

Generalised Parton Distributions from Lattice Feynman-Hellmann Techniques

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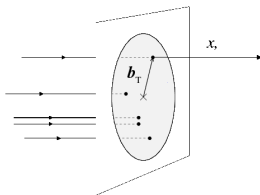
26th July, 2021



Why are we interested in generalised parton distributions?

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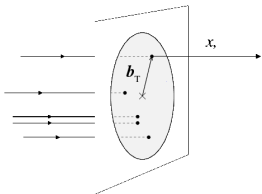
Spatial distribution



Encode spatial distribution of quarks and gluons in a highly boosted hadron.

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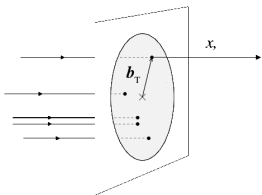
Spin structure

$$\frac{1}{2} \int_{-1}^1 dx \{Z\} \Big|_{\text{pol. DIS}} + \int_{-1}^1 dx \{Z\} \Big|_{\text{GPDs}} = \frac{1}{2}$$

Access components of proton's spin ! offers solution to 'proton spin puzzle'.

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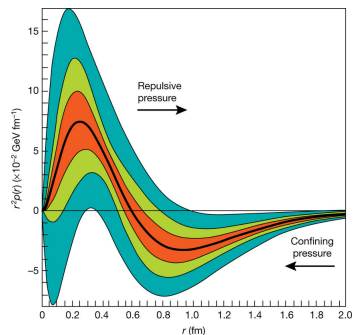
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pol. DIS GPDs

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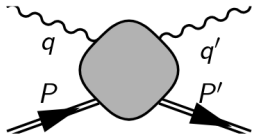
Pressure distribution



Measurement of confining pressure [CLAS data].

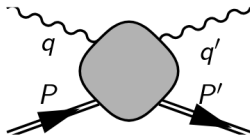
GPDs: Definition and Properties

Compton amplitude

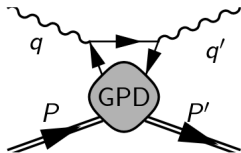


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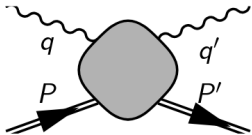


Factorisation for large $j(q^0)^2 j$:

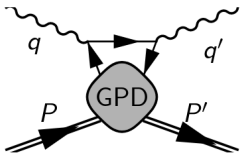


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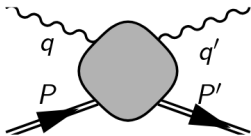


Formal definition

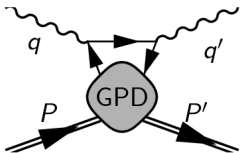
$$\begin{aligned}
 & \int \frac{d^4x}{2} e^{i x \cdot z} \langle P' | \int_{-1}^1 dx \left[\frac{1}{2} \left(\gamma^0 + \not{x} \right) \gamma^j \right] \left[\frac{1}{2} \left(\gamma^0 + \not{x} \right) \gamma^j \right] | P \rangle = H^q(x; ; t) u(P^0) u(P) \\
 & + E^q(x; ; t) u(P^0) \frac{i}{2M} \not{x} \not{P} u(P) :
 \end{aligned}$$

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Formal definition

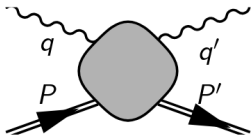
$$\int \frac{d^4x}{2} e^{i x \cdot z} \langle P' | \int_{-1}^1 dz \frac{\text{light-cone matrix elem}}{h P^0 j_q(n=2) \mathbb{A}_q(n=2) j P i} \{ \dots \} | P \rangle = H^q(x; ; t) u(P^0) \mathbb{A} u(P) + E^q(x; ; t) u(P^0) \frac{i}{2M} \frac{n \cdot (P^0 - P)}{2M} u(P):$$

H^q and E^q helicity-conserving and -flipping GPDs

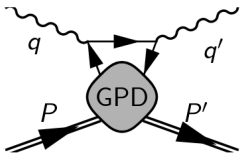
$t = (P^0 - P)^2$ is momentum transfer ! how 'off-forward'

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Factorisation for large $j(q^0)^2j$:



