

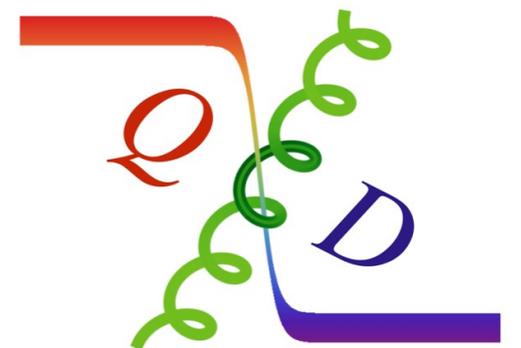
# The contribution of QCD trace anomaly to hadron mass

[arXiv: 2101.04942.](https://arxiv.org/abs/2101.04942)

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# Outline

① Introduction to trace anomaly of QCD and

**Hadron mass decomposition**

② Numerical results

③ Summary

# QCD Trace Anomaly

- **Scale transformation(dilatations):**

$$x \rightarrow xe^{-\sigma}$$

$$\phi(x) \rightarrow e^{-D\sigma}\phi(xe^{-\sigma})$$

$D$  is the mass dimension of field  $\phi$

- **Mass term will break down scale symmetry**  $\partial_\mu J^\mu = T^\mu_\mu = m_q \bar{q}q$

$J_\mu$  is the Noether current for scale transformation

$T^\mu_\mu$  is the trace of energy momentum tensor

- **Scale symmetry is broken when quantum corrections are included** Peskin and Schroeder, An Introduction to QFT, Chapter 19

$$(T^\mu_\mu)^a = \frac{\beta_{QCD}}{2g} F^2 + \gamma_m m_q \bar{q}q$$

R. J. Crewther, PRL28(1972) 1421  
M. S. Chanowitz PLB40(1972) 397  
J.Collins et,al. PRD16(1977) 438  
N. K. Nielsen, NPB120 (1977) 212

$\beta_{QCD}$ : beta function of QCD

$\gamma_m$ : Anomalous dimension of quark mass

- **Total trace term of QCD EMT**

$$(T^\mu_\mu) = (T^\mu_\mu)^a + m_q \bar{q}q = \frac{\beta_{QCD}}{2g} F^2 + (1 + \gamma_m) m_q \bar{q}q$$

# Hadron Mass Decomposition

X. Ji, PRL 74,1071(1995)

- Hadron energy can be decomposed into

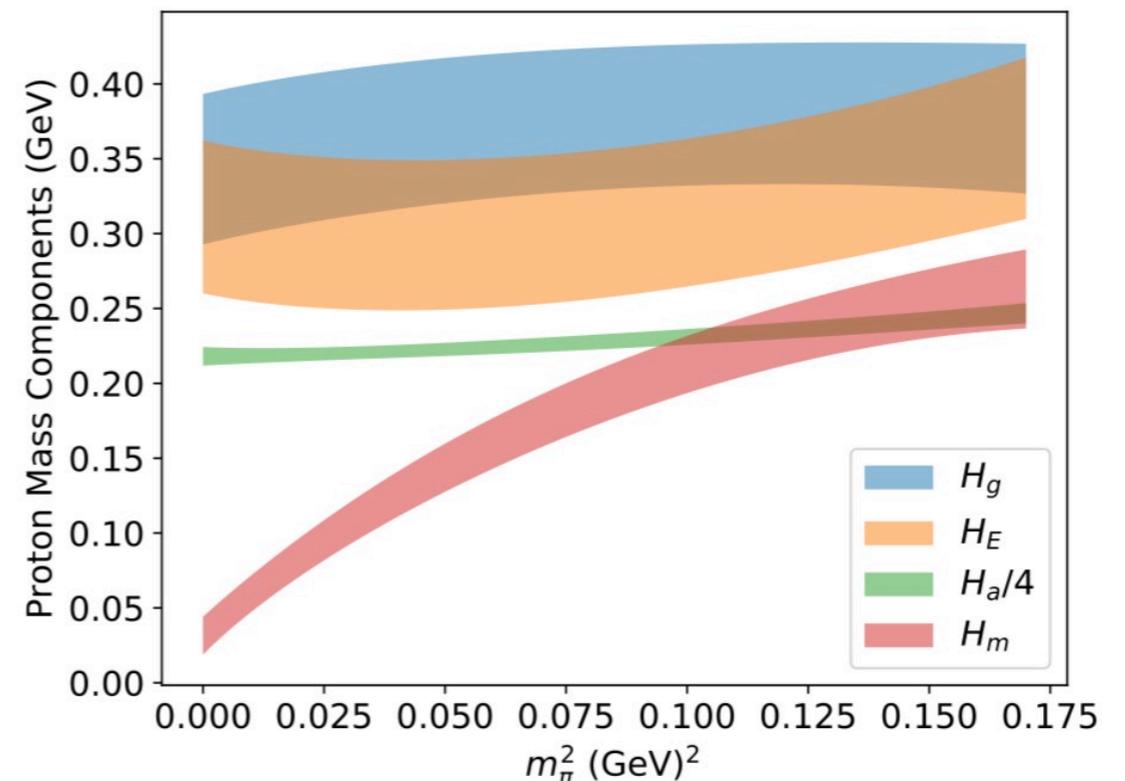
$$M_H = T^{00} = H_g + H_E + H_m + \frac{1}{4}H_a$$

