Generalised Parton Distributions from Lattice Feynman-Hellmann Techniques

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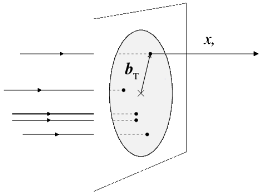

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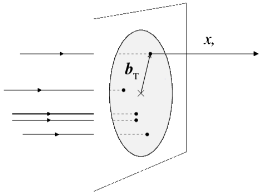

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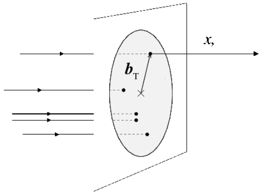

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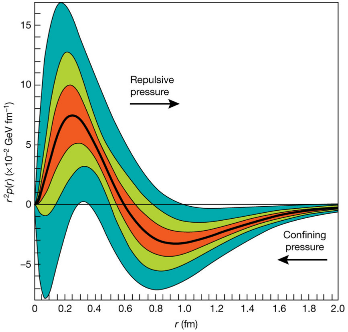
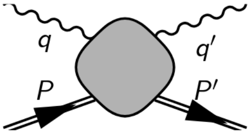
Pressure distribution



- Measurement of confining pressure [CLAS data].

GPDs: Definition and Properties

Compton amplitude



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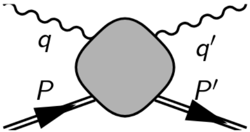

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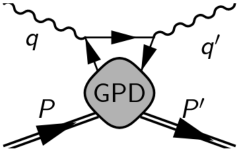

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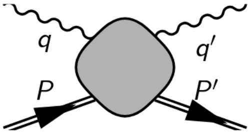

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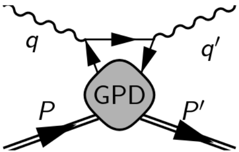

Formal definition

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \overbrace{\langle P' | \bar{\psi}_q(-\lambda n/2) \not{n} \psi_q(\lambda n/2) | P \rangle}^{\text{light-cone matrix elem}} = H^q(x,\xi,t) \bar{u}(P') \not{n} u(P)$$

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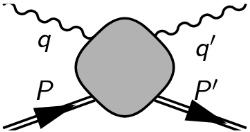

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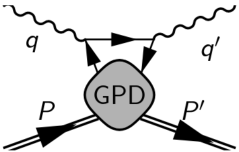

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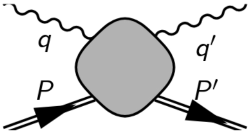

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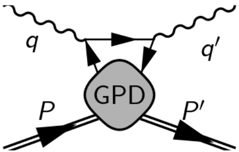

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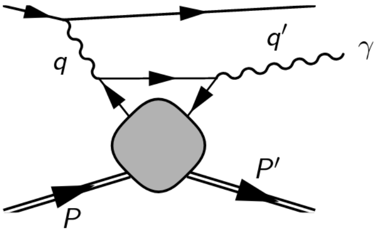
Forward limit

$H^q(x,\xi,t) \xrightarrow{t\to 0} q(x)$, the regular parton distribution function (PDF).

Measurement of GPDs

Deeply virtual Compton scattering



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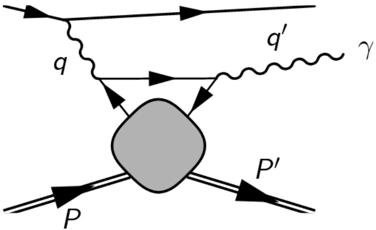

Measurement

- Past and ongoing: HERA, COMPASS, and JLab
- In future: EIC (Brookhaven), Chinese EIC, and (proposed) large HEC

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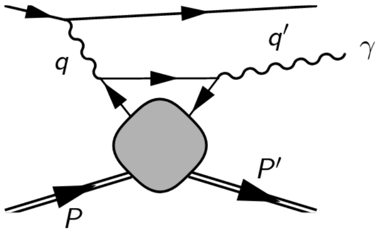

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- Deconvolution problem: $\mathcal{H}(\xi, t) = \mathrm{Conv}(H(x, \xi, t))$
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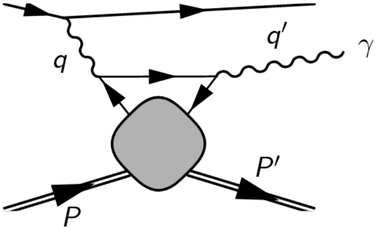

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Upshot: lattice QCD calculations of GPDs are of great interest.

