MOM and SMOM renormalization using the chiral fermion

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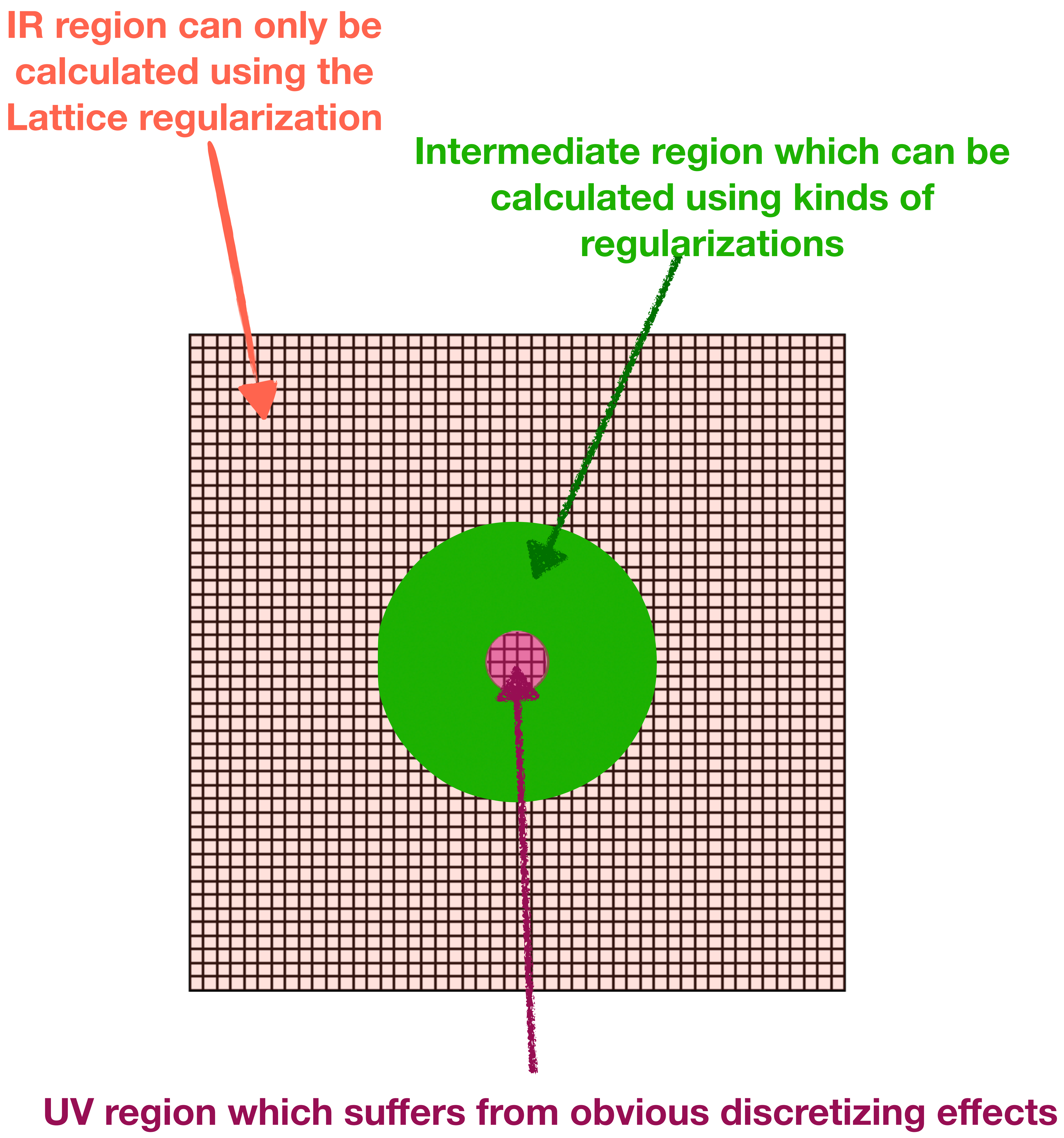
Renormalization

- The phenomenological studies are carried out under the modified minimum subtraction scheme ($\overline{\mathrm{MS}}$).
- But it requires the modification of the dimension and then can not be done on the lattice!
- Thus an intermediate regularization-independent scheme is essential to convert the lattice result to $\overline{\mathrm{MS}}$ scheme.