$H_g$ : Gluon kinematic Energy

$H_E$ : Quark kinematic Energy

$H_m$ : Quark mass

$H_a$ : Trace anomaly



Y.B. Yang et.al. ( $\chi$ QCD Collaboration)PRL121(2018)

- Hadron invariant mass can be decomposed as

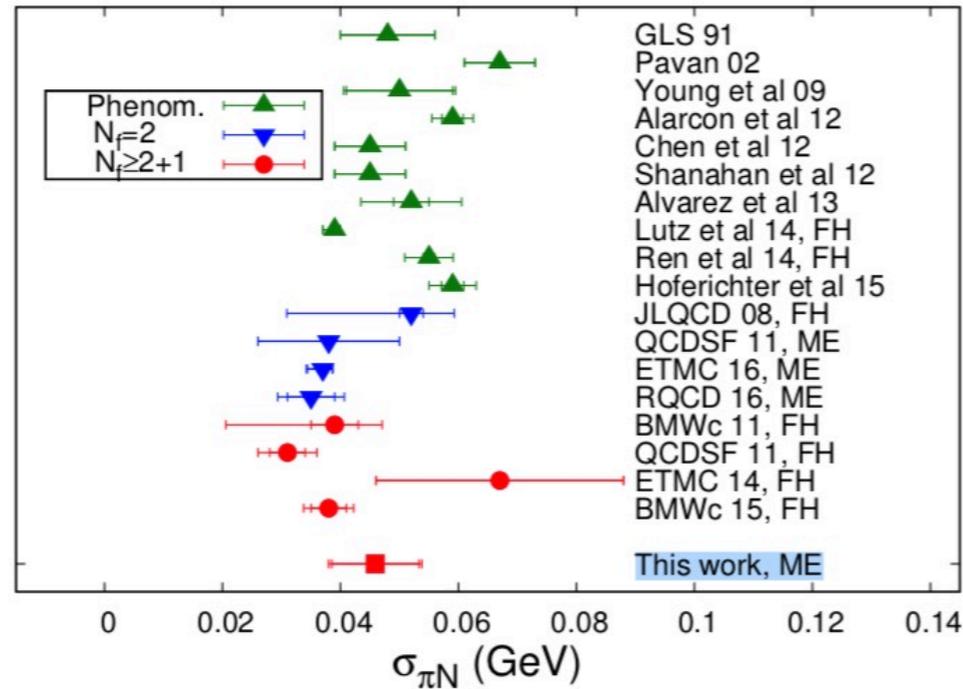
M.A. SHIFMAN et.al. PLB78(1978)

$$M = -\langle \hat{T}_{\mu\mu} \rangle = \langle H_m \rangle + \langle H_a \rangle.$$

