Double parton distributions in the nucleon from lattice simulations

July 28, 2021

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The 38th International Symposium on Lattice Field Theory

LATTICE 21

JULY 26-30 2021, ZOOM/GATHER@MIT

RQCD

SFB TR55

DESY

Introduction

Motivation:

- Double parton scattering processes (DPS), e.g. Double Drell-Yan (DDY), are important standard model contributions in LHC experiments, especially after high-luminosity upgrade
- Approximation via Pocket formula:

$$\sigma_{\mathrm{DPS},i_1 i_2,j_1 j_2} \approx \frac{1}{C} \frac{\sigma_{\mathrm{SPS},i_1 j_1} \sigma_{\mathrm{SPS},i_2 j_2}}{\sigma_{\mathrm{eff}}}$$

- More fundamental description by double parton distributions (DPDs) :

$$\frac{\mathrm{d}\sigma_{\mathrm{DPS},i_1 i_2,j_1 j_2}}{\mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_1' \mathrm{d}x_2'} \propto \int \mathrm{d}^2\boldsymbol{y}\; F_{i_1 i_2}(x_1, x_2, \boldsymbol{y}) F_{j_1 j_2}(x_1', x_2', \boldsymbol{y})$$

- So far, DPDs unknown from experiments, non-perturbative objects, access via lattice simulations
- Results for the pion *[arXiv:1807.03073]*, *[arXiv:2006.14826]*

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This talk: Mellin Moments of DPDs for the nucleon from Lattice QCD

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Points of interest:

- Size of DPDs and polarization effects
- strength of parton-parton correlations
- Validity of factorization assumptions

Double Parton Distributions

- Light cone coordinates for a given 4-vector x^μ: $x^\pm = (x^0 \pm x^3)/\sqrt{2}$, $\boldsymbol{x} = (x^1, x^2)$
- Consider a proton rapidly moving in 3-direction, i.e. $p^+ \sim Q \gg \Lambda \sim m$, $\boldsymbol{p} = \mathbf{0}$, $p^- \sim \Lambda^2/Q$

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Definition of proton DPDs for quarks *[arXiv:1111.0910]*

$$F_{ab}(x_1, x_2, \boldsymbol{y}) := 2p^+ \int \mathrm{d}y^- \left[\prod_{j=1,2} \int \frac{\mathrm{d}z_j^-}{2\pi} e^{i x_j p^+ z_j^-} \right] \times$$
$$\times \tfrac{1}{2} \textstyle\sum_\lambda \langle p, \lambda | \, \mathcal{O}_a(y, z_1^-) \mathcal{O}_b(0, z_2^-) \, | p, \lambda \rangle \big|_{y^+=0}$$

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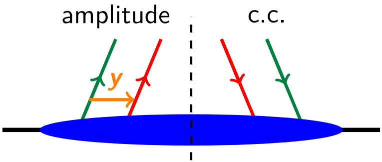

Joint probability to find quark a with momentum $x_1 p^+$ and quark b with momentum $x_2 p^+$ at transverse distance $\boldsymbol{y}$ ($|x_1| + |x_2| \leq 1$)

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Light cone operators

$$\mathcal{O}_a(y, z^-) = \bar{q}(y - \tfrac{z}{2}) \Gamma_a q(y + \tfrac{z}{2}) \Big|_{\boldsymbol{z}=\boldsymbol{0}, z^+=0}$$

- $\bar{q}$, q quark operators for certain flavor (light-like distance z^-)
- Γ_a quark polarization

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Twist-2 components: Quark polarizations

operators	twist-2 comp.	polarization
$V_q^\mu = \bar{q}\gamma^\mu q$	$V_q^+ = \mathcal{O}_q$	$q : q^\uparrow + q^\downarrow$ (unpolarized)
$A_q^\mu = \bar{q}\gamma^\mu\gamma_5 q$	$A_q^+ = \mathcal{O}_{\Delta q}$	$\Delta q : q^\uparrow - q^\downarrow$ (longitudinal)
$T_q^{\mu\nu} = \bar{q}i\sigma^{\mu\nu}\gamma_5 q$	$T_q^{+j} = \mathcal{O}_{\delta q}^j$	$\delta q^j : q^{\uparrow,j} - q^{\downarrow,j}$ (transverse)

