Strange and charm contributions to nucleon charges and moments

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Nucleon charges and moments

Nucleon charges and moments provide important information for nucleon structure.

- scalar charge: dark matter x-section, nucleon σ term
- axial charge: proton spin contribution from quarks spins, spin-dependent dark matter search.
- tensor charge: quark EDM contribution to nEDM, probes novel tensor interactions
- unpolarized first moment: quark momentum fraction

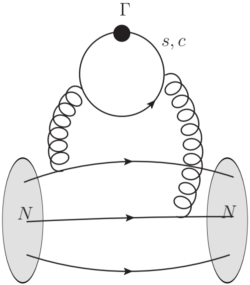
High precision determination of the scalar charges and tensor charges are important in BSM search. As the precision of light quark results improves, the contribution from strange and charm quarks also need to be precisely determined.

Operators used in the calculation

nucleon charges can be calculated on lattice from the 3pt correlators with a quark bilinear operator insertion:

- scalar charge: $g_S = \langle N|\bar{q}q|N\rangle$
- axial charge: $g_A = \langle N|\bar{q}\gamma_5\gamma_i q|N\rangle$
- tensor charge: $g_T = \langle N|\bar{q}\sigma_{ij}q|N\rangle$
- unpolarized first moment:
  $\langle x\rangle_q = \frac{1}{M_N}\langle N|O^q_{44}|N\rangle = \frac{1}{M_N}\langle N|\bar{q}(D_4\gamma_4 - \frac{1}{3}D_i\gamma_i)q|N\rangle$

for strange ($q = s$) and charm ($q = c$), there is only disconnected contraction. More details in [Bhattacharya, et al. PRD.2015]



Correlators from loop calculations

- 2pt correlators: $C_{\text{2pt}}^{\mathcal{P}}(\tau) = \langle 0|\mathcal{P}\chi(\tau)\chi(0)|0\rangle$
  - $\mathcal{P}^{\text{unp}} = (1+\gamma_4)/2$ for unpolarized projection
  - $\mathcal{P}^{\text{pol}} = (1+\gamma_4)(1+i\gamma_5\gamma_i)/2$ for polarized projection
- loop correlators: $C_{\text{loop}}^{O} = \sum_n \text{Tr}\left[\hat{O}S(n)\right]$
- Spin projectors for 3pt correlators:
  $\mathcal{P}^{unp}$ for g_S and O_{44}^{q}, $\mathcal{P}^{pol}$ for g_A and g_T

$$C_{\text{3pt}}^{\text{O}}(t,\tau) = \left\langle \left(C_{\text{2pt}}^{\mathcal{P}}(\tau) - \langle C_{\text{2pt}}^{\mathcal{P}}(\tau)\rangle \right) \left(C_{\text{loop}}^{O}(t) - \langle C_{\text{loop}}^{O}(t)\rangle \right) \right\rangle$$

- Data displayed using the ratio of 3pt to 2pt: $R(t,\tau) = \frac{C_{\text{3pt}}^{O}(t,\tau)}{C_{\text{2pt}}^{\mathcal{P}^{unp}}(\tau)}$

Lattice setup

- MILC 2+1+1-flavor lattices [Bazavov, et al., PRD.2013]
(We thank the MILC collaboration for sharing their lattices)
- HYP smearing
- clover-on-HISQ action
- strange mass are tuned to physical value using M_{η_s}
- charm mass are tuned to physical value using $\frac{M_{\eta_c}+3M_{J/\psi}}{4M_{\Omega}}$
- more data-taking is on-going (aiming at 1000 cfgs)

a (fm)	M_π (MeV)	$L^3 \times T$	N^c_{conf}	N^s_{conf}	N^c_{src}	N^s_{src}	$\frac{N^{\text{2pt}}_{\text{src}}}{\text{cfg}}$
0.06	320(2)	$48^3 \times 144$	469	87	4000	4000	32
0.09	313(3)	$32^3 \times 96$	633	889	6000	6000	32
0.09	138(1)	$64^3 \times 96$	721	310	4000	4000	32
0.12	310(3)	$24^3 \times 64$	983	897	4000	8000	32
0.12	228(2)	$32^3 \times 64$	902	869	5000	5000	32
0.15	321(4)	$16^3 \times 48$	n/a	1795	n/a	2000	32

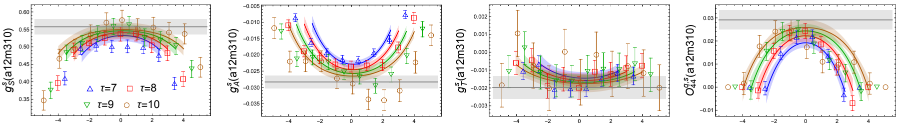
2pt and 3pt fits and ratio plots

2-state fit of the correlators:

$$C_{2\text{pt}}^{\mathcal{P}^{unp}}(\tau) = |A_0|^2 e^{-E_0\tau} + |A_1|^2 e^{-E_1\tau}$$

