Pion and Nucleon Gluon Distribution Function on Lattice

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in collaboration with Huey-Wen Lin
based on work: 2007.16113, 2104.06372
and on-going work

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``CAREER: Constraining Parton Distribution Functions for New-Physics Searches''

Gluon PDF

Studying the parton distribution functions (PDFs) is important to characterize the structure of the hadron and nonperturbative QCD.

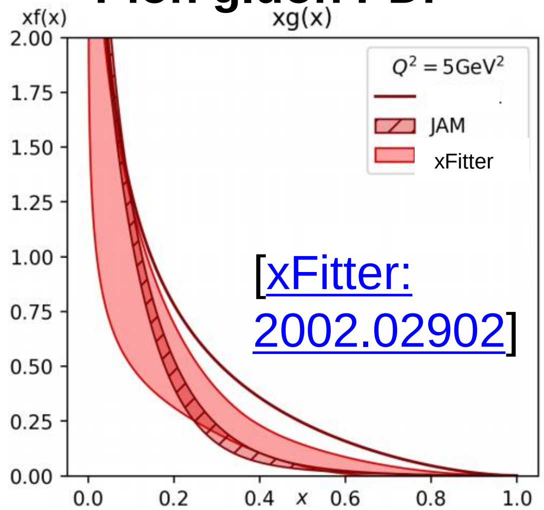
Global fits use different experimental data and fit strategies

- The error at large x-region is large comparing to the valence quark PDFs
- Current gluon PDFs from different global analyses vary by the input experiment data

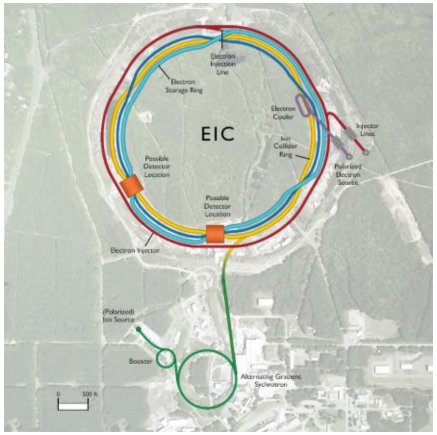
To improve the gluon PDFs,

- Experimentally, for example, there are Electron-Ion Collider (EIC) will aim at gluon PDF
- Theoretically, lattice QCD is an independent approach to calculate gluon PDF

Pion gluon PDF



U.S.-based EIC



[BNL News]
https://www.bnl.gov/newsroom/news.php?a=116998

Pseudo-PDF

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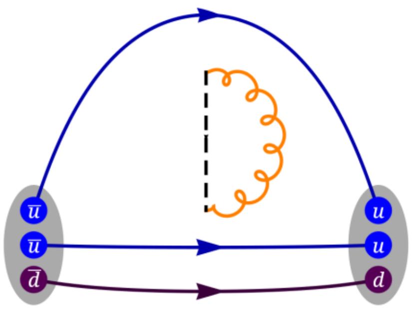
The reduced Ioffe-time distribution (ITDs) definition [Orginos at el, 1706.05373],

$$\mathcal{M}(\nu, z^2) = \frac{M(\nu, z^2)/M(\nu, 0)}{M(0, z^2)/M(0,0)}$$

where $M(\nu, z^2) = \langle 0 | O_g(z) | 0 \rangle$, Ioffe-time $\nu = zP_z$.

$$O_g(z) = \sum_{i \neq z,t} O(F^{ti}, F^{ti}; z) - \sum_{i \neq z,t} O(F^{ij}, F^{ij}; z),$$

$$O(F^{\mu\nu}, F^{\rho\sigma}; z) = F^{\mu\nu}(z) U(z, 0) F^{\rho\sigma}(0)$$



The gluon pseudo-PDF matching condition,

[Balitsky at el, 1910.13963]

$$\mathcal{M}(\nu, z^2) = \int_0^1 dx \frac{xg(x, \mu^2)}{\langle x \rangle_g} \left(R_1(x\nu, z^2\mu^2) + R_2(x\nu)\right)$$

evolution — scheme conversion

The evolved ITD (EITD) definition and matching from EITD to $xg(x, \mu^2)$,

$$G(\nu, z^2, \mu) = \mathcal{M}(\nu, z^2) + \int_0^1 du\, R_1(u, z^2\mu^2)\, \mathcal{M}(u\nu, z^2)$$

$$= \int_0^1 dx \frac{xg(x,\mu^2)}{\langle x \rangle_g} R_2(x\nu)$$

Lattice setups

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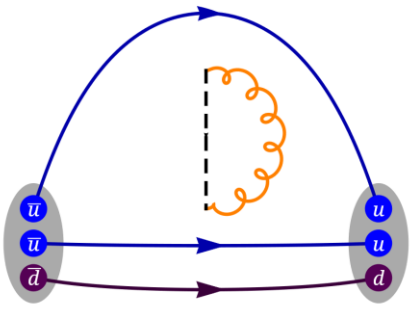
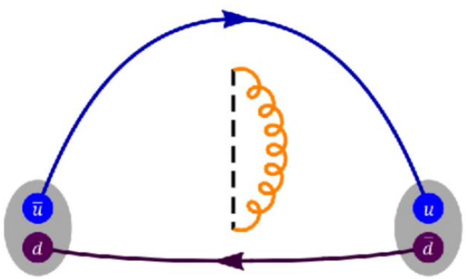
We use clover valence fermions on $N_f = 2+1+1$ highly improved staggered quarks (HISQ) lattices generated by the MILC Collaboration [MILC 1212.4768].