Double Parton Distributions

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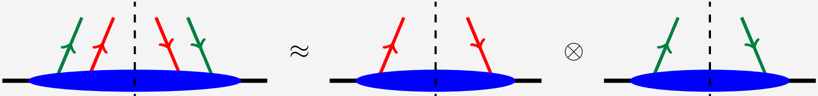
Factorization assumption I

$$\langle p | \mathcal{O}_a(y, z_1) \mathcal{O}_b(0, z_2) | p \rangle \approx \int \frac{\mathrm{d}^2 \boldsymbol{p}' \mathrm{d}p'^+}{(2\pi)^3 2p'^+} \langle p | \mathcal{O}_a(y, z_1) | p' \rangle \langle p' | \mathcal{O}_b(0, z_2) | p \rangle$$

$$\Rightarrow \quad F_{ab}(x_1, x_2, \boldsymbol{y}) \approx \int \mathrm{d}^2 \boldsymbol{B} \; f_a(x_1, \boldsymbol{B} + \boldsymbol{y}) \; f_b(x_2, \boldsymbol{B})$$



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Factorization assumption II

For the pocket formula *[arXiv:1111.0469]*:

$$F_{ab}(x_1, x_2, \boldsymbol{y}) \approx f_a(x_1) f_b(x_2) T(\boldsymbol{y})$$

with **unique** $T(\boldsymbol{y})$

Double parton distributions on the lattice

Accessible quantities

$$F_{ab}(x_i, \boldsymbol{y})$$

$$\Big\uparrow\ p^+ \int \mathrm{d}y^- \,\mathrm{d}z_i^- \, e^{-iz_i x_i p^+} \quad (y^+ = 0,\ \text{twist-2})$$

$$\langle p|\, \mathcal{O}_a(y, z_1^-) \mathcal{O}_b(0, z_2^-)\, |p\rangle$$

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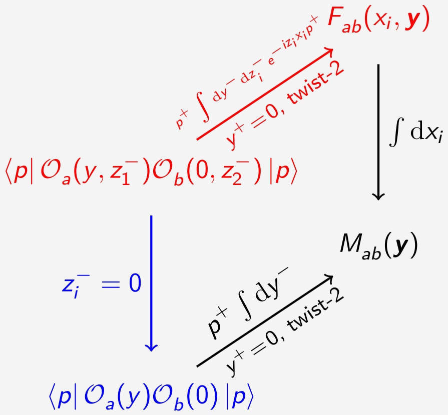
$y^+ = 0$, twist-2

$\langle p | \, \mathcal{O}_a(y, z_1^-) \mathcal{O}_b(0, z_2^-) \, | p \rangle$

not accessible on the lattice
if $z_i^- > 0$

Accessible quantities



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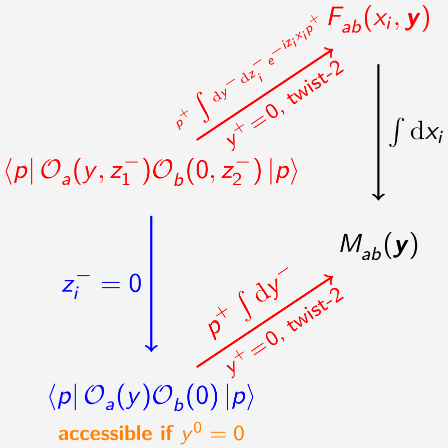