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- range of Q^2 values: $(0.5 - 11\ \mathrm{GeV}^2)$
- non-leading-twist structures (K. U. Can)
- subtraction function (E. Sankey, Wednesday 10:30pm EST)
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Focus of this talk: determining (generalised) parton distributions from the (off-forward Compton) scattering amplitude.

Off-forward Compton amplitude from Feynman-Hellmann

Basic idea: Off-forward Compton amplitude (OFCA) from 4-pt:

$$T_{\mu\nu} = \sum_{z_\mu} e^{\frac{i}{2}(q+q')\cdot z} \langle P' | T\{j_\mu(z) j_\nu(0)\} | P \rangle.$$

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So

$$\frac{\partial^2}{\partial\lambda_1 \partial\lambda_2} C_{\vec{\lambda}} \Big|_{\vec{\lambda}=0} = M^{-1}\mathcal{J}_3(\vec{q}_1)M^{-1}\mathcal{J}_3(\vec{q}_2)M^{-1} + (1 \leftrightarrow 2).$$

Four-point function with momentum transfer $\rightarrow$ off-forward kinematics.

Off-forward Compton amplitude from Feynman-Hellmann

Perturbed quark propagator(s) into nucleon propagator: $\mathcal{G}^{dd}_{\vec{\lambda}} \simeq \langle C^u C^u C^d_{\vec{\lambda}} \rangle$ (see below), or $\mathcal{G}^{uu}_{\vec{\lambda}} \simeq \langle C^u_{\vec{\lambda}} C^u_{\vec{\lambda}} C^d \rangle$.

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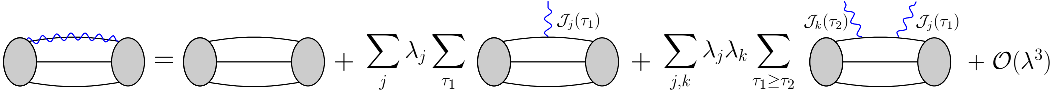

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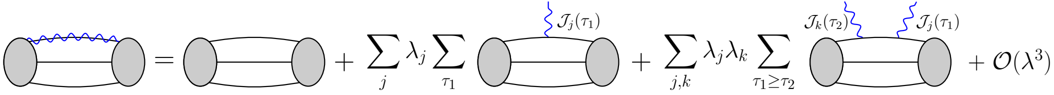

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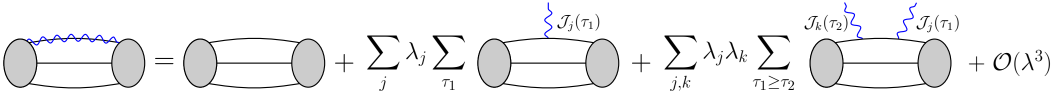

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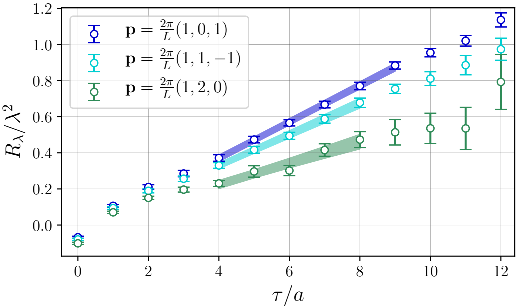
- Linear τ enhancement and GS saturation requires kinematic restrictions.
- For this calculation, time extent same as usual 2-pt propagator.