ensemble	a09m310	a12m220	a12m310	a15m310
a (fm)	0.0888(8)	0.1184(10)	0.1207(11)	0.1510(20)
M_π^{sea} (MeV)	312.7(6)	216.9(2)	305.3(4)	306.9(5)
M_π^{val} (MeV)	313.1(13)	226.6(3)	309.0(11)	319.1(31)
$M_{\eta_s}^{val}$ (MeV)	698.0(7)	696.9(2)	684.1(6)	684.1(6)
$L^3 \times T$	$32^3 \times 96$	$32^3 \times 64$	$24^3 \times 64$	$16^3 \times 48$
N_{meas}^{pion}	N/A	731,200	143,680	21,600
$N_{meas}^{nucleon}$	145,296	N/A	324,160	21,600

nucleon



pion



We thank MILC Collaboration for sharing the lattices used to perform this study.

Extracted bare matrix element

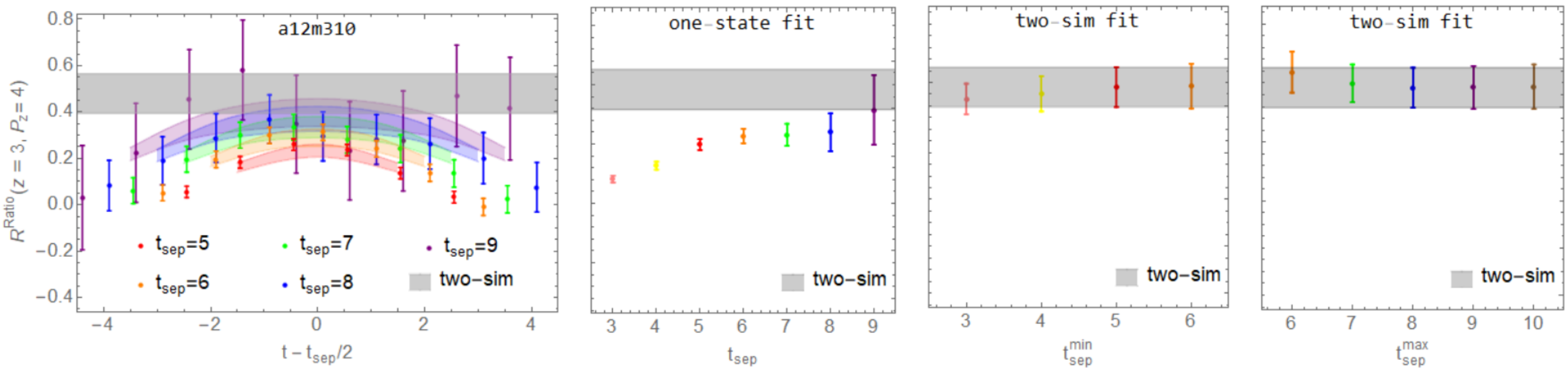
Following the work [Bhattacharya at el, 1306.5435], the correlators C_{3pt} and C_{2pt} can be decomposed as,

$$C_{3pt}(z,P_z;t_{sep},t) = |A_0|^2\langle 0|O|0\rangle e^{-E_0 t_{sep}} + |\mathrm{A}_1||\mathrm{A}_0|\langle 1|O|0\rangle e^{-E_1(t_{sep}-t)}e^{-E_0 t} + |\mathrm{A}_0||\mathrm{A}_1|\langle 0|O|1\rangle e^{-E_0(t_{sep}-t)}e^{-E_1 t} + \cdots$$

$$C_{2pt}(z,P_z;t_{sep}) = |A_0|^2 e^{-E_0 t} + |A_1|^2 e^{-E_1 t} + \cdots$$

where the ground state matrix element is $\langle 0|O|0\rangle$, assuming $\langle 1|O|0\rangle = \langle 0|O|1\rangle$.

a12m310, **nucleon**



- The bare matrix element (ME) extracted from the two-sim fits are stable with the t_{sep} fit ranges choices around our final choice $t_{sep} = [5,9]$