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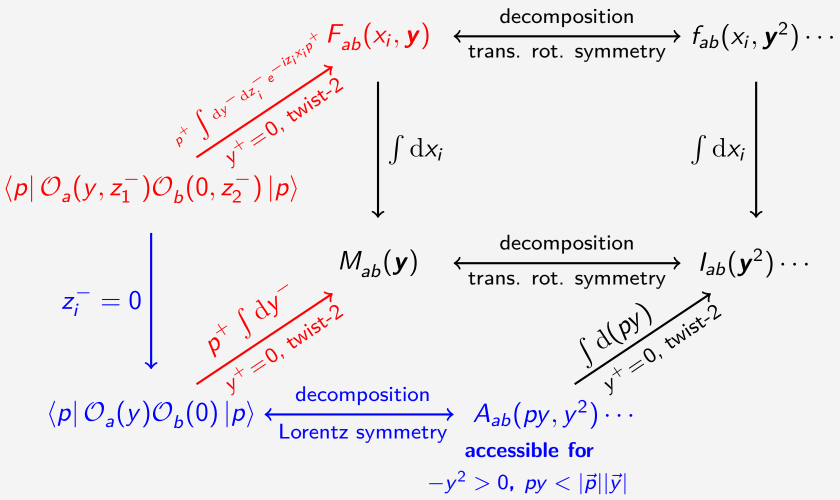

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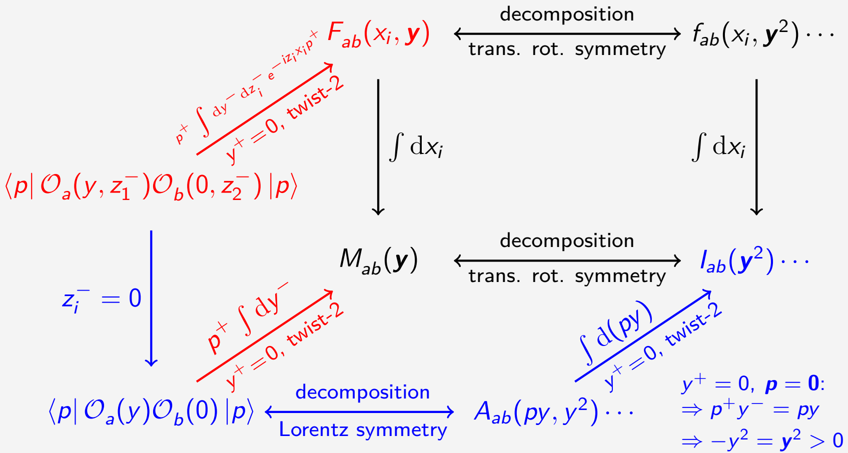
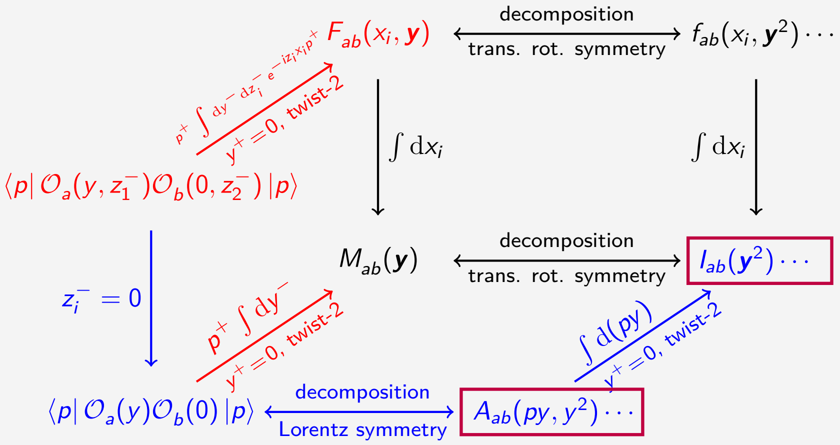


$py < |\vec{p}||\vec{y}|$:

Have to extrapolate in py

Accessible quantities



Results for these quantities

Two-current matrix elements on the lattice

In Euclidean spacetime:

Access via 4-point functions

$$\frac{1}{2}\sum_{\lambda} \langle p,\lambda| \mathcal{O}_{a_1}(y)\mathcal{O}_{a_2}(0) |p,\lambda\rangle \,\Big|_{y^0=0} = 2V\sqrt{m^2+\vec{p}^2}\; \frac{C_{\mathrm{4pt}}^{\vec{p},ij}(t,\tau,\vec{y})}{C_{\mathrm{2pt}}^{\vec{p}}(t)}\Bigg|_{0\ll\tau\ll t}$$

