{11}$$

$$C_{ij} \equiv \langle (y_i - \langle y_i\rangle)(y_j - \langle y_j\rangle)^*\rangle \tag{12}$$

$\chi^2$ invariant under $y' = Ty$ and $f' = Tf$.

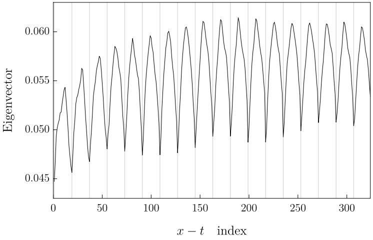
Under a gauge transformation, $(A,\phi) \to (A',\phi')$ with

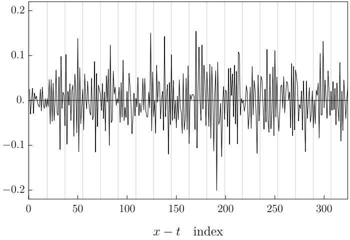
$$\begin{aligned} A'_\mu(x,t) &= A_\mu(x,t) + [\Lambda((x,t)+\hat{\mu}) - \Lambda(x,t)]/a \\ \phi'(x,t) &= e^{iq\Lambda(x,t)}\phi(x,t)\,, \end{aligned} \tag{13}$$

we have

$$G(x,t;A'_\mu,L) = e^{iq[\Lambda(x,t)-\Lambda(x_s,t_s)]}G(x,t;A_\mu,L)\,. \tag{14}$$

The same holds for $G_0$ so $\chi^2$ is invariant under Eq. 13.



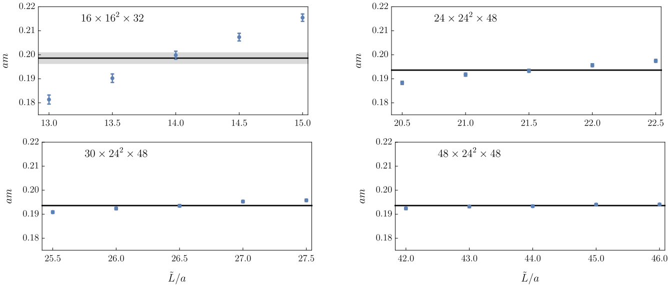




$$
\begin{aligned}
y(t) &= \sum_{x=x_i}^{x_f} L(x,t) G(x,t; A_\mu = 0, L) \\
f(t) &= \mathcal{A} \sum_{x=x_i}^{x_f} L(x,t) G_0(x,t; A_\mu = 0, \tilde{L}) \,.
\end{aligned} \tag{15}
$$

$$
L(x,t) = \prod_{x'=x_i}^{x-a} e^{-iqaA_\mu(x',t)} \,, \tag{16}
$$

Under gauge transformations in Eq. 13:

$$
\begin{aligned}
L'(x,t) &= e^{iq[\Lambda(x_i,t) - \Lambda(x,t)]} L(x,t) \\
y'(t) &= e^{iq[\Lambda(x_i,t) - \Lambda(x_s,t_s)]} y(t)
\end{aligned} \tag{17}
$$



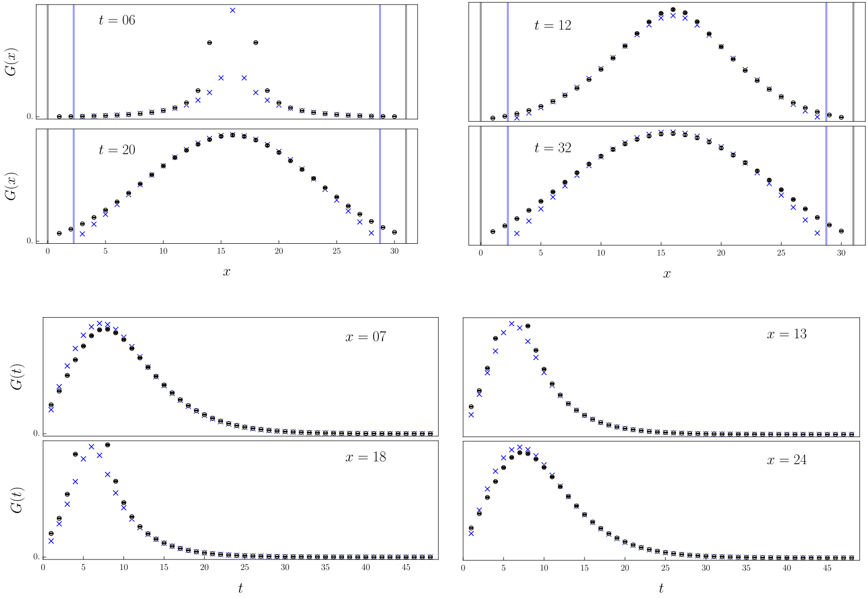

$$y_0(t) = \sum_{x=x_i}^{x_f} L(x,t) G(x,t; A_\mu = 0, L) \qquad y_{\mathcal{E}}(t) = \sum_{x=x_i}^{x_f} L(x,t) G(x,t; A_\mu, L)$$