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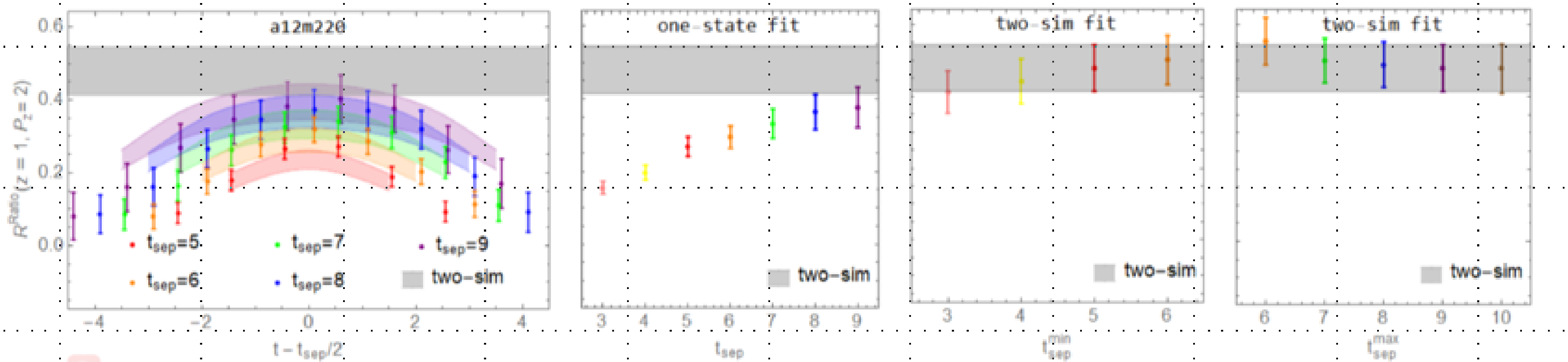
where the ground state matrix element is $\langle 0|O|0\rangle$, assuming $\langle 1|O|0\rangle = \langle 0|O|1\rangle$.

a12m220, **Pion**

Fan at el, 2104.06372



- The bare matrix element (ME) extracted from the two-sim fits are stable with the t_{sep} fit ranges choices around our final choice $t_{sep} = [5,9]$

RITD from different ensembles

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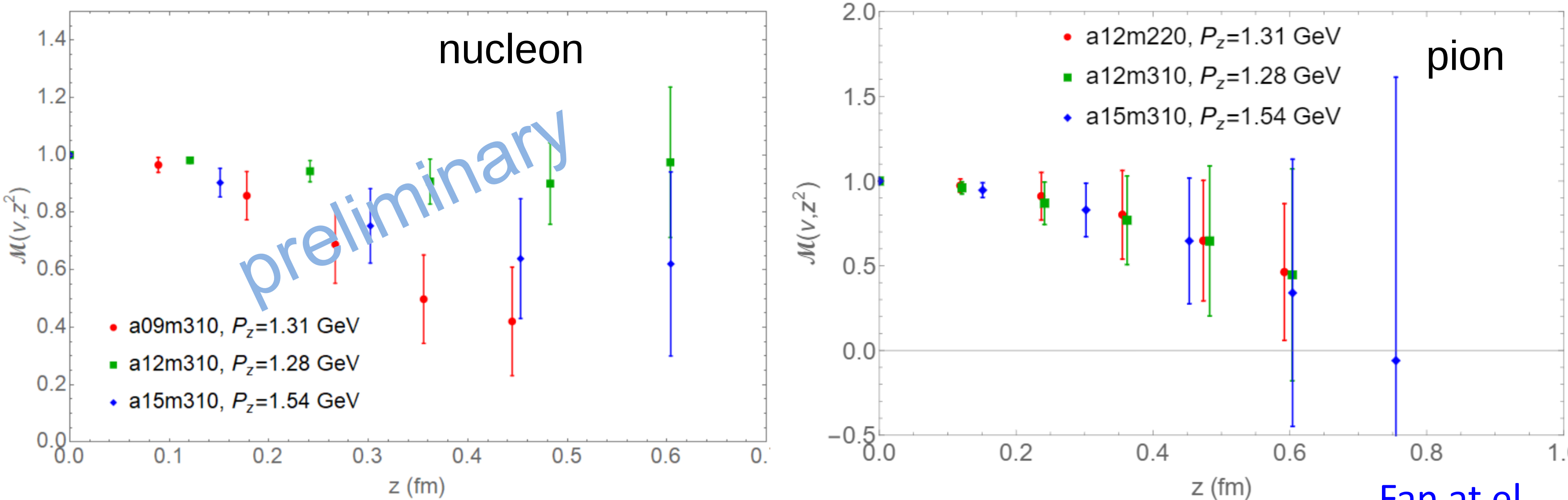
The RITDs are obtained by taking the double-ratio of ground-state matrix element,

$$\mathcal{M}(\nu, z^2) = \frac{M(\nu, z^2)/M(\nu, 0)}{M(0, z^2)/M(0,0)}$$



Pion&nucleon RITDs at boost momentum $P_z \sim 1.3$ GeV

[Fan at el, 2104.06372]

- The RITDs obtained from bare matrix elements have weak Lattice-spacing and pion-mass dependence

