Proton momentum and angular momentum decomposition with overlap fermions

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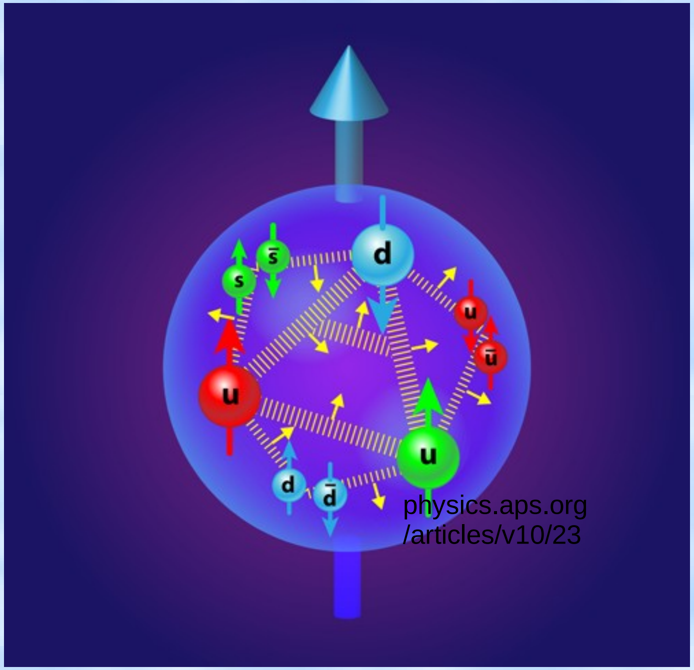
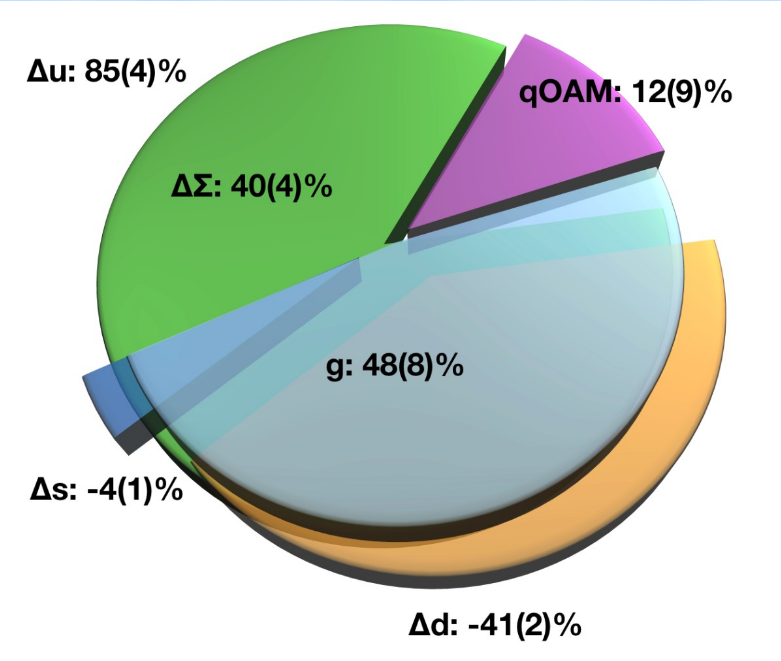
χQCD Collaboration

Motivation

Only 30% of the proton's spin is carried by the spin of quark constituents [1]





[1] C. A. Aidala, S. D. Bass, D. Hasch, and G. K. Mallot, "The Spin Structure of the Nucleon," Rev. Mod. Phys. 85, 655 (2013)
[2] J. Liang, Y.-B. Yang, T. Draper, M. Gong, and K.-F. Liu, Phys. Rev. D 98, 074505 (2018), arXiv:1806.08366
[3] Y.-B. Yang, J. Liang, Y.-J. Bi, Y. Chen, T. Draper, K.-F. Liu, and Z. Liu, Phys. Rev. Lett. 121, 212001 (2018), arXiv:1808.08677