Lattice Signal



- Approximate derivative with

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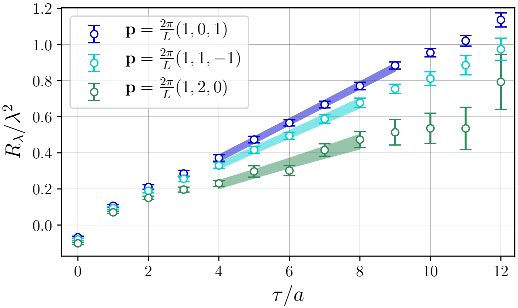

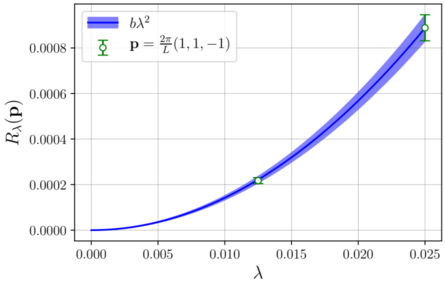

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- After fitting in τ, signal is well-described by quadratic in λ.
- Implies no interference from other powers.

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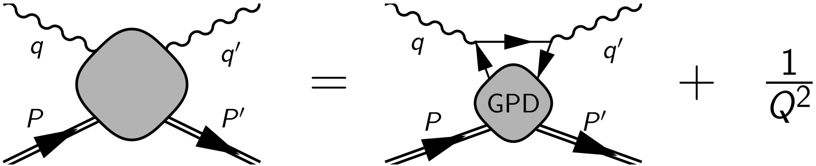

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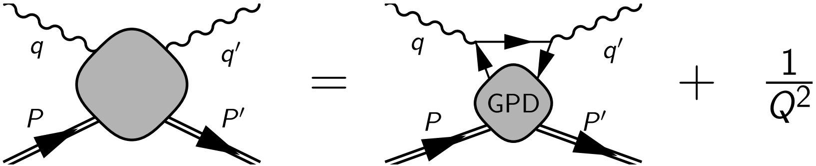

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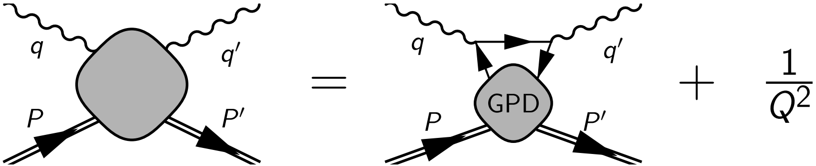

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Operator product expansion

- Not limited to IMF
- Trivial relation between Euclidean and Minkowski

Operator Product Expansion

Final result

$$\overbrace{\sum_{\text{spins}} \Gamma u' T^{33} \bar{u}}^{\text{Feynman-Hellmann}} \propto \sum_{n=2,4,6}^{\infty} \bar{\omega}^n \Big[\underbrace{\overbrace{A^q_{n,0}(t)}^{\text{helicity-conserving}} + \frac{t}{2M(E+M)} \overbrace{B^q_{n,0}(t)}^{\text{helicity-flip}}}_{\text{GPD moments}} \Big] + \mathcal{O}\left(\frac{1}{Q^2}\right).$$

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- Power series in $\bar{\omega} = \frac{2(P+P')\cdot(q+q')}{-(q+q')^2}$
- GPD moments appear as coefficients of $\bar{\omega}$ polynomial:

$$A^q_{n,0}(t) = \int_{-1}^{1} dx x^{n-1} H^q(x,t),$$

$$B^q_{n,0}(t) = \int_{-1}^{1} dx x^{n-1} E^q(x,t).$$

Lattice Details

N_f	$\kappa_l \ \kappa_s$	$L^3 \times T$	a [fm]	M_π [GeV]
$2+1$	0.1209	$32^3 \times 64$	0.07	0.47

Lattice Details

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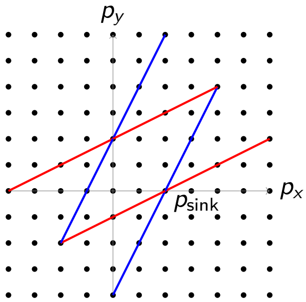