Nucleon and pion results comparison

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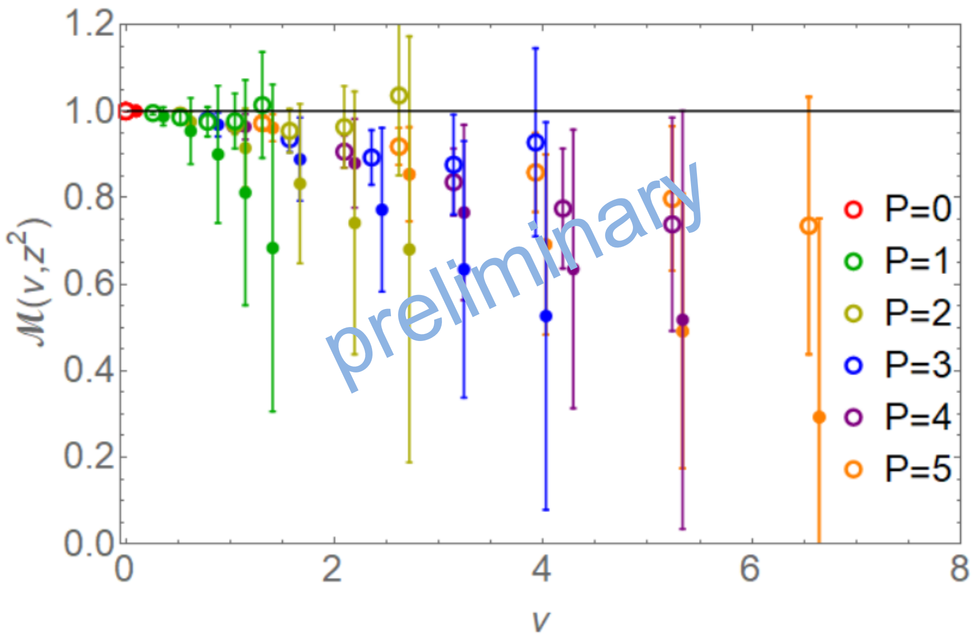
We compare the nucleon and pion RITD under the same measurements and 2-sim fit method.

a12m310, $M_{\pi}^{val} = 305.3(4)\ \mathrm{MeV}$



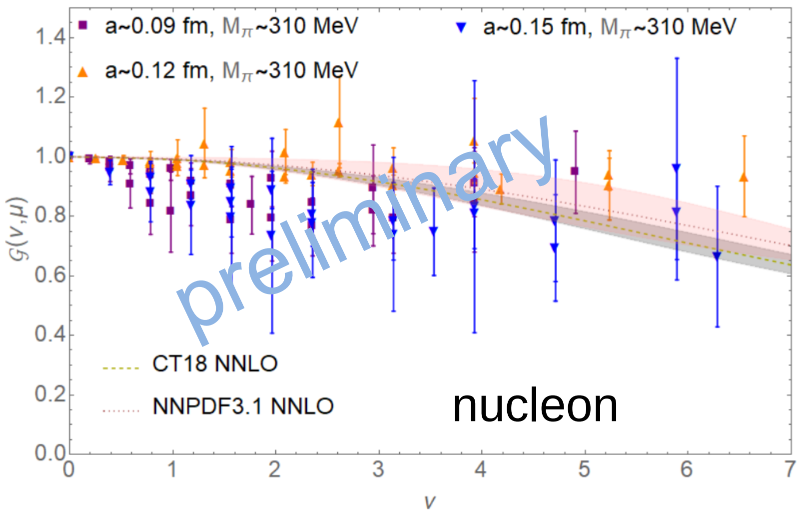
nucleon (open points)
pion (solid points)

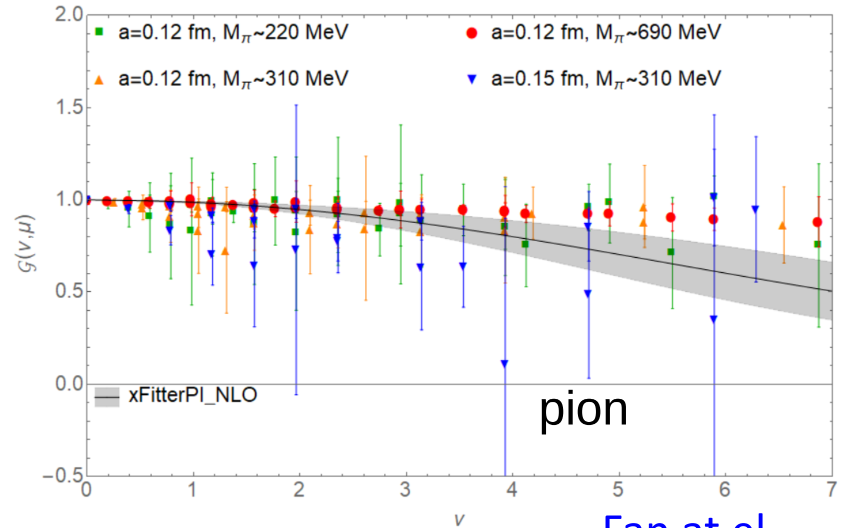
- The nucleon RITDs have smaller relative errors then pion RITD for the same ensembles