Nucleon Energy-momentum tensor

Traceless, symmetric energy-momentum tensor (EMT) between two nucleon state to T1, T2 and T3 form factors

$$\langle p', s' | \mathcal{T}^{\{\mu\nu\}q,g} | p, s \rangle = \frac{1}{2}\bar{u}(p', s')\Big[T_1(q^2)(\gamma^\mu \bar{p}^\nu + \gamma^\nu \bar{p}^\mu) + \frac{1}{2m}T_2(q^2)\left(i(\bar{p}^\mu \sigma^{\nu\alpha} + \bar{p}^\nu(\sigma^{\mu\alpha}))\right) + \frac{1}{m}T_3(q^2)q^\mu q^\nu\Big]^{q,g} u(p, s)$$

T1 and T1+T2 form factors at zero momentum transfer give Momentum and angular momentum fractions

$$\mathcal{T}^{\{4i\}q,g} \quad \Longrightarrow \quad \langle x \rangle^{q,g} = T_1(0)^{q,g} \qquad \langle J \rangle^{q,g} = \frac{1}{2}\left[T_1(0) + T_2(0)\right]^{q,g}$$

X.-D. Ji, Phys. Rev. D 52, 271 (1995), arXiv:hep-ph/9502213

Mixing and Renormalization

T1, T2 and T3 form factors are renormalized by

$$T^{u,d}(\mathrm{CI})^R = Z_{QQ}^{\overline{\mathrm{MS}}}(\mu) T^{u,d}(\mathrm{CI}),$$

$$T^{u,d,s}(\mathrm{DI})^R = Z_{QQ}^{\overline{\mathrm{MS}}}(\mu) T^{u,d,s}(\mathrm{DI}) + \delta Z_{QQ}^{\overline{\mathrm{MS}}}(\mu) \sum_{q=u,d,s} [T^q(\mathrm{CI}) + T^q(\mathrm{DI})] + Z_{QG}^{\overline{\mathrm{MS}}}(\mu) T^g(\mathrm{DI}),$$

$$T^g(\mathrm{DI})^R = Z_{GQ}^{\overline{\mathrm{MS}}}(\mu) \sum_{q=u,d,s} [T^q(\mathrm{CI}) + T^q(\mathrm{DI})] + Z_{GG}^{\overline{\mathrm{MS}}} T^g(\mathrm{DI}),$$

Non-perturbative renormalization constants include mixing calculated in [1]

Lattice	Z_{QQ}	δZ_{QQ}	Z_{QG}	Z_{GQ}	Z_{GG}
32ID	1.25(0)(2)	0.018(2)(2)	0.017(17)	0.57(3)(6)	1.29(5)(9)

[1] Y-B Yang, J. Liang, et al., χQCD Collaboration, Phys. Rev. Lett. 121, 212001 (2018)

Normalization

With momentum and angular momentum conservation, the momentum and angular momentum sum rules are

$$\langle x\rangle^q + \langle x\rangle^g = T_1(0)^q + T_1(0)^g = 1$$

$$J^q + J^g = \frac{1}{2}\{[T_1(0)^q + T_2(0)^q] + [T_1(0) + T_2(0)]^g\} = \frac{1}{2}$$