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with 4-point / 2-point function ($P_+ = \frac{1}{2}(\mathbb{1}+\gamma_4)$):

$$C_{\mathrm{4pt}}^{\vec{p},ij}(t,\tau,\vec{y}) = \sum_{\vec{x}} e^{-i\vec{p}(\vec{z}'-\vec{z})} \left\langle \mathrm{tr}\left\{ P_+ \mathcal{P}^{\vec{p}}(\vec{z}',t)\; \mathcal{O}_i^{q_1 q_2}(\vec{0},\tau)\; \mathcal{O}_j^{q_3 q_4}(\vec{y},\tau)\; \overline{\mathcal{P}}^{\vec{p}}(\vec{z},0)\right\}\right\rangle$$

$$C_{\mathrm{2pt}}^{\vec{p}}(t) = \sum_{\vec{x}} e^{-i\vec{p}(\vec{z}'-\vec{z})} \left\langle \mathrm{tr}\left\{ P_+ \mathcal{P}^{\vec{p}}(\vec{z}',t)\; \overline{\mathcal{P}}^{\vec{p}}(\vec{z},0)\right\}\right\rangle$$

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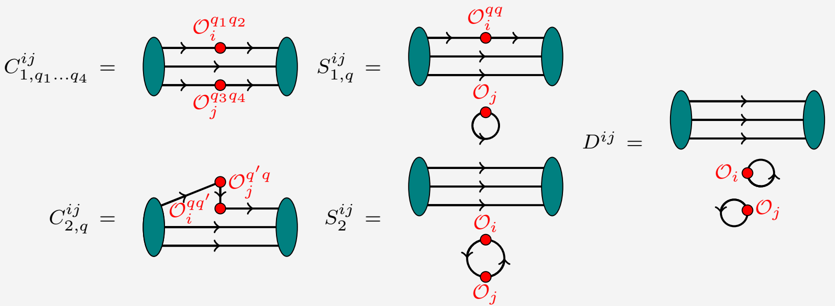
and Proton interpolators:

$$\mathcal{P}^{\vec{p}}(\vec{x},t) = \epsilon_{abc}u_a(x)\left[u_b^T(x)C\gamma_5 d_c(x)\right]\Big|_{x^4=t}$$

$$\overline{\mathcal{P}}^{\vec{P}}(\vec{x},t) = \epsilon_{abc}\;\left[\bar{u}_a(x)C\gamma_5\bar{d}_b^T(x)\right]\bar{u}_c(x)\Big|_{x^4=t}$$

Two-current matrix elements on the lattice

Wick contractions



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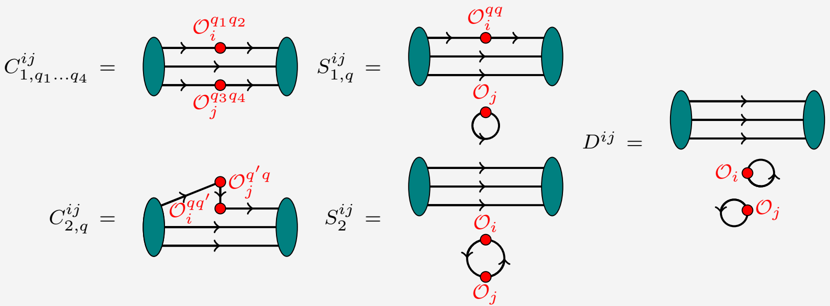

Physical matrix elements

$$\langle p|\, \mathcal{O}_i^{uu}(\vec{0})\mathcal{O}_j^{dd}(\vec{y})\,|p\rangle = C^{ij,\vec{p}}_{1,uudd}(\vec{y}) + S^{ij,\vec{p}}_{1,u}(\vec{y}) + S^{ji,\vec{p}}_{1,d}(-\vec{y}) + D^{ij,\vec{p}}(\vec{y})$$