Evolved Ioffe-time distribution (EITD)

Through the EITD G, the lattice calculated RITD can be connected with gluon PDF with these two steps,

$$G(\nu, z^2, \mu) = \mathcal{M}(\nu, z^2) + \int_0^1 du\, R_1(u, z^2\mu^2)\, \mathcal{M}(u\nu, z^2) = \int_0^1 dx\, \frac{xg(x,\mu^2)}{\langle x \rangle_g} R_2(x\nu)$$





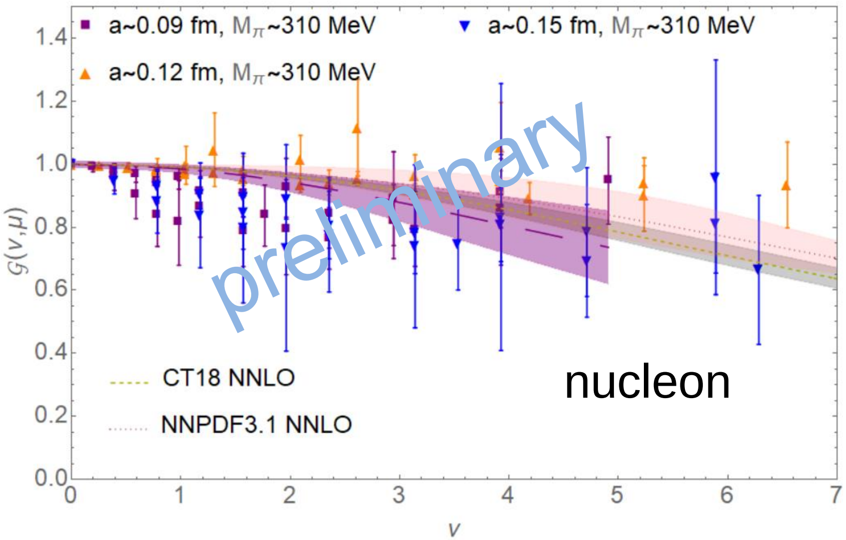
Fan at el, 2104.06372

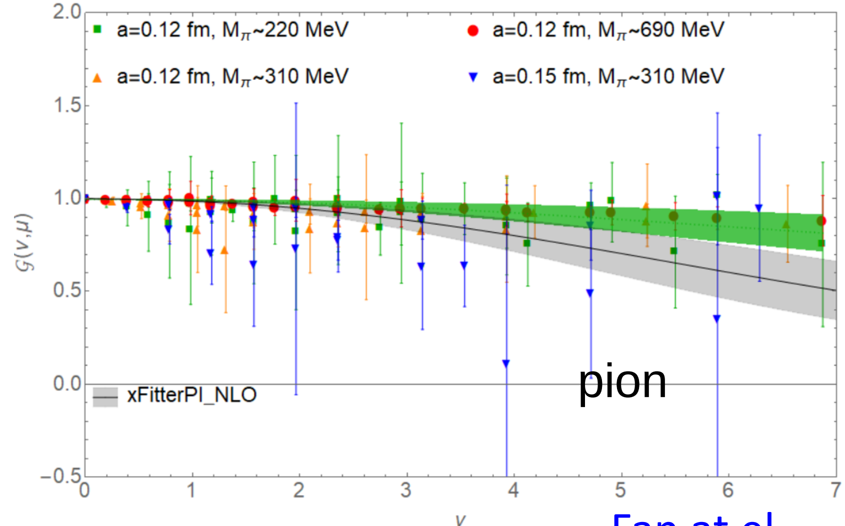
- The EITDs from global fit PDFs go through the EITD points from lattice calculations

Evolved Ioffe-time distribution (EITD)

Functional form light-cone PDF,

$$f_g(x,\mu) = \frac{xg(x,\mu)}{\langle x\rangle_g(\mu)} = \frac{x^A(1-x)^C}{B(A+1,C+1)},$$





[Fan at el, 2104.06372](https://arxiv.org/abs/2104.06372)

- RITD → EITD → Gluon PDF
- Assuming a function form to fit the EITD instead of a direct Fourier transformation