Effective renormalization constant

- Obtain the regularization independent renormalization constant non-perturbatively:

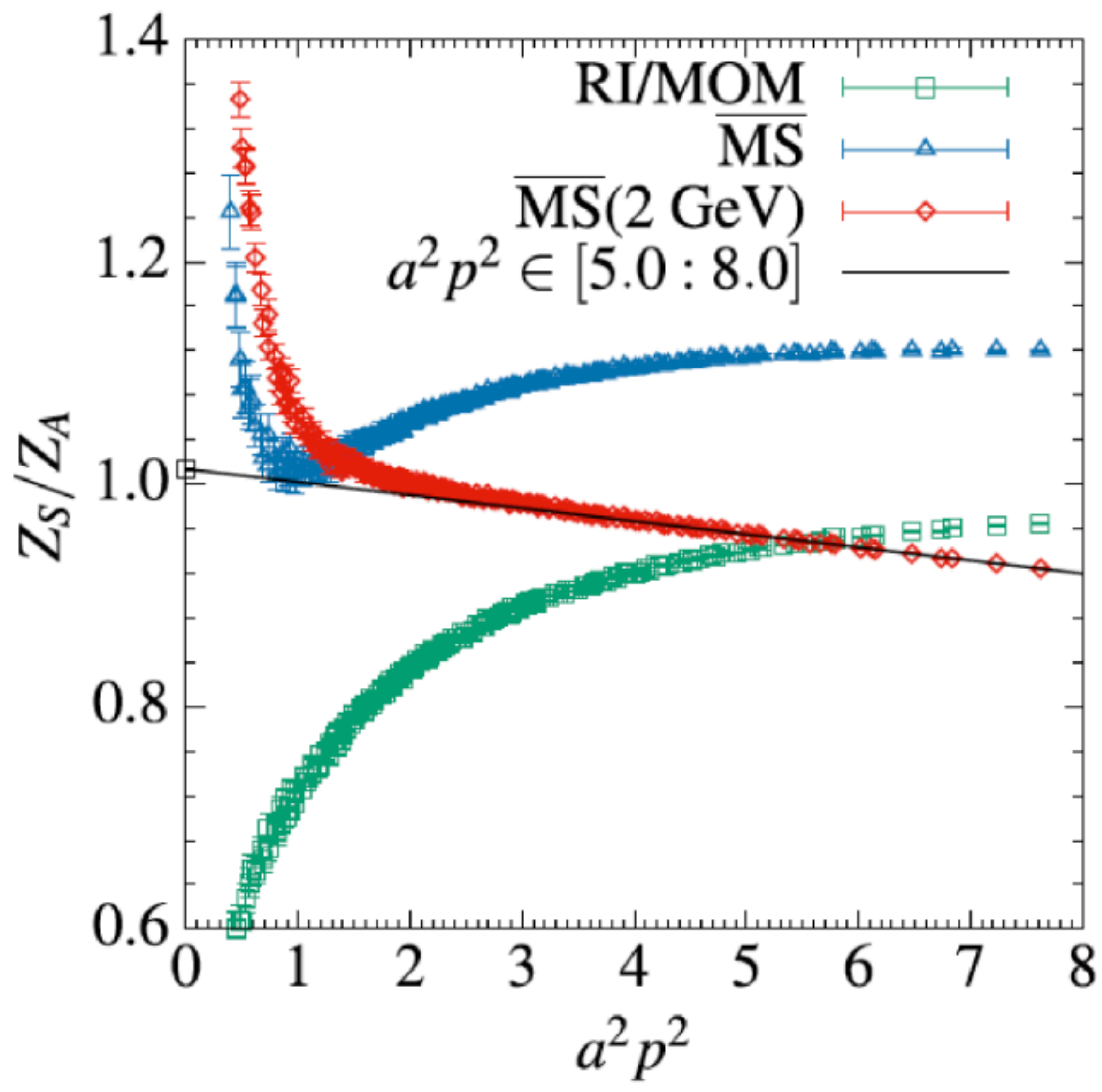
$$Z_S^{\mathrm{MOM}}(Q,a) = 1 - \frac{\alpha_s C_F}{4\pi}[-3\log(a^2Q^2) - \xi + b_S] + \mathcal{O}(\alpha_s^2, a^2Q^2)$$