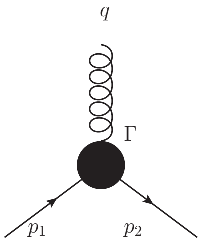
$$C_{3\text{pt}}^{O}(t,\tau) = \Big(|A_0|^2\langle 0|O|0\rangle e^{-E_0 t} e^{-E_0(\tau-t)} + A_0 A_1 \langle 1|O|0\rangle e^{-E_0 t} e^{-E_1(\tau-t)} + A_1 A_0 \langle 0|O|1\rangle e^{-E_1 t} e^{-E_0(\tau-t)} + |A_1|^2\langle 1|O|1\rangle e^{-E_1 t} e^{-E_1(\tau-t)}\Big)$$

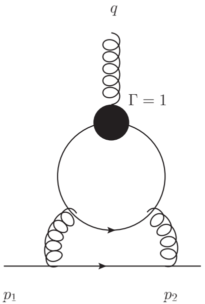
Ratio plots and the fits for charges and moments on a12m310 lattice:



Renormalization of operators

- non-perturbative renormalization factors Z_Γ
- RI-sMOM scheme ($p_1^2 = p_2^2 = (p_2 - p_1)^2$)
- 2-loop matching to $\overline{MS}$ scheme
- 3-loop running to $\mu = 2$ GeV
- remove μ-dependence by fitting to
  $Z_\Gamma(\mu) = Z_\Gamma + c_1\mu^2 + c_2\mu^4$
- mixing for iso-scalar and strange scalar charge (charm on-going)
- mixing for O_{44}^s is not included yet



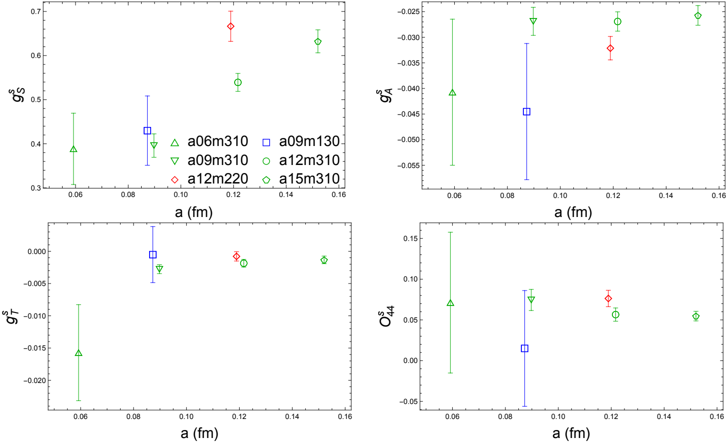


Mixing in scalar charge NPR

- connected contribution: $c_{ff'} \equiv \frac{\delta_{ff'}}{12Z_\psi^f} \, \mathrm{Tr}\left[\langle f|O^{f'}|f\rangle_{conn}\right]$
- disconnected contribution: $d_{ff'} \equiv \frac{-1}{12Z_\psi^f} \, \mathrm{Tr}\left[\langle f|O^{f'}|f\rangle_{disc}\right]$
- renormalization with mixing: $\begin{pmatrix} g_S^{R,u+d} \\ g_S^{R,s} \\ g_S^{R,c} \end{pmatrix} = Z^{RI-sMOM} \cdot \begin{pmatrix} g_S^{bare,u+d} \\ g_S^{bare,s} \\ g_S^{bare,c} \end{pmatrix}$
- Mixing matrix: $Z^{RI-sMOM} = \begin{pmatrix} c_l - 2d_{ll} & -2d_{sl} & -2d_{cl} \\ -d_{ls} & c_s - d_{ss} & -d_{cs} \\ -d_{lc} & -d_{sc} & c_c - d_{cc} \end{pmatrix}^{-1}$
- $d_{ff'}$ is sensitive to the quark loop f', but insensitive to the external quark f.

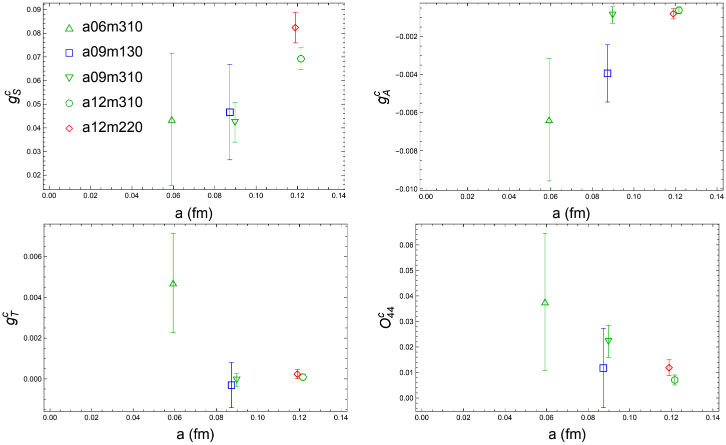
Strange renormalized charges and moments