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$$\int \frac{d^4z}{2} e^{i x \cdot z} \langle \text{light-cone matrix elem} \rangle \Big|_{q(n=2) \rightarrow q'(n=2)jP} = H^q(x; ;t) u(P^0) u(P) + E^q(x; ;t) u(P^0) \frac{i}{2M} \frac{n \cdot (P^0 - P)}{2M} u(P):$$

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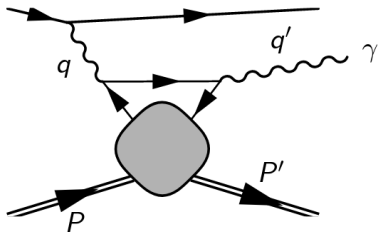
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Forward limit

$H^q(x; ;t) \xrightarrow{t \rightarrow 0} q(x)$; the regular **parton distribution function** (PDF).

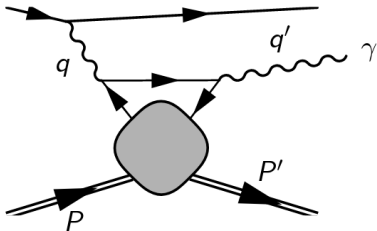
Measurement of GPDs

Deeply virtual Compton scattering



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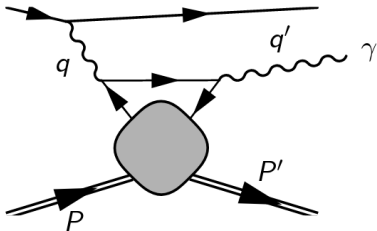
Measurement

Past and ongoing: HERA, COMPASS, and JLab

In future: EIC (Brookhaven), Chinese EIC, and (proposed) large HEC

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Difficulties extracting GPDs from experiment

Deconvolution problem:

$$H(x; t) = \text{Conv}(H(x; t))$$

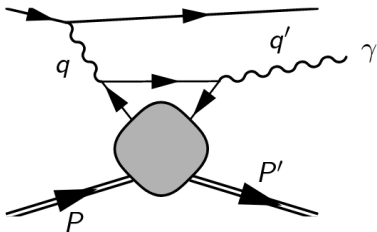
High dimensionality

Can't access full kinematics

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Upshot: lattice QCD calculations of GPDs are of great interest.

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Euclidean spacetime prohibits light-like separations:

$$x^2 = t^2 + \mathbf{x} \cdot \mathbf{x} \quad \mathbf{x} = 0 \quad , \quad t = 0:$$

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From the scattering amplitude

range of Q^2 values: (0.5 – 11 GeV²)
non-leading-twist structures (K. U. Can)
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Wednesday 10:30pm EST)
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Focus of this talk: determining (generalised) parton distributions from the (off-forward Compton) scattering amplitude.

Off-forward Compton amplitude from Feynman-Hellmann

Basic idea: Off-forward Compton amplitude (OFCA) from 4-pt:

$$T = \sum_z e^{\frac{i}{2}(q+q') \cdot z} \langle P^{\prime\prime} | T f_j(z) | P \rangle \langle 0 | g_j | P \rangle$$

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But 4-pt functions computationally expensive.

Feynman-Hellmann: 4-pt from *perturbed* 2-pt.

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Perturbed quark propagator

$$C_{\sim} = \left| \begin{array}{c} M \\ \{Z\} \end{array} \right|_{\text{fermion matrix}} \left| \frac{J_3(q_1)}{Z} \{ \frac{J_3(q_2)}{Z} \} \right|_{\text{background fields}}^1$$

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fermion matrix background fields unperturbed three-point

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So

$$\frac{\partial^2}{\partial a_1 \partial a_2} C_{\sim} \Big|_{\tilde{a}=0} = M \overset{1}{J_3(q_1)} M \overset{1}{J_3(q_2)} M \overset{1}{1} + (1 \text{ } \S \text{ } 2):$$

Four-point function with momentum transfer ! **off-forward kinematics.**

Off-forward Compton amplitude from Feynman-Hellmann

Perturbed quark propagator(s) into nucleon propagator: $G_{\sim}^{dd} \ ' \ hC^u C^u C_{\sim}^d i$ (see below), or $G_{\sim}^{uu} \ ' \ hC_{\sim}^u C_{\sim}^u C^d i$.