# Quark Mass Contribution

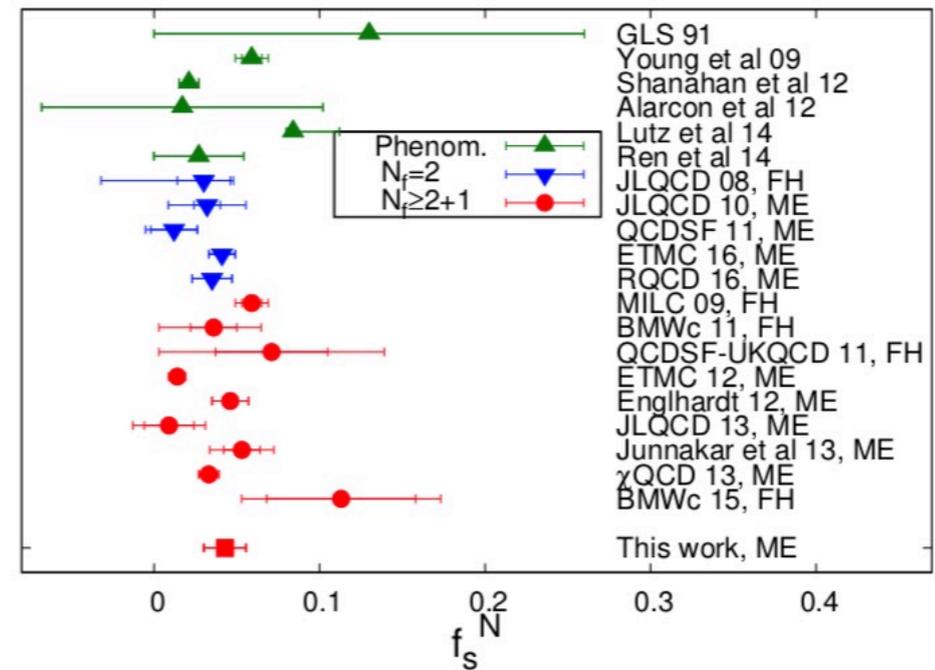
- Quark mass contribution to proton mass (sigma terms)

Y.B. Yang et,al.( $\chi$ QCD Collaboration)PRD(2016),054503



$$H_{m,u+d} = 45.9(7.4)(2.8)MeV$$

Y.B. Yang et,al. ( $\chi$ QCD Collaboration)PRL121(2018)



$$H_{m,s} = 40.2(11.7)(3.5)MeV$$

- The three light quark mass will contribute less than 100MeV to proton mass, according to sum rule:  $M_p = \sum_q \langle H_{m,q} \rangle + H_a$  **Most of proton mass is contributed by trace anomaly!**

# Outline

- ① Introduction to trace anomaly of QCD and hadron mass decomposition

- ② Numerical results

RBC ensemble

Symbol	$L^3 \times T$	$a$ (fm)	$6/g^2$	$m_\pi$	$m_K$	$N_{\text{cfg}}$
24I	$24^3 \times 64$	0.1105(3)	2.13	340	593	203

- ③ Summary

# Calculation Procedure

- To verify the mass sum rule  $M_H = \langle m_q \bar{q}q \rangle_H + \gamma_m \langle m_q \bar{q}q \rangle_H + \frac{\beta}{2g} \langle F^2 \rangle_H$

Our calculation is divided into the following steps:

- ① Calculate the hadron mass and the matrix elements of quark mass, gluon condensate in different hadron, such as in pseudoscalar meson , vector meson and nucleon...
- ② Determine the values of  $\gamma_m$  and  $\beta$ . Since their values are independent of hadron state and quark mass, we can obtain them by solving the following equations

$$M_{PS} - (1 + \gamma_m) \langle m_q \bar{q}q \rangle_{PS} - \frac{\beta}{2g^3} \langle g^2 F^2 \rangle_{PS} |_{m_v=0.48GeV} = 0$$

$$M_V - (1 + \gamma_m) \langle m_q \bar{q}q \rangle_V - \frac{\beta}{2g^3} \langle g^2 F^2 \rangle_V |_{m_v=0.48GeV} = 0$$

**PS: Pseudoscalar meson**  
**V: Vector meson**

- ③ Check the mass sum in different hadron state for different quark mass.