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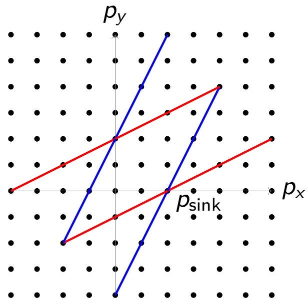

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Two sets of data

1. $t = -1.10\ \text{GeV}^2,\ \bar{Q}^2 = 7.13\ \text{GeV}^2.$
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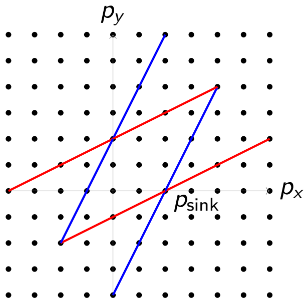

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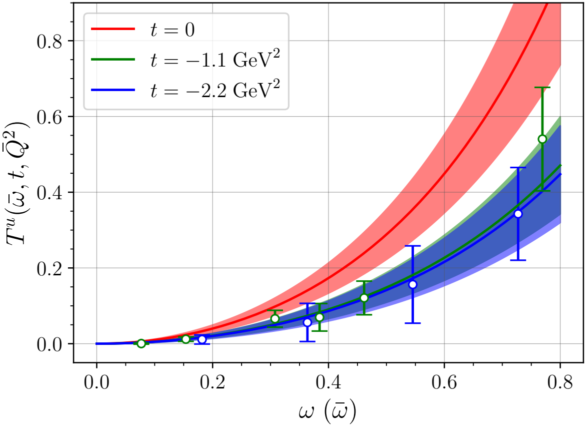
(Different $\bar{Q}^2$ means different systematics)

Compton Amplitude



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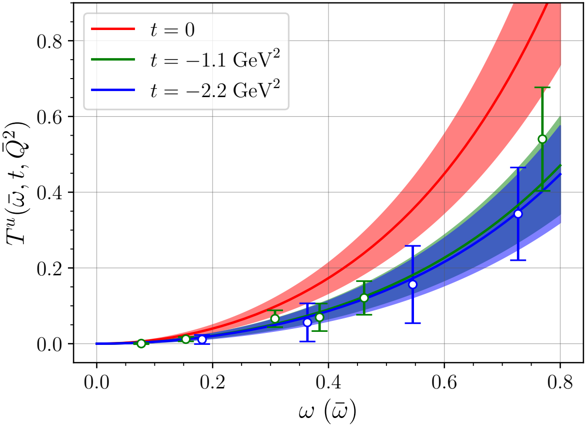

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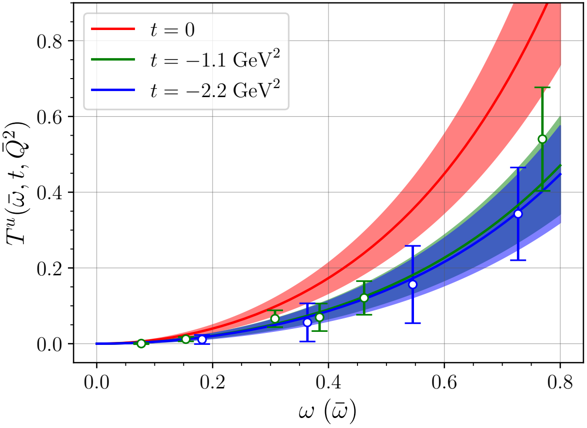

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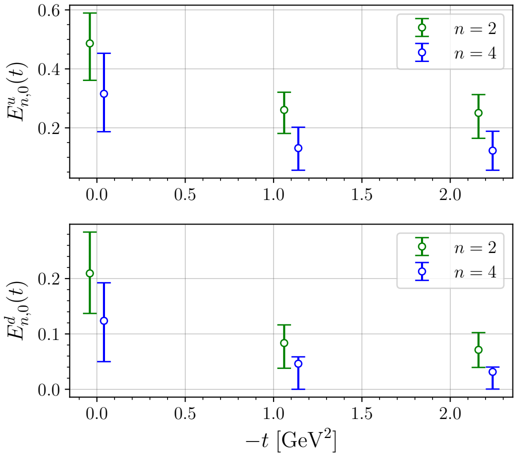
$$T^u(\bar{\omega}, t) = 2 \sum_{n=2,4,6}^{\infty} \bar{\omega}^n \Big[A^u_{n,0}(t) + \frac{t}{2M(E+M)} B^u_{n,0}(t) \Big].$$