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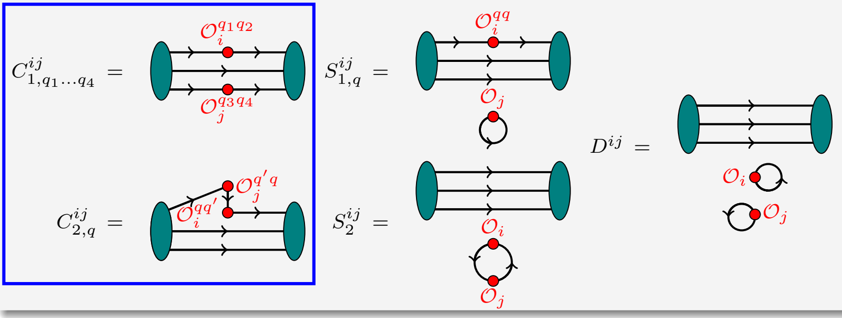

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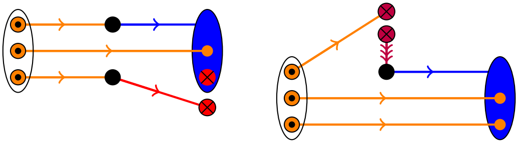

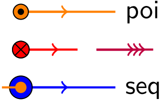

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- Boosted sources (momentum smearing) *[arXiv:1602.05525]*
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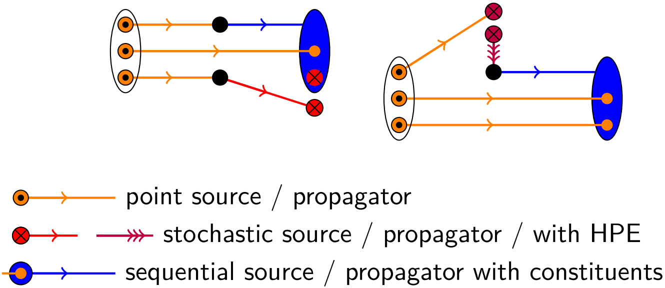
Technical Details



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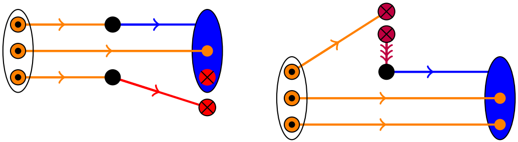

point source / propagator

stochastic source / propagator / with HPE

sequential source / propagator with constituents

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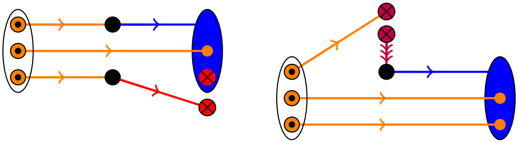

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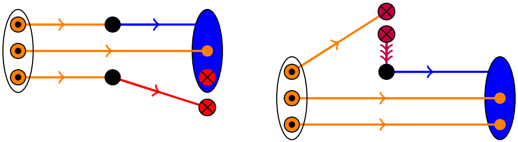

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Lattice Setup

CLS ensembles ($n_f = 2+1$, Wilson fermions, order-a improved *[arXiv:1411.3982]*), start with H102, 990 configs used:

id	β	a[fm]	$L^3 \times T$	$\kappa_{l/s}$	$m_{\pi/K}$[MeV]	$m_\pi L$	conf.
H102	3.4	0.0856	$32^3 \times 96$	0.136865	355	4.9	2037
				0.136549339	441		

- $t_{\rm src} = 48a$ (point sources at random spatial position)
- $t = t_{\rm snk} - t_{\rm src} = \begin{cases} 12a & \vec{p} = \vec{0} \\ 10a & \vec{p} \neq \vec{0} \end{cases}$
- Insertion time $\tau \in [t_{\rm src} + 3a, t_{\rm snk} - 3a]$ for C_1 (fit), else fix $\tau = t_{\rm src} + t/2$
- 6 Momenta up to $|\vec{p}| = \sqrt{12}\frac{2\pi}{La} \approx 1.57$ GeV