# The Values of $\gamma_m$ and $\beta$

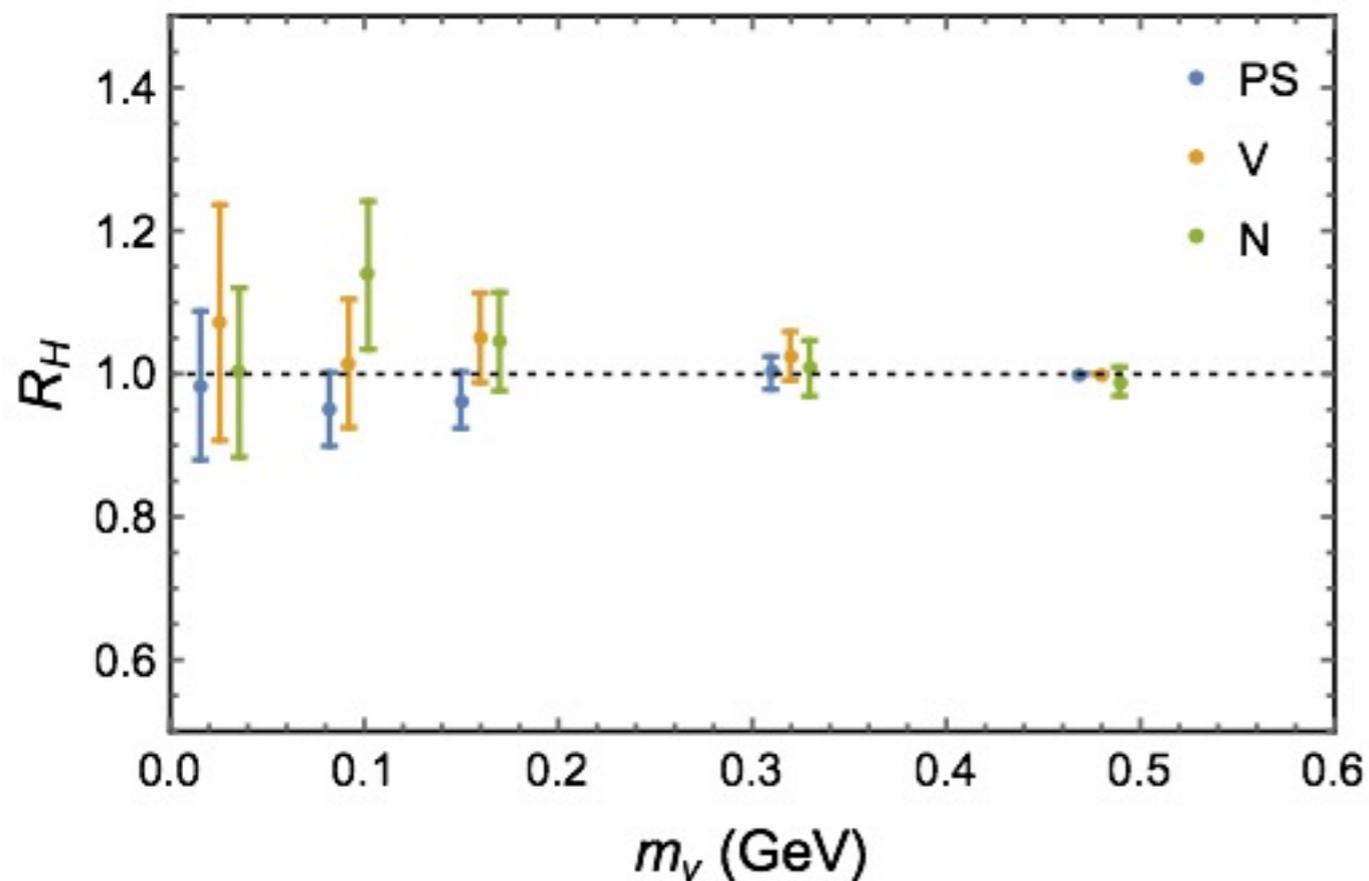
- Comparison of  $\gamma_m$  and  $\frac{\beta}{g^3}$  between our results and other results

$\gamma_m$		$\frac{\beta}{g^3}$	
Our result	4-loop results(MSbar) J.Vermaseren. PLB, 405(1997) 327	Our result	regularization independent leading order
0.38(3)	0.33(1) ( $\mu = 1/a = 1.78\text{GeV}$ )	-0.056(6)	$-\frac{11 + \frac{2N_f}{3}}{(4\pi)^2} = -0.057$