Fitting Moments

Fit our data to

$$f_j(\bar{\omega}) = M_2\bar{\omega}^2 + M_4\bar{\omega}^4 + \ldots + M_{2j}\bar{\omega}^{2j}.$$



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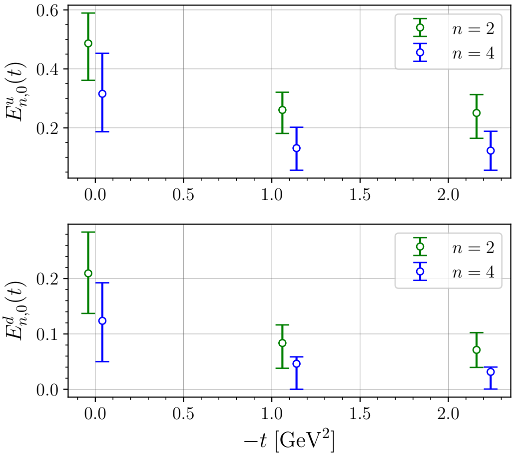

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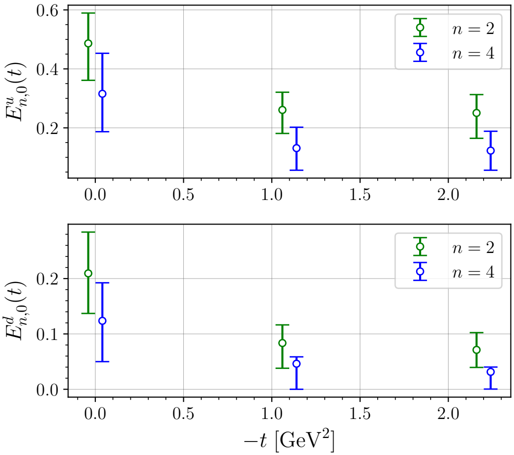

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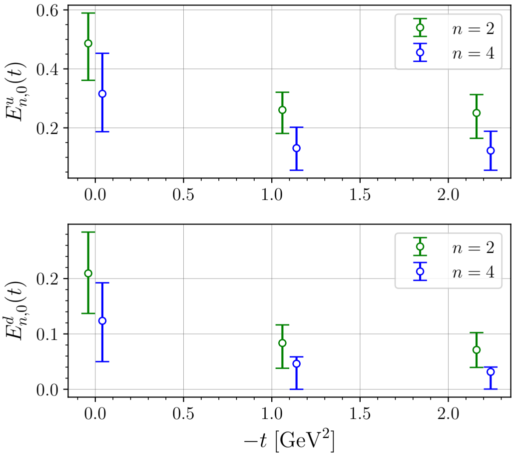
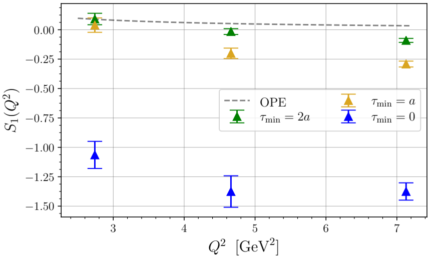
- Fit $j = 3$; limited by number of $\bar{\omega}$ points
- $n = 4$ moments never calculated before!
- $n = 2$ consistent with 3-pt moments at similar pion mass



Systematic Errors

Feynman-Hellmann specific

- Unphysical asymptotic behaviour of subtraction function, $T(\bar{\omega} = 0, t, \bar{Q}^2)$
- Largely accounted for in forward case: see my poster (Wednesday 8:00am) or E. Sankey (Wednesday 10:30pm).