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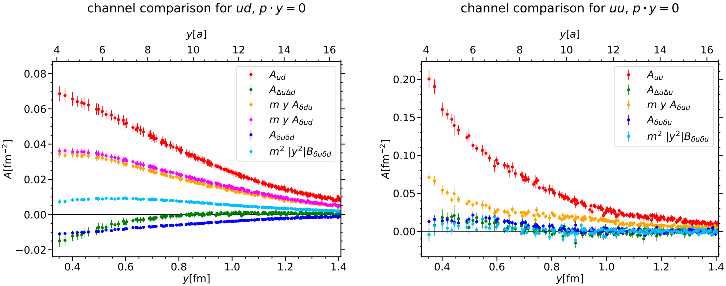
Renormalization for $\beta = 3.4$, including conversion to $\overline{\rm MS}$ at $\mu = 2$GeV *[arXiv:2012.06284]*:

	V	A	T
Z	0.7128	0.7525	0.8335

Results: Polarization dependence

Invariant functions $A(py = 0, y^2)$, connected graphs only (notation $y = \sqrt{-y^2}$, $y^2 = y^\mu y_\mu$):



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- *ud*: Clear contributions from all polarized channels (large for δud, δdu)
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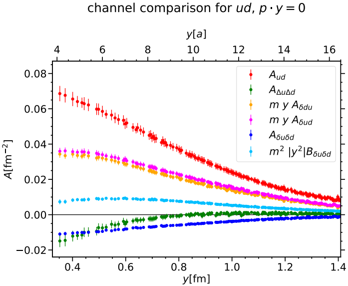

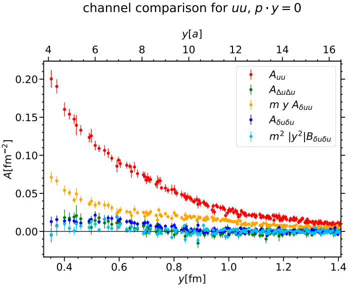

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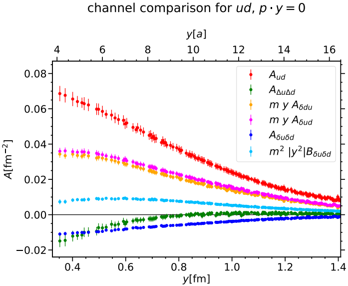

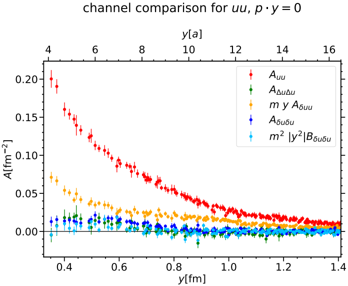

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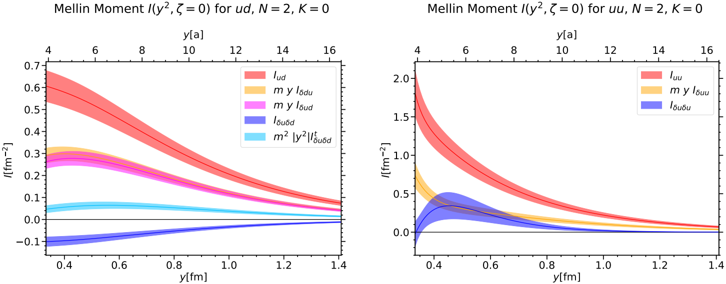
DPD moments $I(\boldsymbol{y}^2)$ (notation $y = |\boldsymbol{y}|$):



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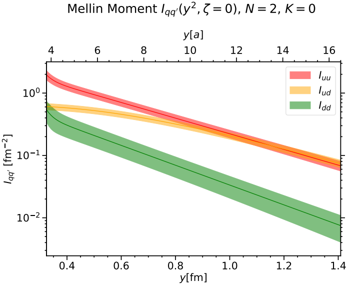
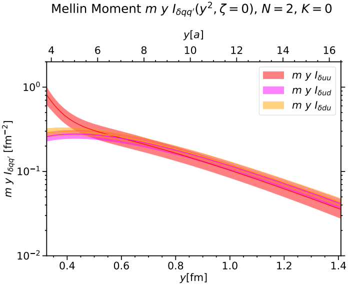
Results: Flavor dependence