# Numerical Results

F. He, P. Sun and Y. Yang, arXiv: 2101.04942.

- **Verify Sum rules:**  $M_H = \langle H_m \rangle_H + \langle H_a \rangle_H$



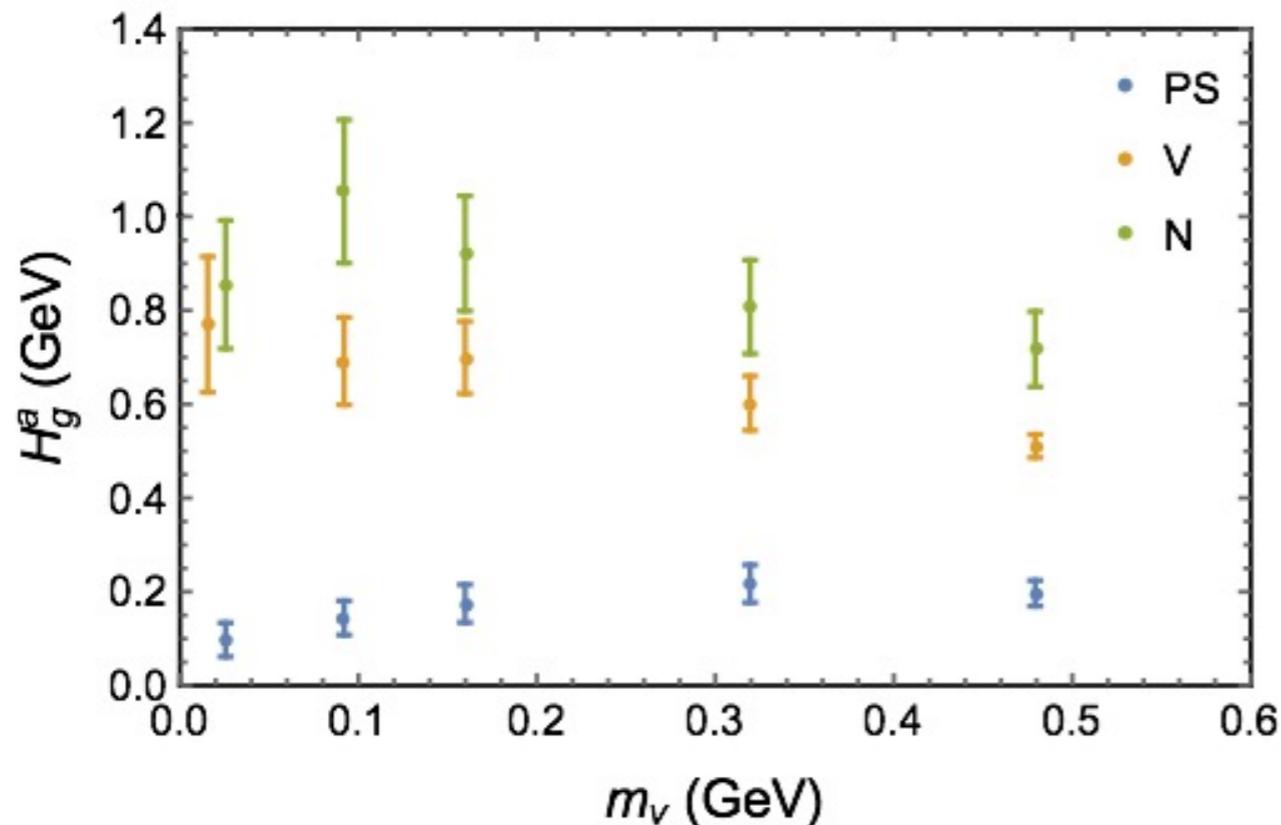
$$R_H = \frac{\langle H_m \rangle_H + \langle H_a \rangle_H}{M_H}$$

**We checked the trace anomaly sum rule. The ratio of sum rules to hadron mass is plotted,  $R_H$  in all the cases is consistent with one within the uncertainties.**

# Numerical Results

F. He, P. Sun and Y. Yang, arXiv: 2101.04942.

The contribution of gluon to hadron mass ( $H_g^a = \frac{\beta}{2g} \langle F^2 \rangle_H$ )



1. The contribution of gluon part in trace anomaly to the pseudoscalar meson mass is always much smaller than that in the other hadrons, especially around the chiral limit.
2. The ratio of gluon trace anomaly in pseudoscalar meson is also smaller.