$$f_0(t) = \mathcal{A} \sum_{x=x_i}^{x_f} L(x,t) G_0(x,t; A_\mu = 0, \tilde{L}, am) \tag{18}$$

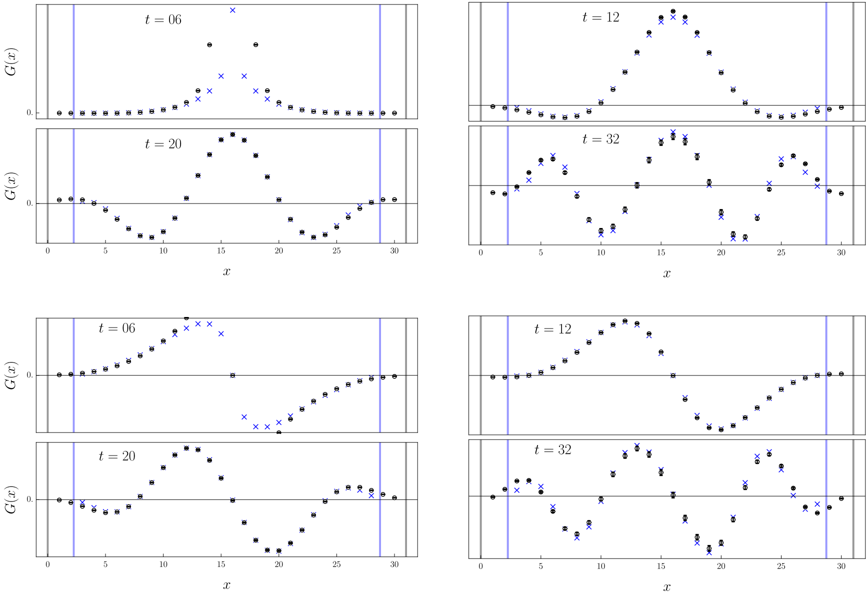
$$f_{\mathcal{E}}(t) = (\mathcal{A} + \Delta\mathcal{A}) \sum_{x=x_i}^{x_f} L(x,t) G_0(x,t; A_\mu, \tilde{L}, am + \delta am)$$

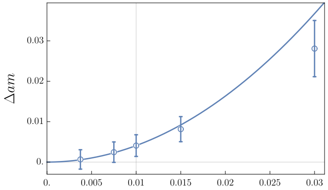
$$C = \begin{pmatrix} C_{00} & C_{0\mathcal{E}} \\ C_{\mathcal{E}0} & C_{\mathcal{E}\mathcal{E}} \end{pmatrix} \tag{19}$$

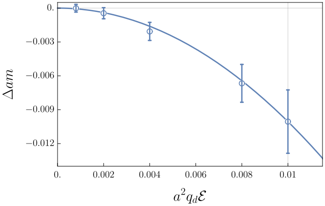
# Zero field wave functions



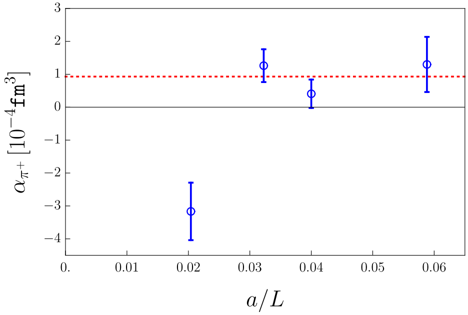
# Wave functions in the presence of the electric field






$$\delta am = \frac{1}{2}\alpha_{\pi^\pm} a\mathcal{E}^2 \tag{20}$$




Phys.Rev.D 104, 014510

# Conclusion

- LQCD offers the opportunity to compute and understand hadron polarizabilities from quark-gluon dynamics.
- Neutral hadron calculations produced the results for $\alpha_n$ that are consistent with $\chi$PT calculations.
- Charged hadrons present new challenges because they are accelerated by electric fields.
- We constructed a fitting function that incorporates the finite-volume effects for charged hadrons and gives results that are gauge-invariant.
- It can be used to extract electric polarizabilities for charged pions.