- Calculate the matching coefficient perturbatively and obtain the result at $\overline{\text{MS}}$ scale Q:

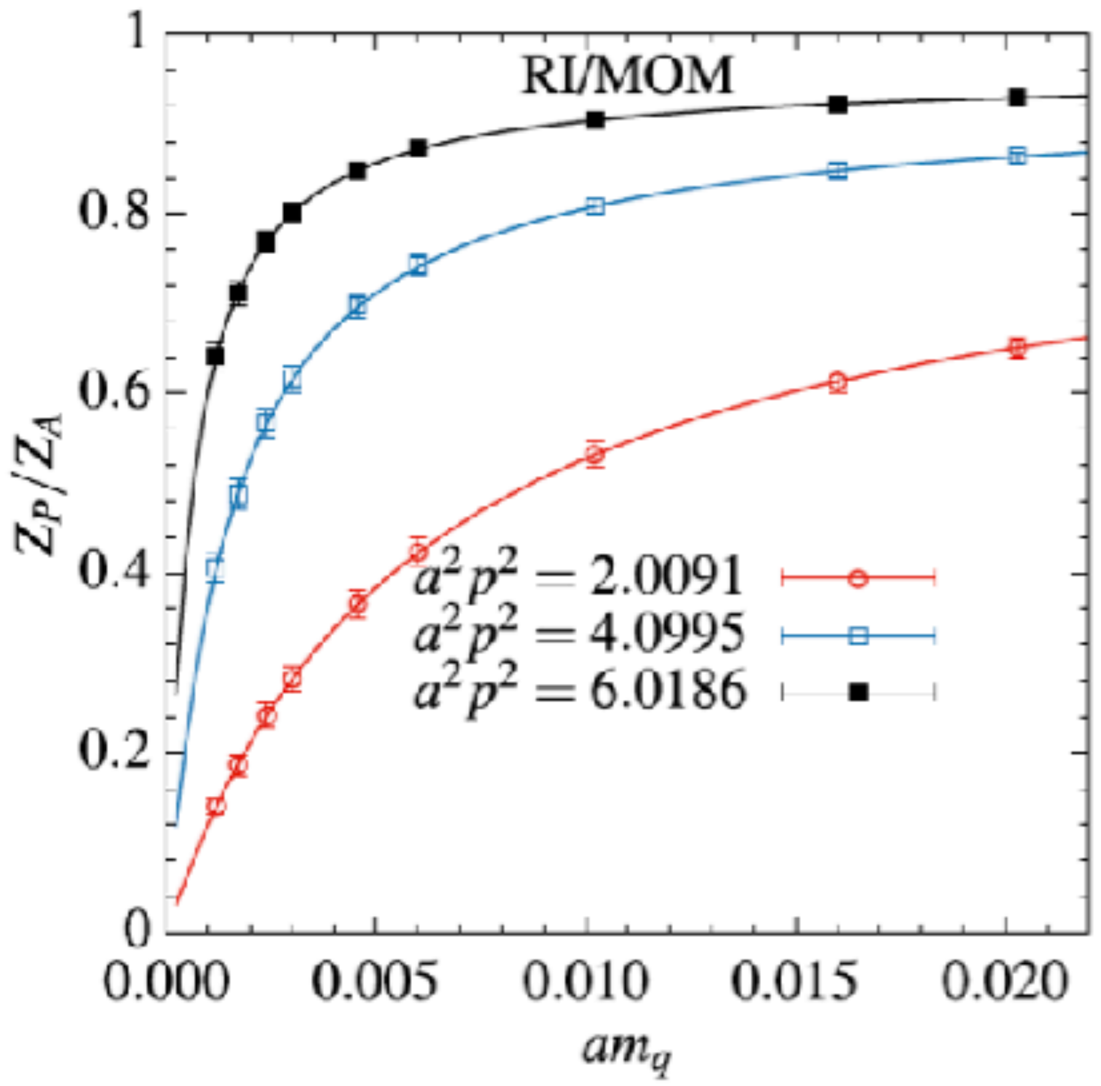
$$Z_S^{\overline{\text{MS}}}(Q,a) = 1 - \frac{\alpha_s C_F}{4\pi}[-3\log(a^2Q^2) - 5 + b_S] + \mathcal{O}(\alpha_s^2, a^2Q^2)$$