Off-forward Compton amplitude from Feynman-Hellmann

Perturbed quark propagator(s) into nucleon propagator: $G_{\sim}^{dd} \cdot h C^u C^u C^d i$ (see below), or $G_{\sim}^{uu} \cdot h C_{\sim}^u C_{\sim}^u C^d i$.

$$\begin{aligned}
 \text{Diagram with wavy line} &= \text{Diagram without wavy line} + \sum_j \lambda_j \sum_{\tau_1} \text{Diagram with } \mathcal{J}_j(\tau_1) \\
 &+ \sum_{j,k} \lambda_j \lambda_k \sum_{\tau_1 \geq \tau_2} \text{Diagram with } \mathcal{J}_k(\tau_2) \text{ and } \mathcal{J}_j(\tau_1) + \mathcal{O}(\lambda^3)
 \end{aligned}$$

Off-forward Compton amplitude from Feynman-Hellmann

Perturbed quark propagator(s) into nucleon propagator: $G_{\sim}^{dd} \sim \langle h C^u C^u C^d i \rangle$ (see below), or $G_{\sim}^{uu} \sim \langle h C_{\sim}^u C_{\sim}^u C^d i \rangle$.

$$\text{Diagram} = \text{Diagram} + \sum_j \lambda_j \sum_{\tau_1} \text{Diagram} + \sum_{j,k} \lambda_j \lambda_k \sum_{\tau_1 \geq \tau_2} \text{Diagram} + \mathcal{O}(\lambda^3)$$

The diagram shows a sequence of terms representing the expansion of a perturbed quark propagator into a nucleon propagator. The first term is a simple cylinder with a blue wavy line on top. The second term is a cylinder with a blue wavy line labeled $J_j(\tau_1)$ on top. The third term is a cylinder with two blue wavy lines labeled $J_k(\tau_2)$ and $J_j(\tau_1)$ on top, with $\tau_1 \geq \tau_2$.

Feynman-Hellmann relation

$$\frac{\partial^2}{\partial \lambda_1 \partial \lambda_2} \frac{G_{\sim}(\lambda; \mathbf{p})}{G_0(\lambda; \mathbf{p})} \Big|_{\lambda=0} \sim \frac{\partial}{\partial \lambda} \frac{e^{i(\mathbf{q}_1 + \mathbf{q}_2) \cdot \mathbf{z}} \langle h N(\mathbf{p}) j T f j_3(\mathbf{z}) j_3(0) g j N(\mathbf{p} - \mathbf{q}_1 - \mathbf{q}_2) i \rangle}{2E_N(\mathbf{p})}$$

$\underbrace{\hspace{15em}}_{\text{discretisation of } \mathbf{z} = \mathbf{z} = 3 \text{ OFCA}}$

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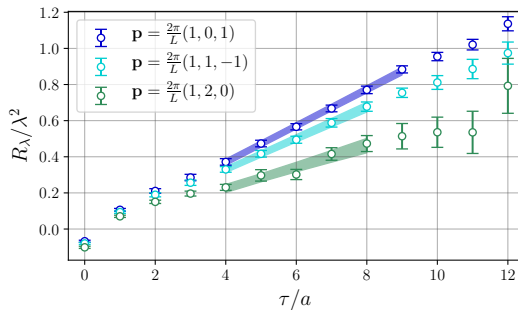
$$\frac{\partial^2}{\partial \lambda_1 \partial \lambda_2} \frac{G_{\sim}(\lambda; \mathbf{p})}{G_0(\lambda; \mathbf{p})} \Big|_{\lambda=0} \sim \frac{\partial^2}{\partial \lambda_1 \partial \lambda_2} \frac{1}{2E_N(\mathbf{p})} \int \frac{d^3z}{(2\pi)^3} e^{i(\mathbf{q}_1 + \mathbf{q}_2) \cdot \mathbf{z}} h N(\mathbf{p}) j T f j_3(z) j_3(0) g j N(\mathbf{p} - \mathbf{q}_1 - \mathbf{q}_2) i :$$

$\int \frac{d^3z}{(2\pi)^3}$ discretisation of $\int d^3z = 3 \text{ OFCA}$

Linear enhancement and GS saturation requires kinematic restrictions.

For this calculation, time extent same as usual 2-pt propagator.