We will use these two sum rules as the normalization conditions for the local current

$$N_q \langle x\rangle^q_R + N_g \langle x\rangle^g_R = 1$$

$$N_q J^q_R + N_g J^g_R = \frac{1}{2}$$

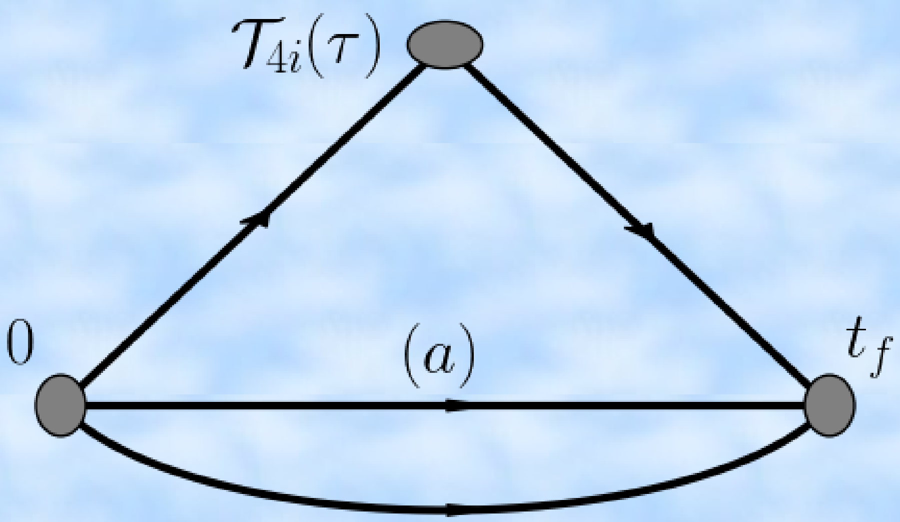
As T2 is too small and noisy

$$N_q = N_g = N$$

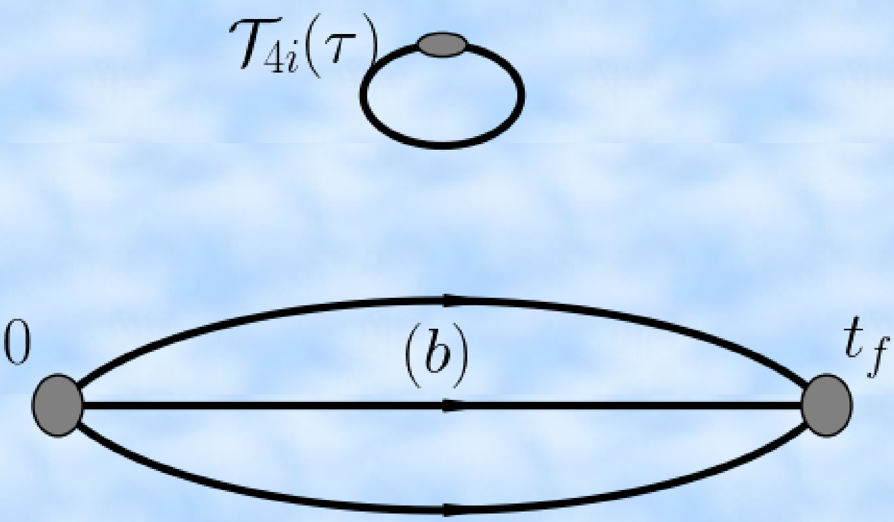
M. Deka, T. Doi, Y-B Yang, et. al., χQCD collaboration, PRD91, 014505 (2015)

Lattice Simulations

- Lattice
  - 32ID--Domain Wall 2+1 Lattice, $32^3 * 64$, a=0.143 fm, Pion 172 MeV
  - Overlap Fermions with six different valence quark masses



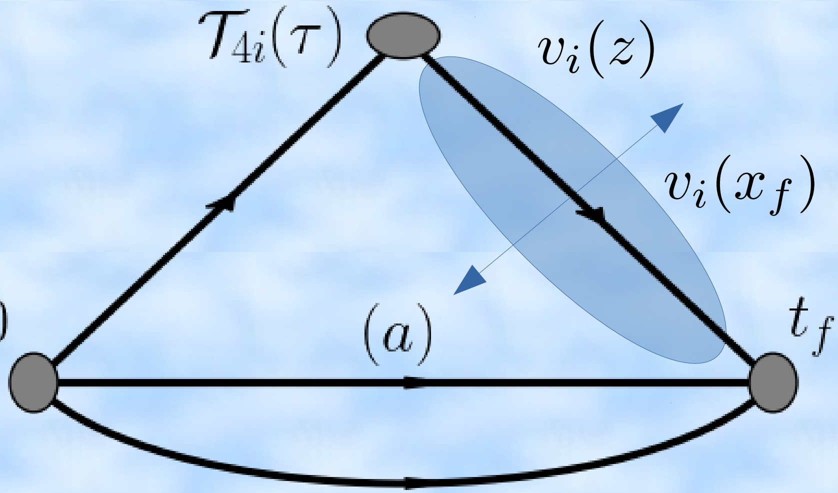
Connected insertions (CI)
for light quarks