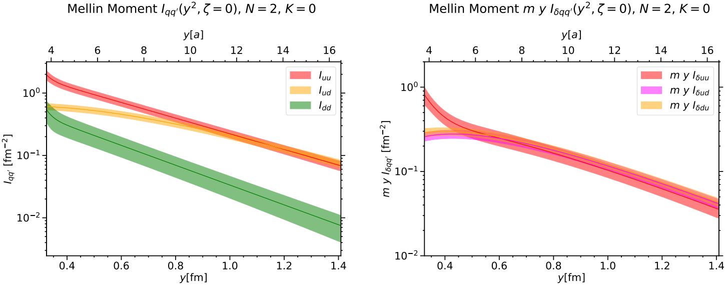
- Clear flavor dependence observable
- Reminder: Assumption for the pocket formula:

$$F_{ab}(x_1, x_2, \boldsymbol{y}) = f_a(x_1) f_b(x_2) T(\boldsymbol{y}) \quad \Rightarrow \quad I_{ab}(\boldsymbol{y}^2) = C_{ab} T(\boldsymbol{y}^2)$$

with **unique** $T(\boldsymbol{y})$

- **Clearly not fulfilled**

Results: Flavor dependence



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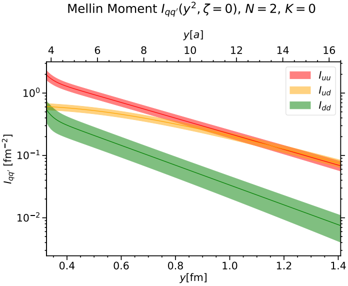

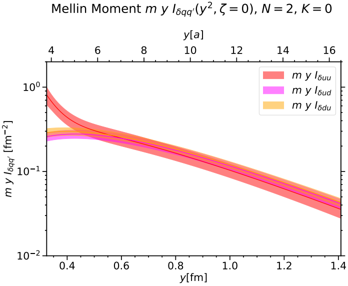

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Factorization tests

Factorization in terms of impact parameter distributions $f_q(x, \boldsymbol{b})$:

$$F_{qq'}(x_1, x_2, \boldsymbol{y}) \approx \int \mathrm{d}^2\boldsymbol{b}\; f_q(x_1, \boldsymbol{b} + \boldsymbol{y})\; f_{q'}(x_2, \boldsymbol{b})$$

Factorization tests

For the Mellin moments

$$I_{qq'}(\boldsymbol{y}) \approx \int \frac{\mathrm{d}r}{2\pi}\; rJ_0(ry)\left[F_1^q(-\boldsymbol{r}^2)F_1^{q'}(-\boldsymbol{r}^2) + \frac{\boldsymbol{r}^2}{4m^2}F_2^q(-\boldsymbol{r}^2)F_2^{q'}(-\boldsymbol{r}^2)\right]$$

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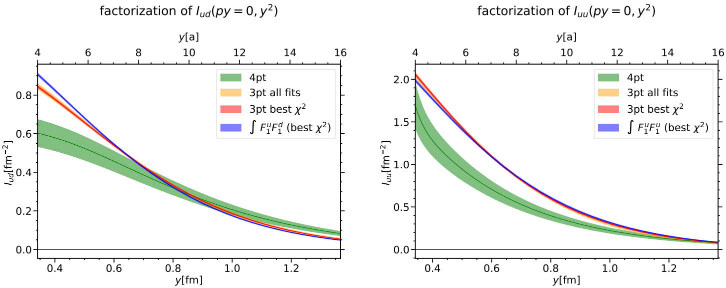

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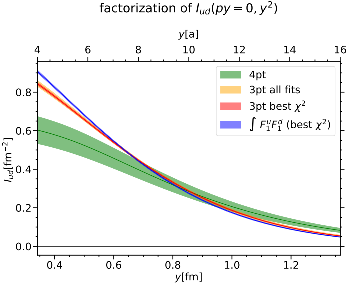

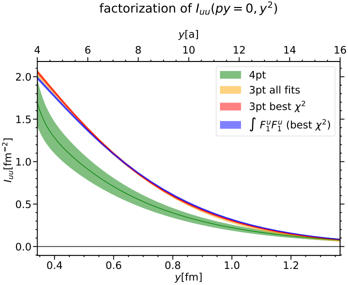

Comparable size but deviations are visible

Thank you for your attention!