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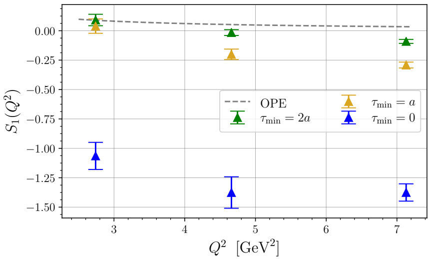

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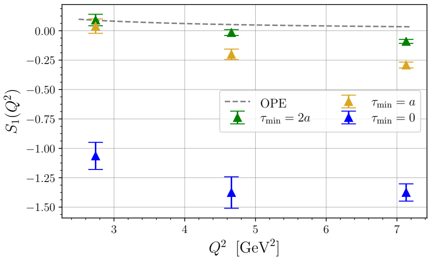

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General lattice

- Excited state contamination
- Finite volume (PRD103, 034507 (2021))
- Quark mass

Conclusion and Outlook

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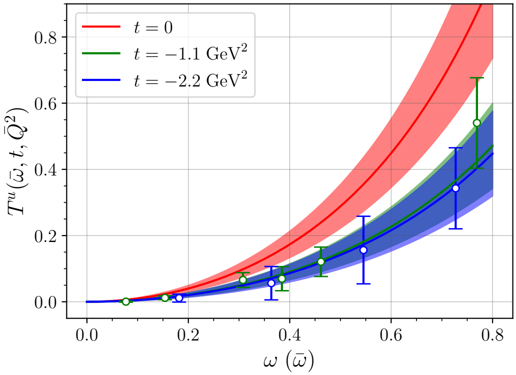
- Highly exploratory calculation—first of its kind
- Calculate the OFCA
- Fit GPD moments—first look at $n = 4$



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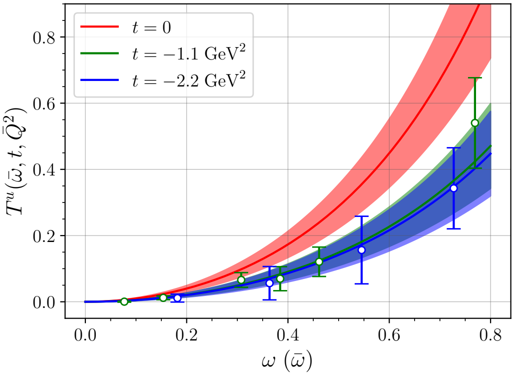
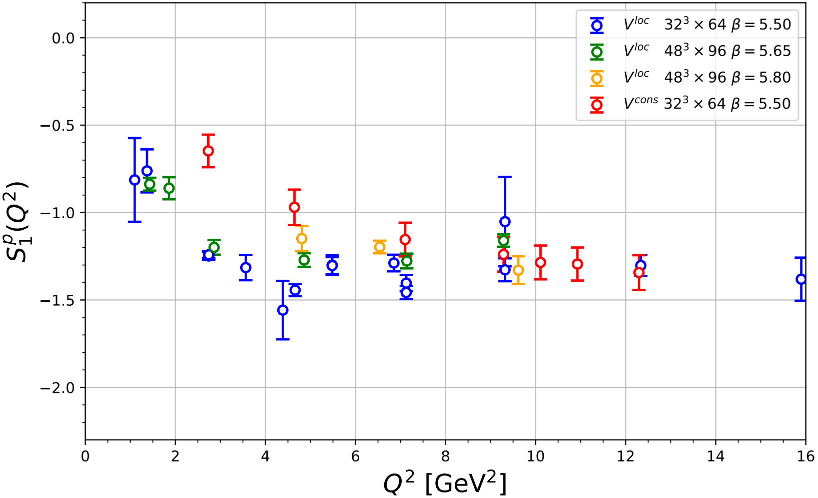
Outlook

- Separating A and B moments
- Investigate and control systematics
- Greater kinematic spread
- Non-perturbative contributions: subtraction function, power-suppressed structure
- Constrain GPDs: higher moments, fitting models, inverse Mellin transform



Subtraction Term (Forward)



Subtraction Term (Off-Forward)