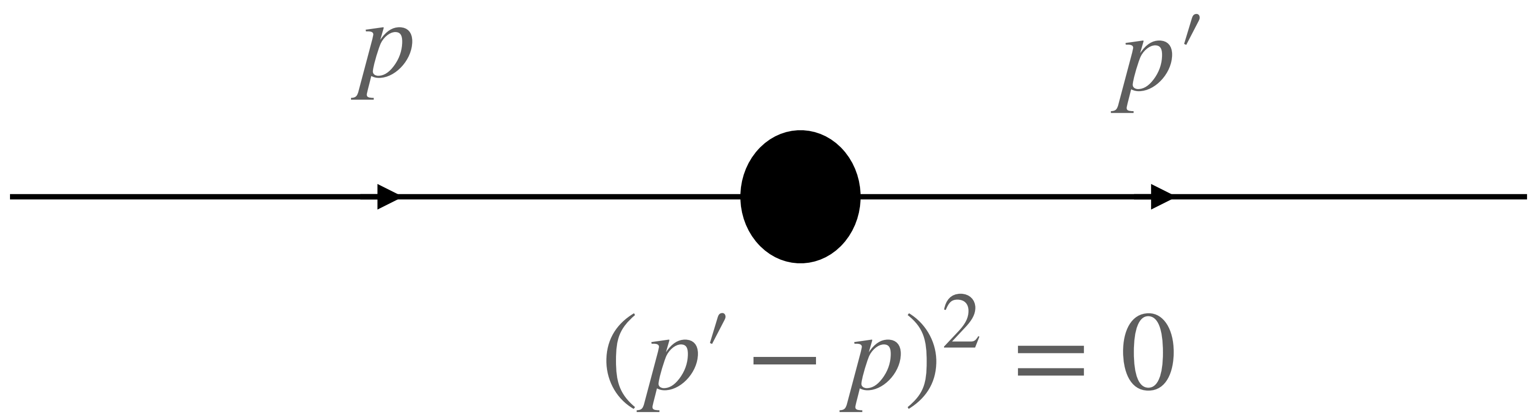
- Obtain the result at $\overline{\text{MS}}$ scale μ with the scale evolution:

$$Z_S^{\overline{\text{MS}}}(\mu,a) = 1 - \frac{\alpha_s C_F}{4\pi}[-3\log(a^2\mu^2) - 5 + b_S] + \mathcal{O}(\alpha_s^2, a^2Q^2)$$