Fit form comparison

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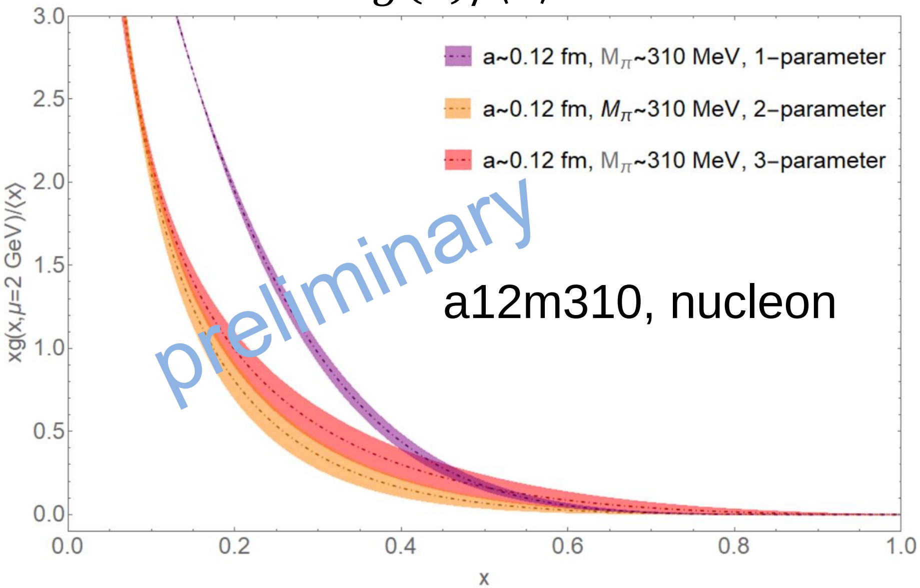
The 1-, 2-, 3-parameter fit forms,

$$xg(x) \sim (1-x)^C$$
$$\sim x^A(1-x)^C$$
$$\sim x^A(1-x)^C(1+D\sqrt{x})$$

$xg(x)/\langle x\rangle$



$xg(x)/\langle x\rangle$

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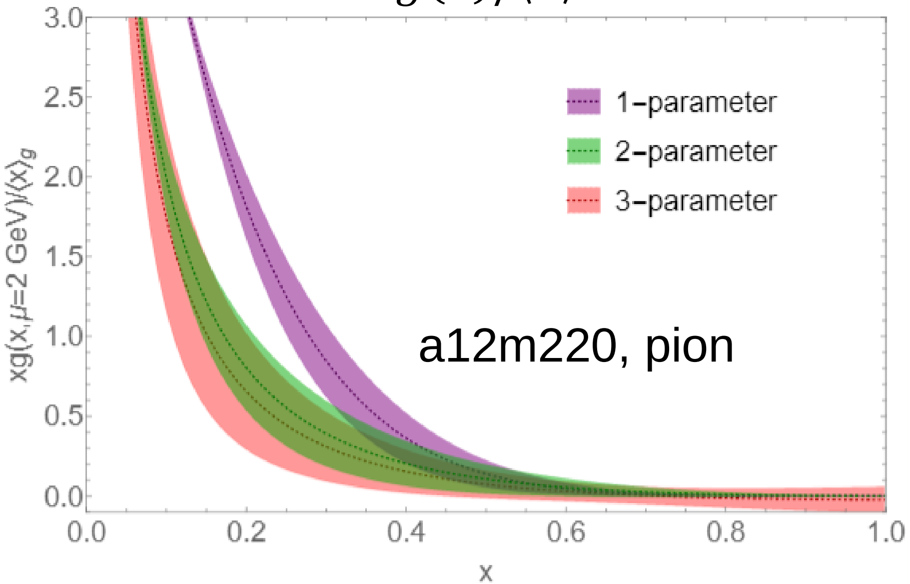


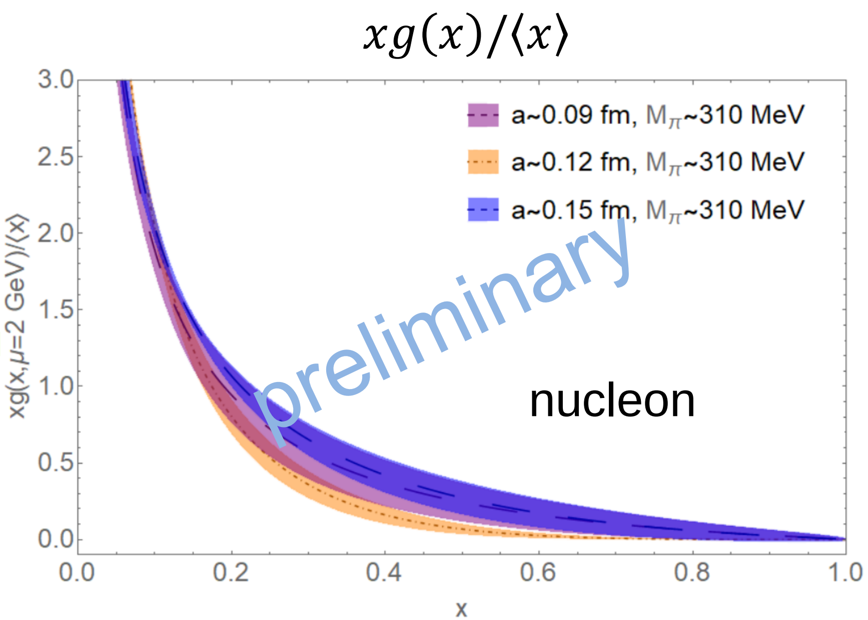
- We conclude that 1-parameter fit on lattice data here is not quite reliable, and the fit results converge at the 2- and 3-parameter fits