Disconnected insertions (DI)
quarks and glue

CI 3pt with FFT

The stochastic-sandwich method with low mode substitution used to estimate the propagator from current to sink can be improved further with FFT



$$S(z|x_{\rm f}) = S^L(z|x_{\rm f}) + S^H(z|x_{\rm f}),$$

$$S^L(z|x_{\rm f}) = \sum_{\lambda_i \leq \lambda_c} \frac{1}{\lambda_i + m} v_i(z) v_i^\dagger(x_{\rm f})$$

The corresponding "low mode" part of the 3pt can be written as

$$C^{u/d,}_{\rm CI,3pt} = \langle \sum_{\lambda_i \leq \lambda_c} {\rm Tr}[\frac{1}{\lambda_i + m} G_i^L(\vec{q}, \tau) F_i^{L,u/d}(\vec{p}_f, t_f)] \rangle$$

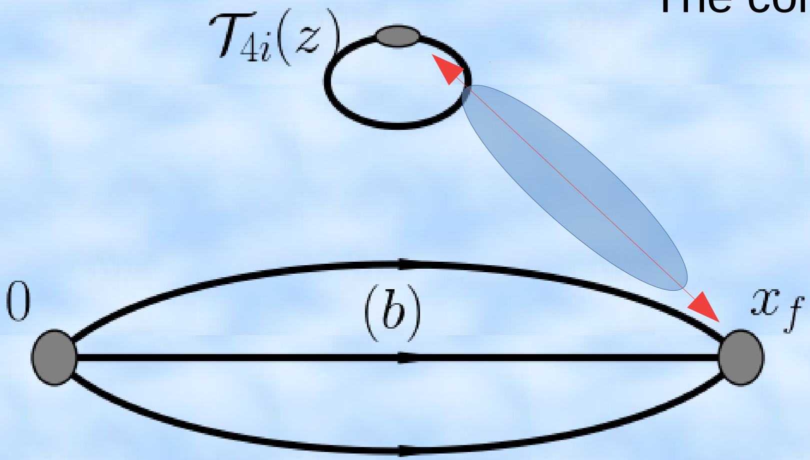
Similar separations could also be done with the stochastic "high mode" part

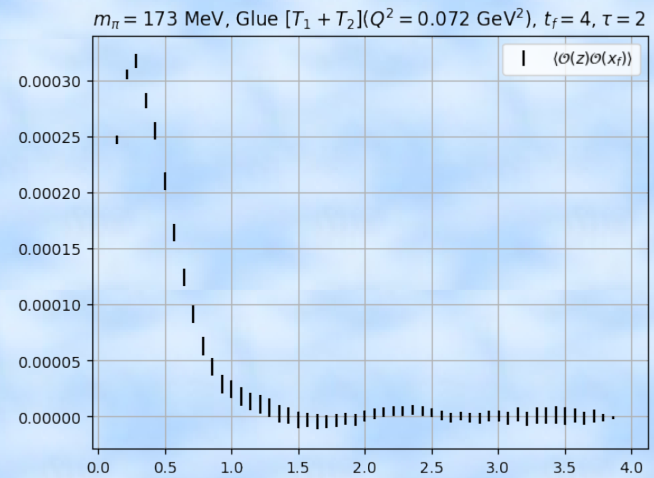
G. Wang, J. Liang, T. Draper, K.-F. Liu, and Y.-B. Yang (chiQCD), arXiv:2006.05431