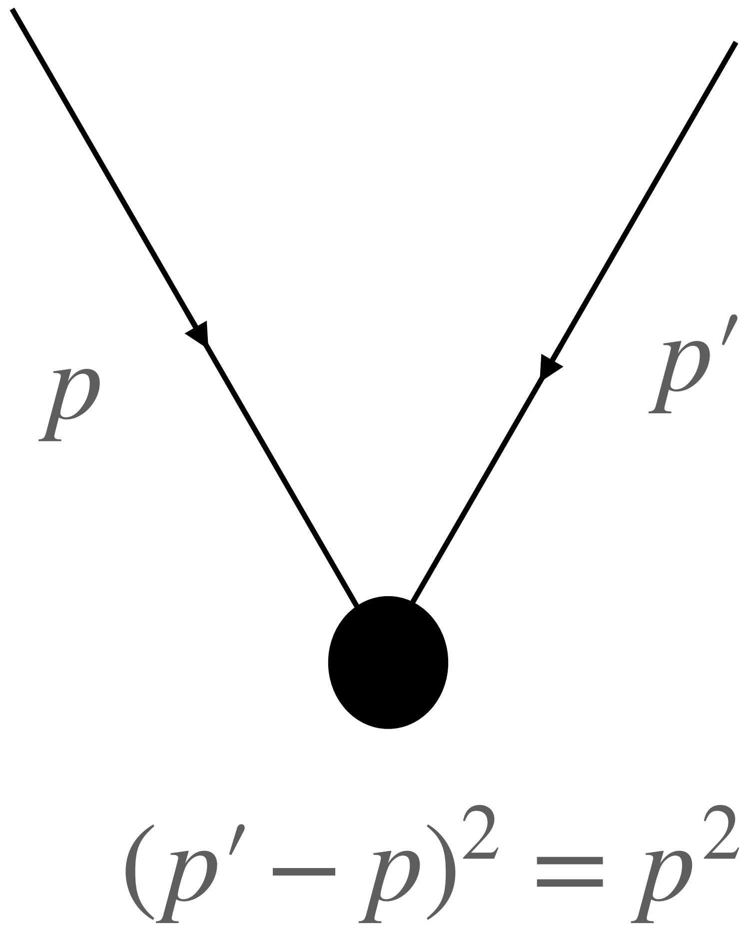
MOM v.s. SMOM

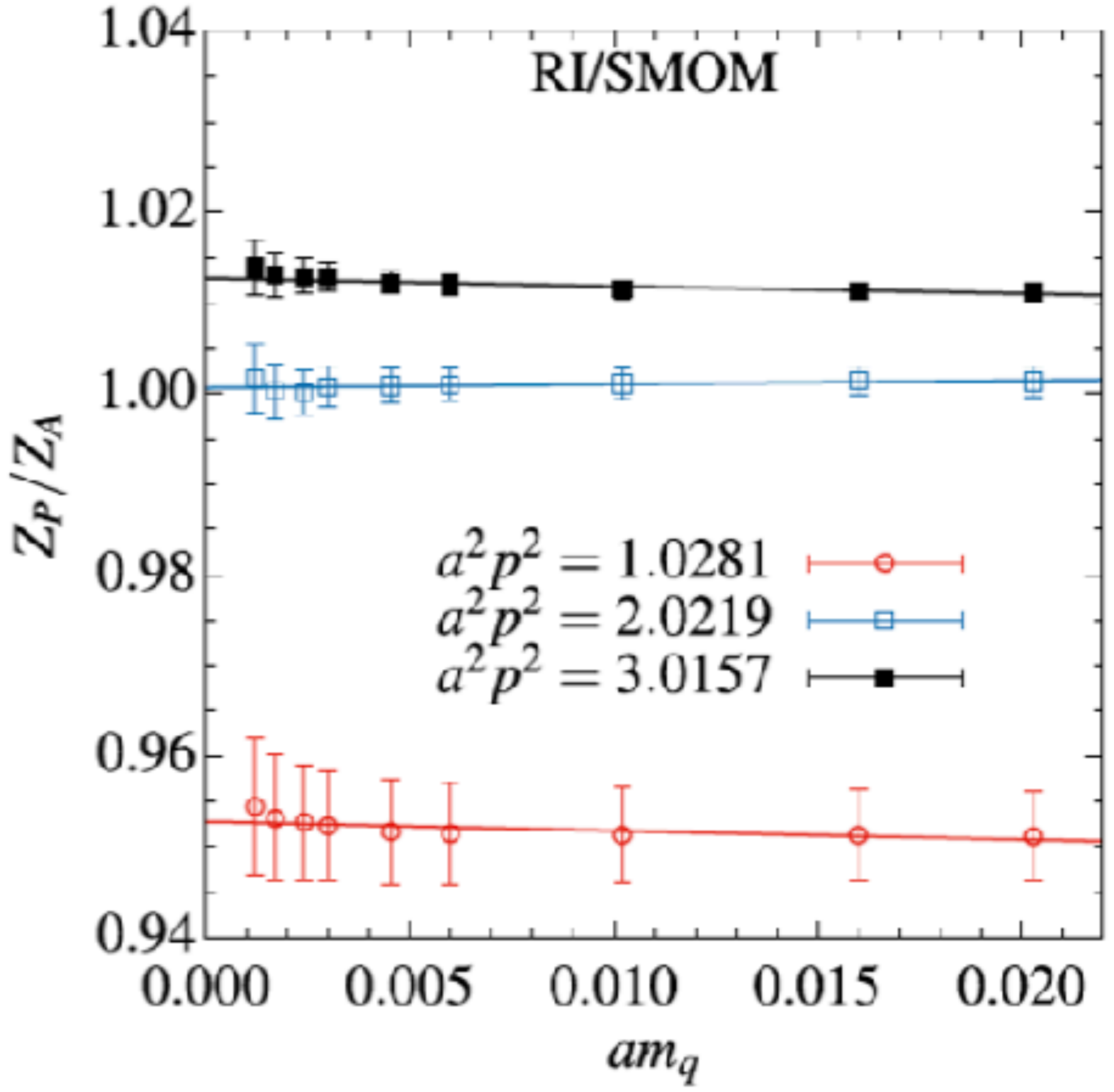




- **MOM scheme**: The zero momentum transfer at the current can introduce additional mixing/non-perturbative effect;

- **SMOM scheme**: Requiring the momentum transfer to be non-zero can avoid such a problem.