Hadron mass at the unitary point:

$$M_N = 1.16 \text{ GeV}$$

$$M_V = 0.881 \text{ GeV}$$

$$M_{PS} = 0.34 \text{ GeV}$$

# The density of gluon trace anomaly

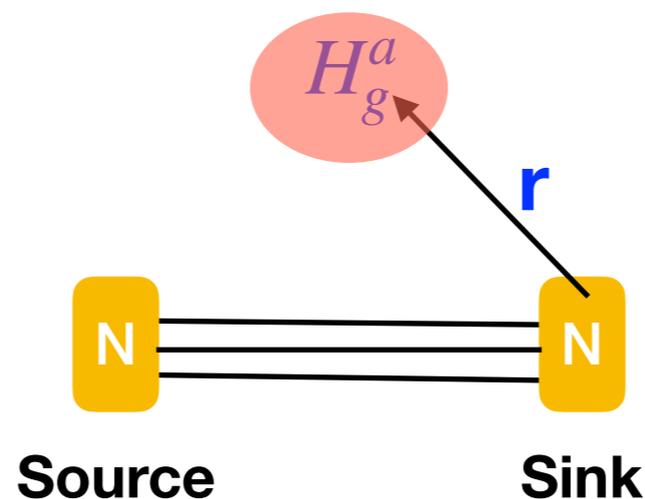
- The density of gluon trace anomaly density can be expressed as

$$\rho_H(|r|) = \frac{\langle \sum_{\vec{y}} \mathcal{H}(t_f, \vec{y}) H_a^g(t, \vec{y} + \vec{r}) \sum_{\vec{x}} \mathcal{H}^\dagger(0, \vec{x}) \rangle}{\langle \sum_{\vec{y}} \mathcal{H}(t_f, \vec{y}) \sum_{\vec{x}} \mathcal{H}^\dagger(0, \vec{x}) \rangle} \Big|_{t, t_f - t \rightarrow \infty},$$