DI parts

Cluster-decomposition error (CDER) [1] technique are used for DI parts

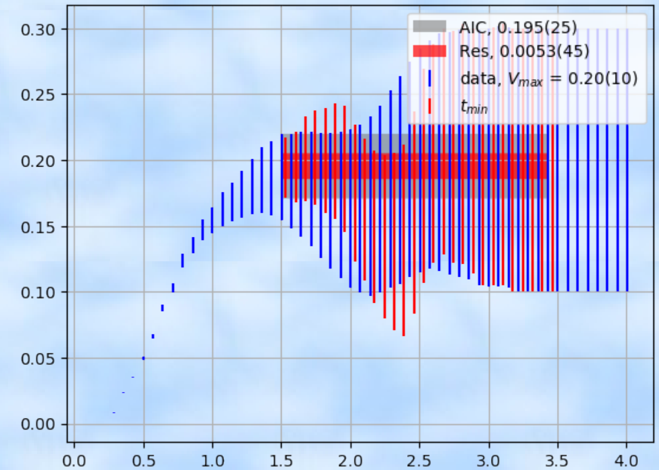
The correlations satisfy [2] $\langle \mathcal{O}(z)\mathcal{O}(x_f)\rangle_s \leq A r^{-\frac{2}{3}} e^{-Mr}$





Use Akaike information criterion (AIC) weight factors to average different fit results

$$AIC = exp\left[-\frac{1}{2}(\chi^2 - 2n_{\mathrm{dof}})\right]$$

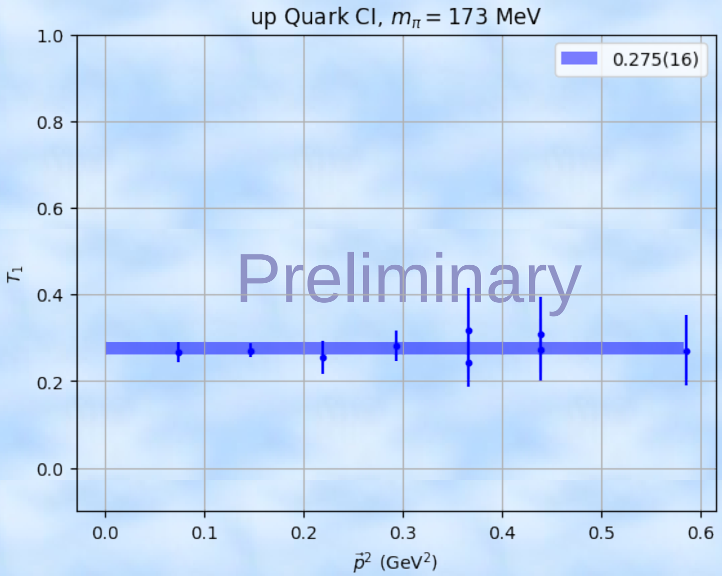


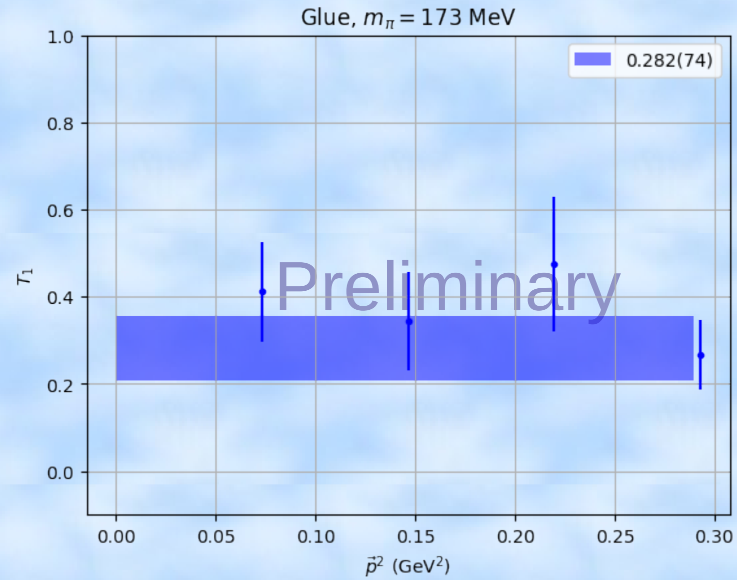
The residue of the correlators from 1.5 fm