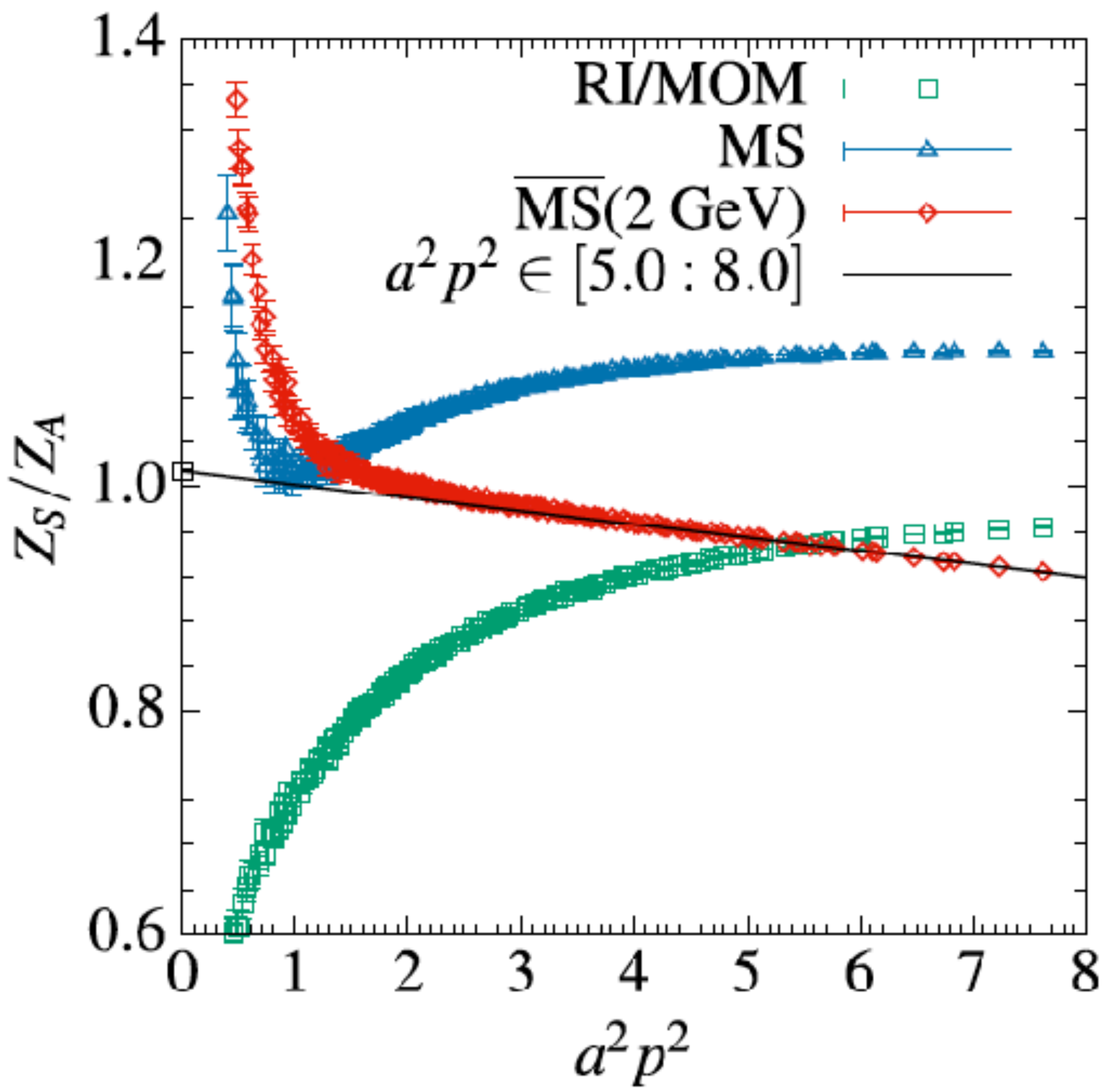


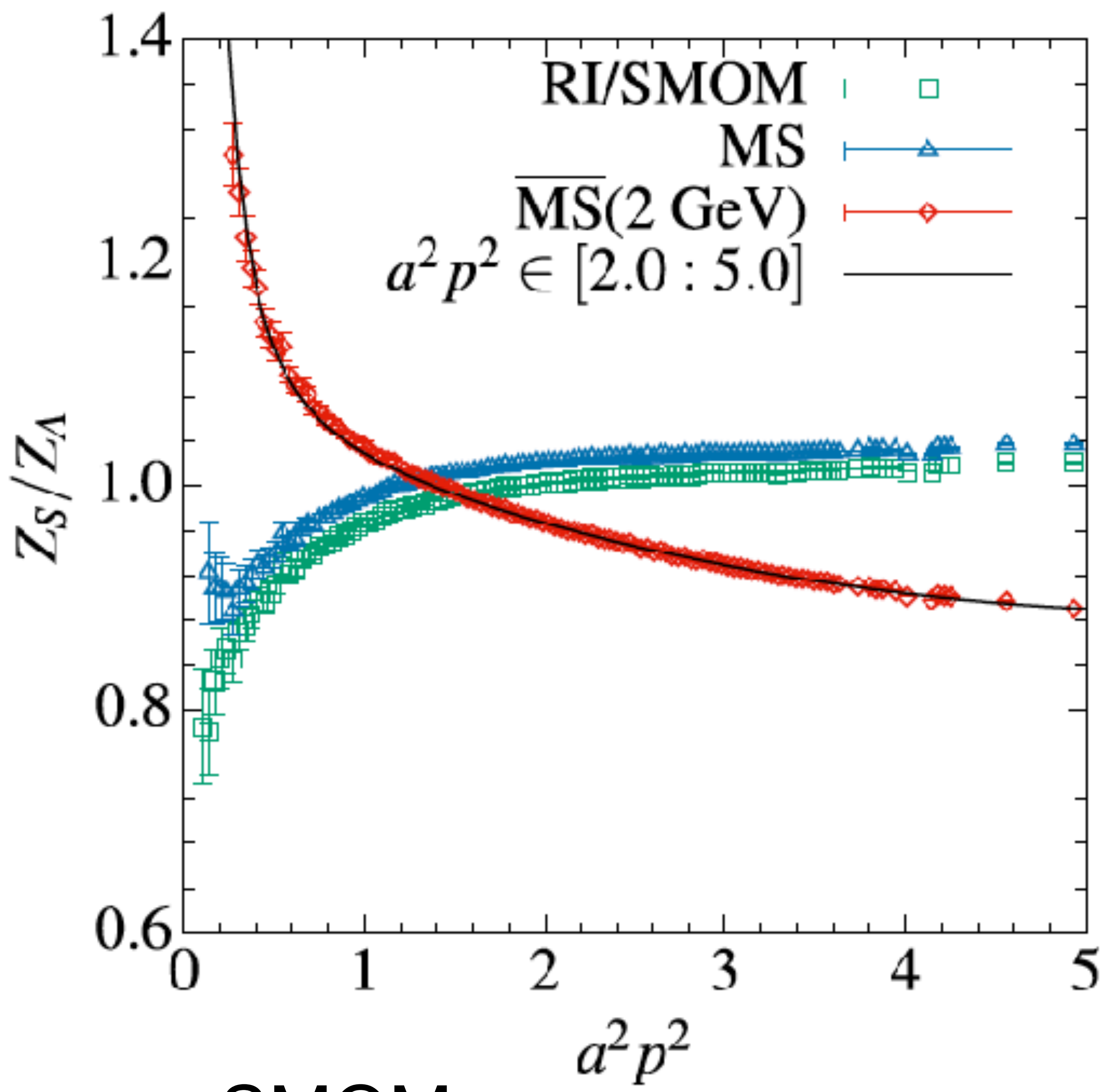


Y. Aoki, et.al, PRD78(2009)054520, 0712.1061
C. Sturm. et.al, PRD90(2009)014501, 0901.2599

Our previous SMOM result

Yujiang Bi, et.al, χQCD, PRD97(2018)094501, 1710.08678





- a=0.114 fm
- $48^3 \times 96$
- Overlap on DWF
- $m_\pi^{\mathrm{sea}} = 139$ MeV