Strange contributions calculated on different lattices:



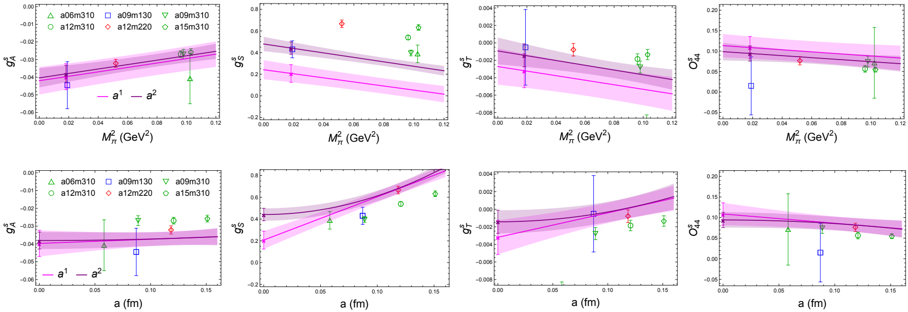
Charm renormalized charges and moments

Charm contributions from different lattices:



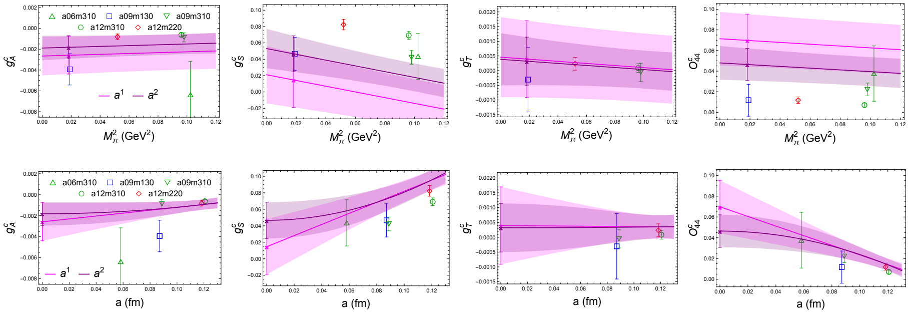
Extrapolate with: $g(M_\pi, a) = g^{\text{phy}}(1 + c_i a^i + d_2(M_\pi^2 - (M_\pi^{\text{phy}})^2)),\ i = 1, 2.$

Strange chiral and continuum extrapolation



- M_π dependence is non-negligible
- a dependence large in g_S. Linear and quadratic fits have close χ^2.
- a dependence in g_A, g_T, O_{44}^S is not clear
- a and a^2 fits are consistent within 1σ, except for g_S

Charm chiral and continuum extrapolation fit



- M_π dependence is not significant
- a and a^2 extrapolations are consistent within 1σ
- scalar charge reduces under continuum extrapolation
- momentum fraction becomes larger under continuum extrapolation

Comparison with previous lattice results

strange	$g_S^{s,R}$	$g_A^{s,R}$	$g_T^{s,R}$	$\langle x \rangle_s$
This work (preliminary)	$0.13 \sim 0.50$	$-0.0397(73)$	$-0.0032(19)$	0.109(27)
PNDME'18		$-0.053(8)$	$-0.0027(16)$	
⋆ETMC'19 Nf=4	0.454(16)	$-0.0458(73)$	$-0.00268(58)$	0.052(12)
⋆ETMC'19 Nf=2	0.371(38)	$-0.061(17)$	$-0.0041(12)$	
χQCD'18		$-0.035(9)$		0.051(26)
⋆JLQCD'18	0.15(16)	$-0.046(28)$	$-0.012(18)$	

charm	$g_S^{c,R}$	$g_A^{c,R}$	$g_T^{c,R}$	$\langle x \rangle_c$
This work (preliminary)	0.015(34)	$-0.0026(18)$	0.0004(13)	0.070(25)
⋆ETMC'19 Nf=4	0.075(17)	$-0.0098(34)$	$-0.00024(16)$	0.019(9)
⋆ETMC'19 Nf=2	0.059(18)	$-0.0065(51)$	$-0.0060(37)$	

⋆ from single lattice spacing

- The axial and tensor charges are consistent with previous results
- Our momentum fraction (without mixing) is much larger

Summary and outlook

Summary:

- We present the calculation of strange and charm contribution to nucleon charges and momentum fraction
- The extrapolation of scalar charge need further study and increased statistics on finer lattices.
- Our results on axial charge and tensor charge are consistent with previous lattice results.
- The calculation of momentum fraction needs more data and evaluation of mixing with gluon and other flavors.

Outlook:

- More measurements on fine lattices and light pion masses to better constrain the extrapolation.
- Include charm in scalar charge mixing.
- NPR mixing with gluons and quarks in the momentum fraction.