Lattice Signal

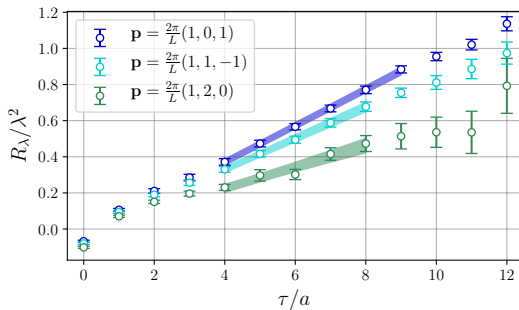


Approximate derivative with

$$R = \frac{G ; + G ; \quad G ; \quad G ;}{G_0} ;$$

Good linear fit: slope is OFCA

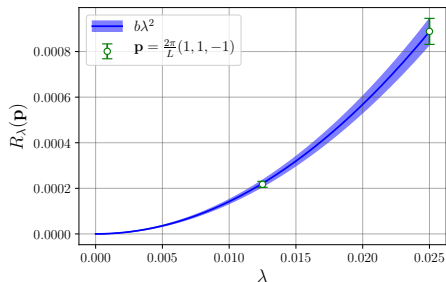
Lattice Signal



Approximate derivative with

$$R = \frac{G_{\mathbf{p}} + G_{-\mathbf{p}}}{G_0} ; \quad G_{\mathbf{p}} ; \quad G_{-\mathbf{p}} ; \quad :$$

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After fitting in λ , signal is well-described by quadratic in λ .

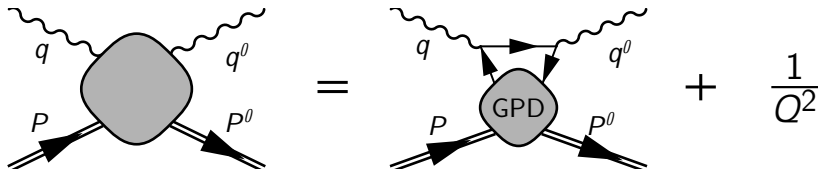
Implies no interference from other powers.

GPDs from the Compton amplitude

Now that we can calculate the OFCA, how do we access GPDs?

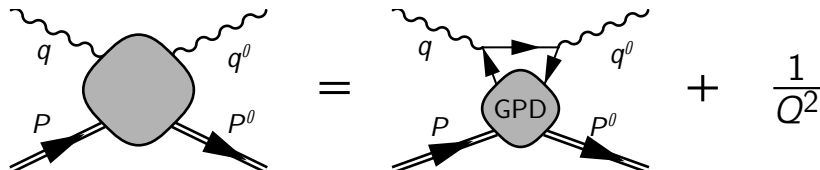
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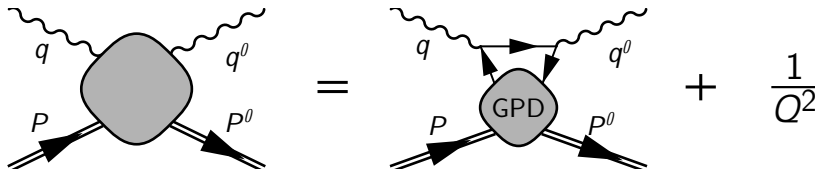
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Not suitable for lattice

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Existing work

Use infinite momentum frame !

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Not suitable for lattice

Operator product expansion

Not limited to IMF

Trivial relation between Euclidean and Minkowski

Lattice Details

N_f	l_s	L^3	T	a [fm]	M [GeV]
2 + 1	0.1209	32^3	64	0.07	0.47

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2 + 1	0:1209	32^3	64	0:07	0:47

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$$t = \frac{4p_{\text{sink}} (\varrho_1 + \varrho_2)}{(\varrho_1 + \varrho_2)^2}; \quad / \varrho_1^2 \quad \varrho_2^2$$

We choose zero-skewness: $j\varrho_1j = j\varrho_2j$

$$t = (\varrho_1 \quad \varrho_2)^2; \quad Q^2 = \frac{1}{4}(\varrho_1 + \varrho_2)^2$$

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t determines how **off-forward**, Q^2 how **perturbative**

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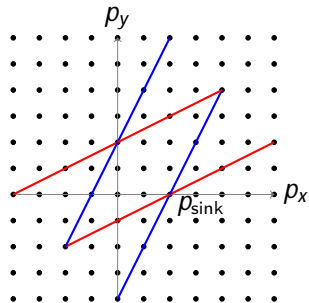
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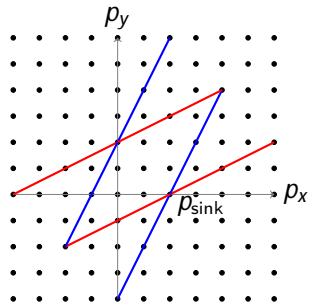
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t determines how **off-forward**, Q^2 how **perturbative**



Two sets of data

- ① $t = 1:10 \text{ GeV}^2$, $Q^2 = 7:13 \text{ GeV}^2$.
- ② $t = 2:20 \text{ GeV}^2$, $Q^2 = 6:03 \text{ GeV}^2$.