$$Res = \sum_{r > r_{\mathrm{cut}}}^{r < r_{\mathrm{max}}} A r^{-\frac{3}{2}} e^{-Mr}$$

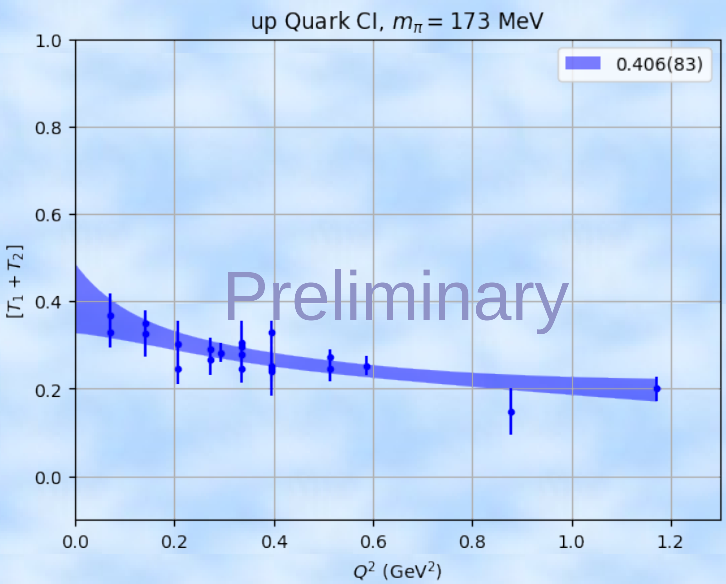
[1] K.-F. Liu, J. Liang, and Y.-B. Yang, Phys. Rev. D 97, 034507 (2018), arXiv:1705.06358
[2] H. Araki, K. Hepp, and D. Ruelle, Helv. Phys. Acta 35, 164 (1962)

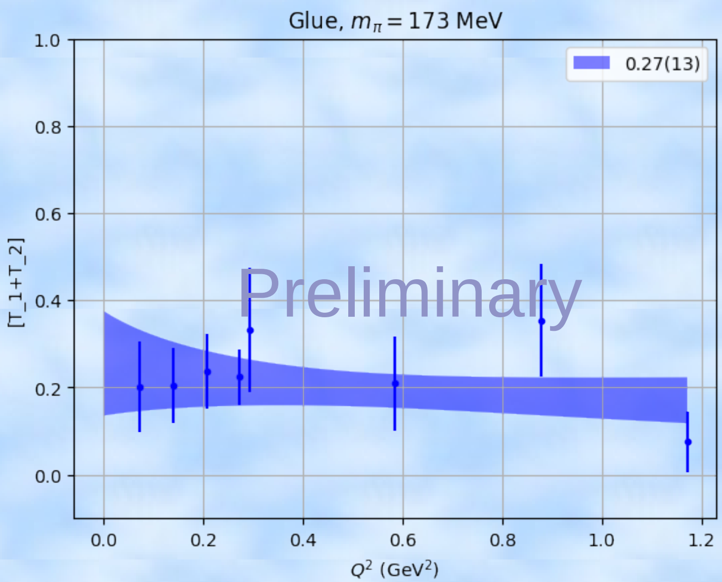
T1(0) and T1+T2 form factors





Averaged over results from different nucleon initial momenta for T1(0)