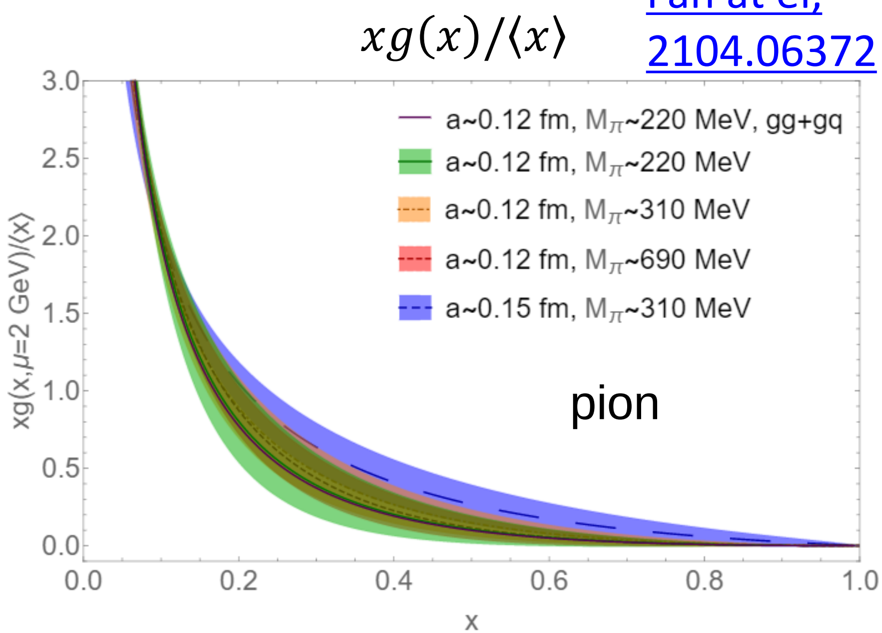
Comparing the different ensembles results

Nucleon $xg(x)/\langle x\rangle$ from different ensembles with 3 different lattice spacings

Pion $xg(x)/\langle x\rangle$ from different ensembles with 2 different lattice spacings and 3 pion masses

Fan at el, 2104.06372





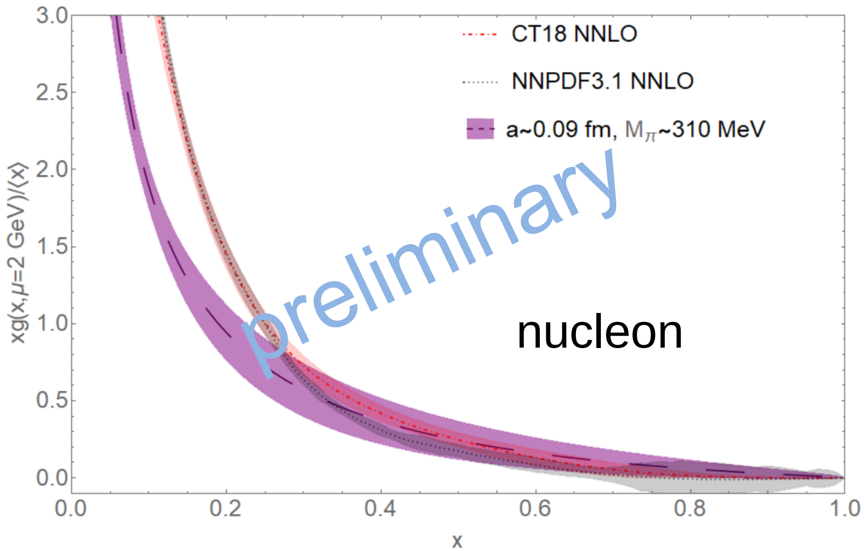
- The different lattice spacings and pion masses $xg(x)/\langle x\rangle$ results are consistent with each other within one sigma error

Comparing with the global fit results

Nucleon $xg(x)/\langle x\rangle$ comparing with the CT18 and NNPDF3.1 NNLO unpolarized gluon PDFs.

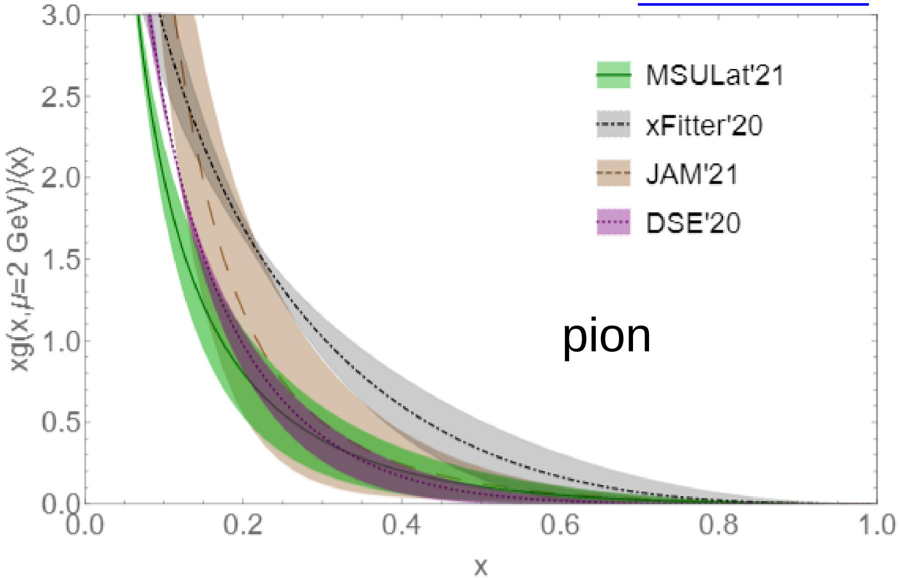
Pion $xg(x)/\langle x\rangle$ comparing with the DSE, JAM and xFitter pion gluon PDFs.