Lattice Details

N_f	l_s	L^3	T	a [fm]	M [GeV]
2 + 1	0:1209	32^3	64	0.07	0.47

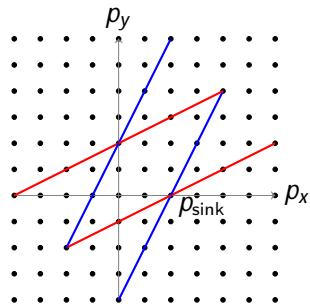
Momentum scalars

$$l = \frac{4p_{\text{sink}} (\vartheta_1 + \vartheta_2)}{(\vartheta_1 + \vartheta_2)^2}; \quad \nearrow \vartheta_1^2 \quad \vartheta_2^2$$

We choose zero-skewness: $j\vartheta_1j = j\vartheta_2j$

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(Different Q^2 means **different systematics**)

Compton Amplitude

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Red curve is forward Compton amplitude: $t = 0$, $Q^2 = 7.13 \text{ GeV}^2$ (PRD102, 114505 (2020)).

Recall:

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$$T^u(!; t) = 2 \sum_{n=2,4,6} !^n A_{n;0}^u(t) + \frac{t}{2M(E+M)} B_{n;0}^u(t) :$$

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Fit our data to

$$f_j(!) = M_2!^2 + M_4!^4 + \dots + M_{2j}!^{2j}:$$

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Extract the linear combination of moments

$$M_n(t) \quad E_{n;0}(t) = A_{n;0}(t) + \frac{t}{4M^2} B_{n;0}(t):$$

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Fit $j = 3$; limited by number of t points

$n = 4$ moments never calculated before!

$n = 2$ consistent with 3-pt moments at similar pion mass

Systematic Errors

Feynman-Hellmann specific

Unphysical asymptotic behaviour of
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Largely accounted for in forward case: see
my poster (Wednesday 8:00am) or
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General lattice

Excited state contamination

Finite volume (PRD103, 034507 (2021))

Quark mass

Conclusion and Outlook

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Highly exploratory calculation first of its kind

Calculate the OFCA

Fit GPD moments first look at $n = 4$

Conclusion and Outlook

Conclusion

Highly exploratory calculation—first of its kind

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Outlook

Separating A and B moments

Investigate and control systematics

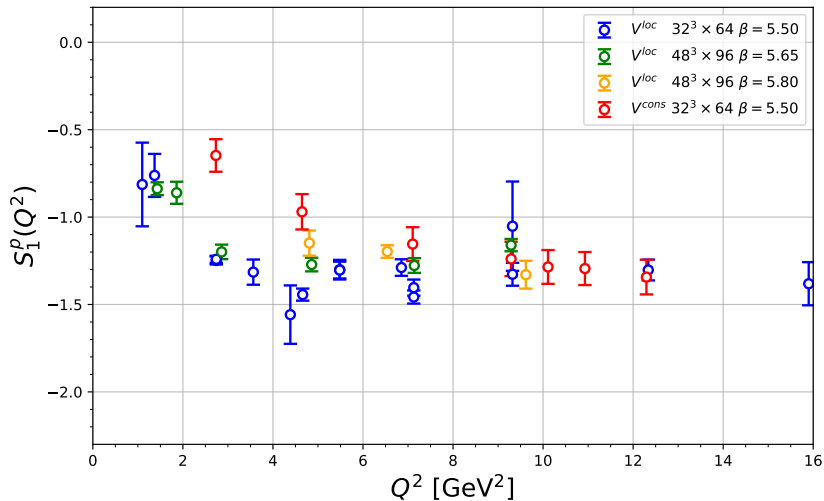
Greater kinematic spread

Non-perturbative contributions:

subtraction function, power-suppressed structure

Constrain GPDs: higher moments, fitting models, inverse Mellin transform

Subtraction Term (Forward)



Subtraction Term (Off-Forward)