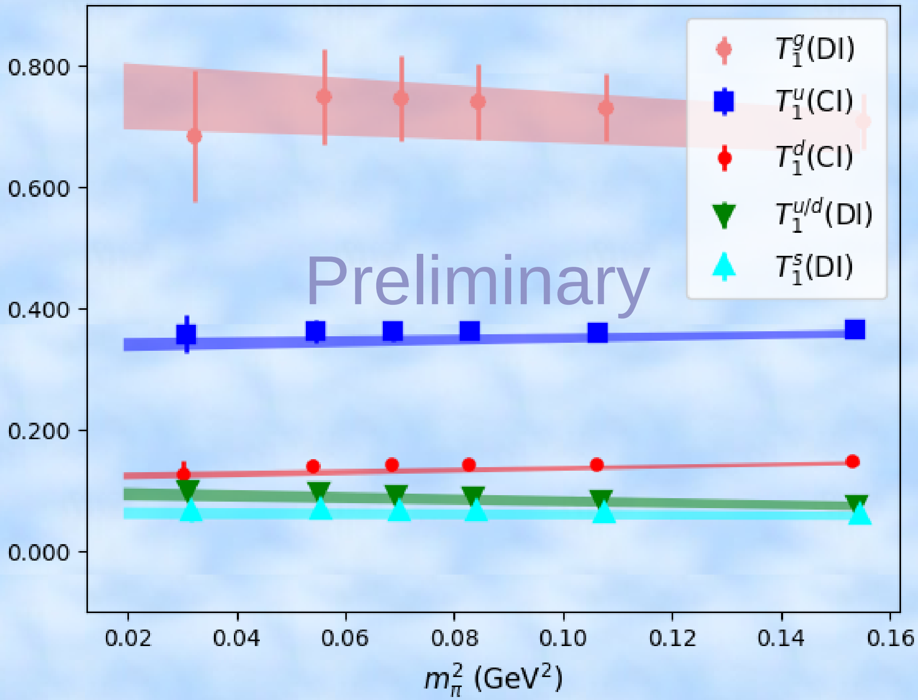


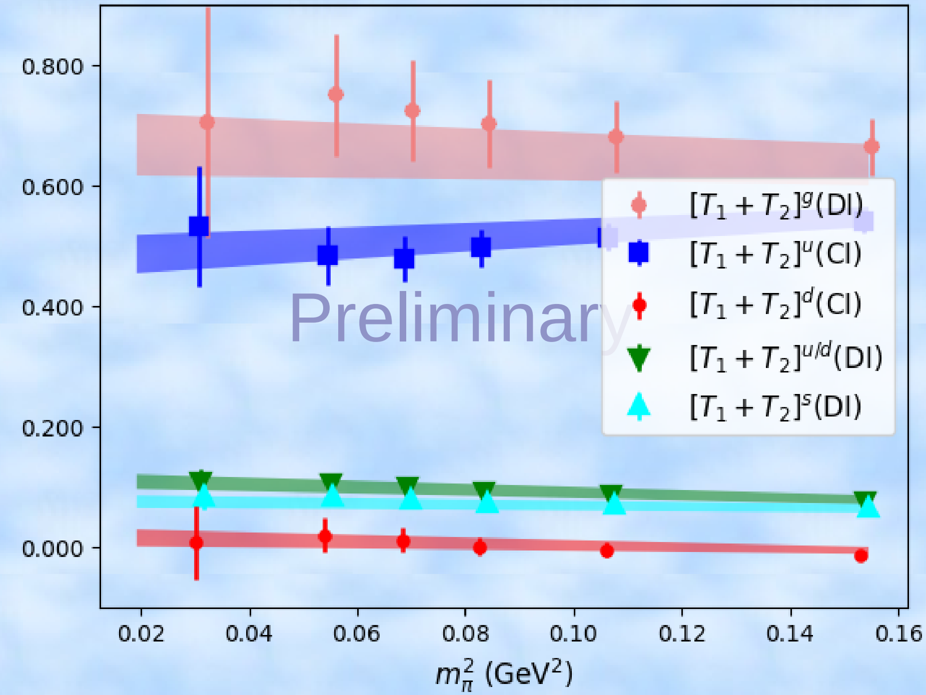


z-expansion fit to extrapolate to zero momentum transfer

T1 and T1+T2 form factors Summary

Simple linear extrapolations of each constituents under current statistics

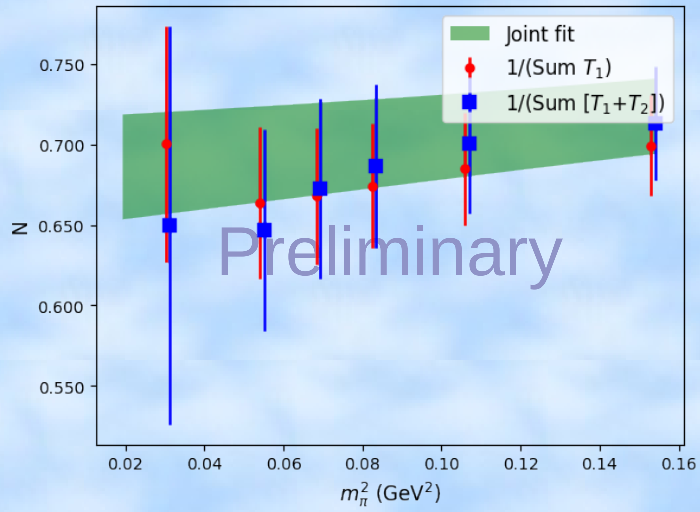




The sum of renormalized T1 and T1+T2 form factors are consistent with each other at all pion masses

$$N \langle x \rangle_R^q + N \langle x \rangle_R^g = 1$$

$$N\, J_R^q + N\, J_R^g = \frac{1}{2}$$



Momentum and Angular momentum fractions

Summary table of the CI and DI parts for quark and gluon constituents

Preliminary

	u(CI)	d(CI)	$[u+d]$(CI)	u/d(DI)	s(DI)	glue
$\langle x \rangle$	0.233(12)	0.0841(51)	0.317(15)	0.0637(58)	0.0419(56)	0.514(20)
$2J$	0.333(23)	0.0097(96)	0.343(24)	0.0739(73)	0.0512(62)	0.458(22)

Compare with previous calculation and phenomenological global fit results

Preliminary

	u	d	$[u-d]$	s	glue
$\langle x \rangle$	0.296(12)	0.1477(69)	0.149(10)	0.0419(56)	0.514(20)
$\langle x \rangle_{[1]}$	0.307(35)	0.160(48)	0.151(40)	0.051(26)	0.482(84)
$\langle x \rangle_{\rm CT14}$	0.348(5)	0.190(5)	0.158(6)	0.035(9)	0.416(9)

[1] Y-B Yang, J. Liang, et al., χQCD Collaboration, Phys. Rev. Lett. 121, 212001 (2018)
[2] S. Dulat, et al., Phys. Rev. D, 93(3):033006, (2016)
[3] M. Deka, T. Doi, Y-B Yang, et. al., χQCD collaboration, PRD91, 014505 (2015)

Further Calculations

- FFT and low mode substitution has been successfully used with stochastic-sandwich method for CI to reach better statistics
- CDER technique greatly increase DI statistics with systematic errors under control
- A complete calculation of proton momentum and angular momentum fractions at several overlap valence pion masses has been done on one Lattice
- Extend the calculation to other lattice spacing and volumes to extrapolate to physical limit
- Extend to all kinematics to obtain T1, T2, T3 form factors at different Q^2

Thank You