$xg(x)/\langle x\rangle$



$xg(x)/\langle x\rangle$

Fan at el, 2104.06372



- Our smallest lattice spacing a09m310 PDF result is consistent with global fit PDFs at x>0.3 region
- Our smallest pion mass a12m220 PDF result is consistent with global fit PDFs at x>0.2 region

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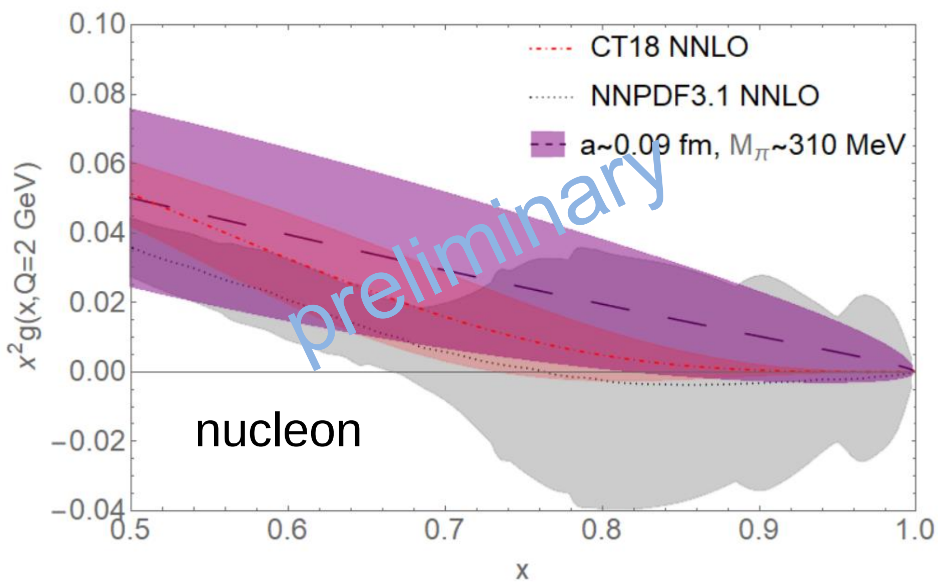
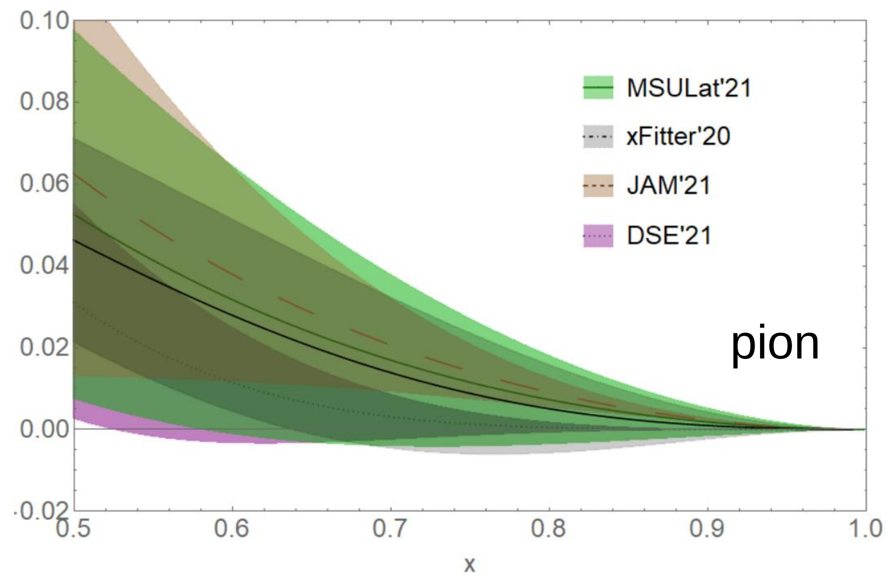
$x^2g(x)/\langle x\rangle$



- Consistent but slightly larger than the global fit PDFs

$x^2g(x)/\langle x\rangle$



- Despite the differences in the fit form, the large-x behaviors are quite consistent

Conclusion and outlook

- We extract the pion and nucleon x-dependent gluon PDF.
- The pion mass and lattice spacing dependents are weak under the current statistics.
- There are systematics yet to be studied for nucleon gluon PDF (quark contribution, finite ν in EITDs)