Cluster decomposition method:  
K. F. Liu et al., PRD,97(2018) 034507

C. Bouchard et al., PoS Lattocce2016,170  
X. Feng et al., PRD,101(2020) 051502

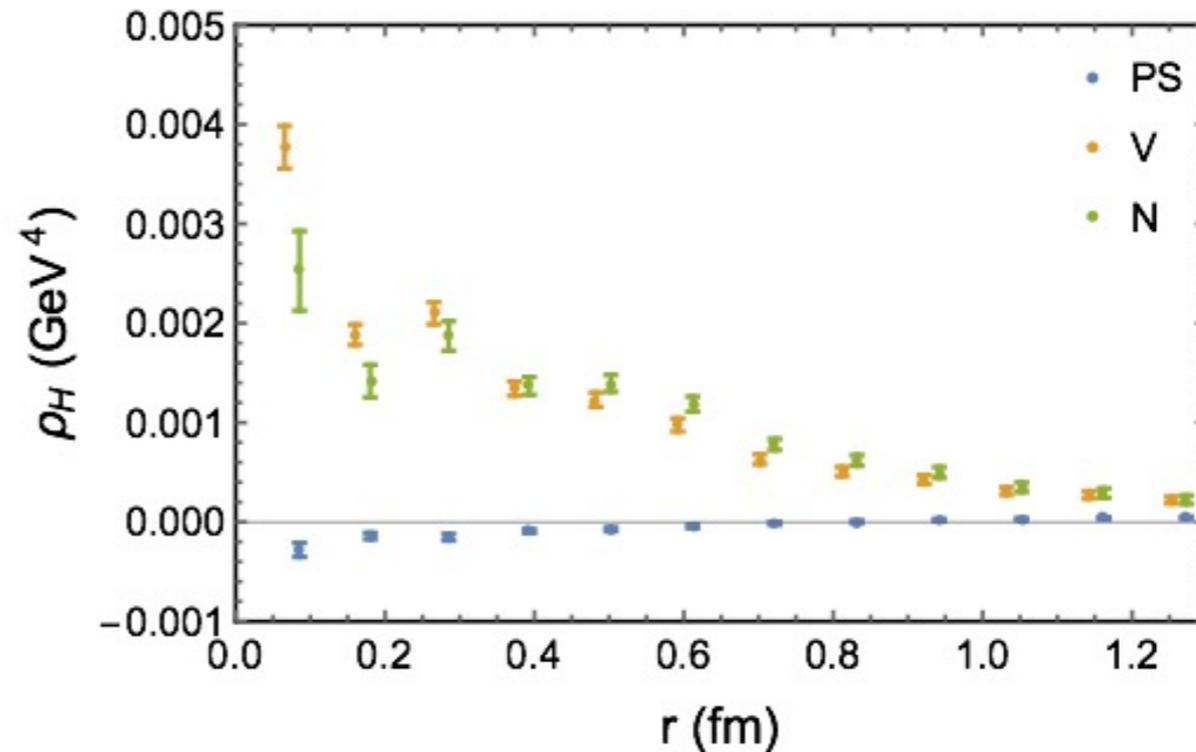
- Schematic description



# Numerical Results

F. He, P. Sun and Y. Yang, arXiv: 2101.04942.

- The density of gluon trace anomaly (  $\int d^3r \bar{\rho}_H(r) = \frac{\beta}{2g} \langle F^2 \rangle_H$  ) at the unitary point ( $M_\pi = 340 MeV$ )



- The density of gluon trace anomaly in pseudoscalar meson is negative, make the total trace anomaly of gluon is smaller than that in other hadron.

# Summary

- **Summary**

- ① **We calculate the contribution of quark mass and trace anomaly in different hadron, we also verify the mass sum rule, the hadron mass obtained from sum rule is consistent with its ground state mass.**
- ② **We determine the values of  $\gamma_m$  and  $\beta/g^3$ ,  $\gamma_m$  is comparable with the perturbative result,  $\beta/g^3$  is perfectly consistent with the regularization independent leading ordering term.**
- ③ **We find the gluon trace anomaly contribute most of the hadron masses, except the pion case.**
- ④ **The density of gluon trace anomaly in pseudoscalar meson is negative near the center and the magnitude is smaller than that in other hadron.**

**Thanks for your attention!**

# Backup

# The Effect of Heavy quark

- Trace term of ETM

$$T_{\mu}^{\mu} = \frac{\beta_{QCD}}{2g} F^2 + \sum_l m_l (1 + \gamma_{m,l}) \bar{l}l + \sum_h m_h (1 + \gamma_{m,h}) \bar{h}h$$

- The heavy quark terms can be changed into

M.A. SHIFMAN et,al. PLB78(1978)

$$m_h \bar{h}h \rightarrow -\frac{2}{3} \frac{\alpha}{8\pi} n_h F^2 + O(1/m_h)$$

- The final expression of trace term is

$$T_{\mu}^{\mu} = \frac{\tilde{\beta}}{2g} F^2 + \sum_l m_l (1 + \gamma_{ml}) \bar{l}l + O(1/m_h)$$

# Trace anomaly in perturbation theory

Y. Hatta et al., *JHEP* 12 (2018) 008  
 R. Tarrach, *NPB* 196 (1982) 45-61

- The trace of ETM in d dimension

$$T_\alpha^\alpha = -2\epsilon \frac{F^2}{4} + \bar{\psi} i \overleftrightarrow{D} \psi = -2\epsilon \frac{F^2}{4} + m\bar{\psi}\psi.$$

In d dimension

- Renormalization of FF.

$$F^2 = \left(1 - \frac{\beta}{g} \frac{1}{\epsilon}\right) (F^2)_R - 2 \frac{\gamma_m}{\epsilon} (m\bar{\psi}\psi)_R$$

The bare operator FF is divergent

$$\begin{aligned} T_\mu^\mu &= -2\epsilon \frac{F^2}{4} + m\bar{q}q \\ &= \underbrace{\frac{\beta(g)}{2g} (F^2)_R + \gamma_m (m\bar{q}q)_R}_{\text{from } (T_g)_\mu^\mu} + \underbrace{(m\bar{q}q)_R}_{\text{from } (T_q)_\mu^\mu} \end{aligned}$$

For the bare ETM, the anomaly entirely comes from the gluon part