- MOM:
  - Linear a^2p^2 dependence
  - Larger matching correction:

$$\frac{Z_S^{\overline{\mathrm{MS}}}}{Z_S^{\mathrm{MOM}}}(4\mathrm{GeV}) = 1 + 0.092 + 0.047 + 0.028 + \mathcal{O}(\alpha_s^4)$$

- SMOM:
  - Non-linear a^2p^2 dependence,
  - Smaller matching correction:

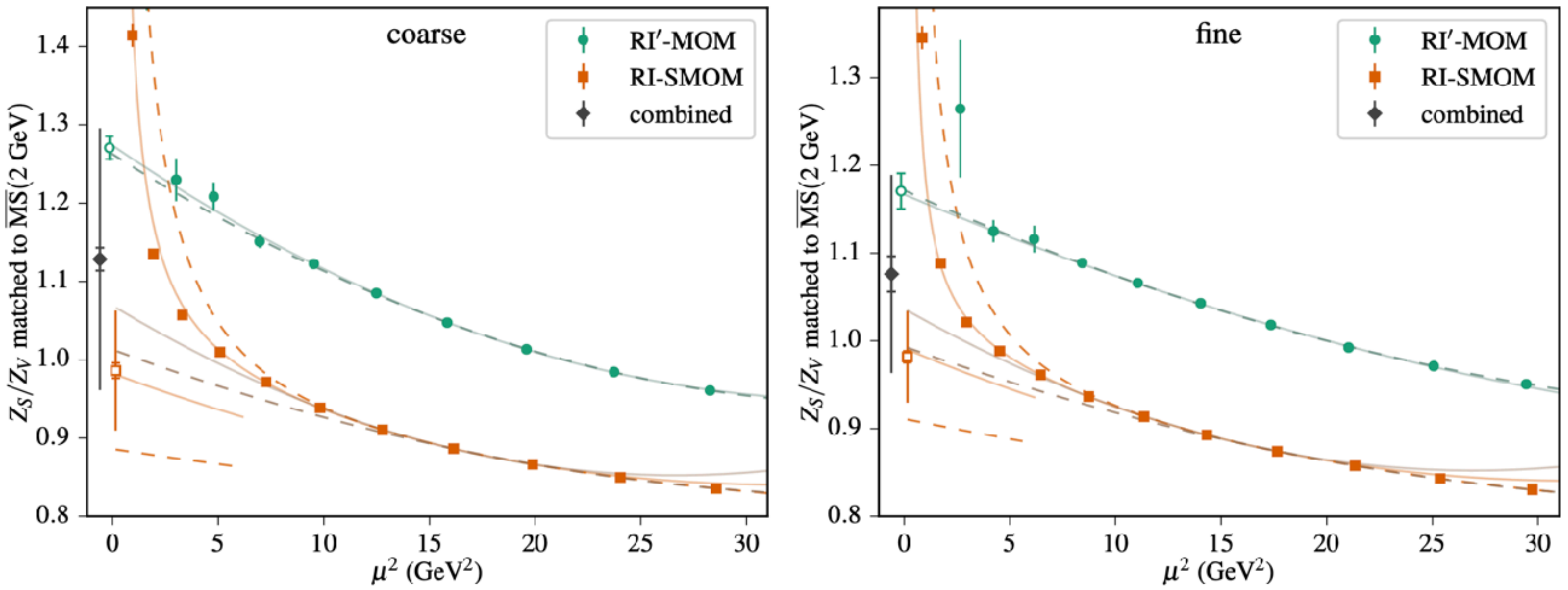
$$\frac{Z_S^{\overline{\mathrm{MS}}}}{Z_S^{\mathrm{SMOM}}}(4\mathrm{GeV}) = 1 + 0.011 + 0.003 + \mathcal{O}(\alpha_s^3)$$

Observation from the other group

- Similar issue is also observed using the clover fermion;
- While the different between the MOM and SMOM scheme seems to be even larger.

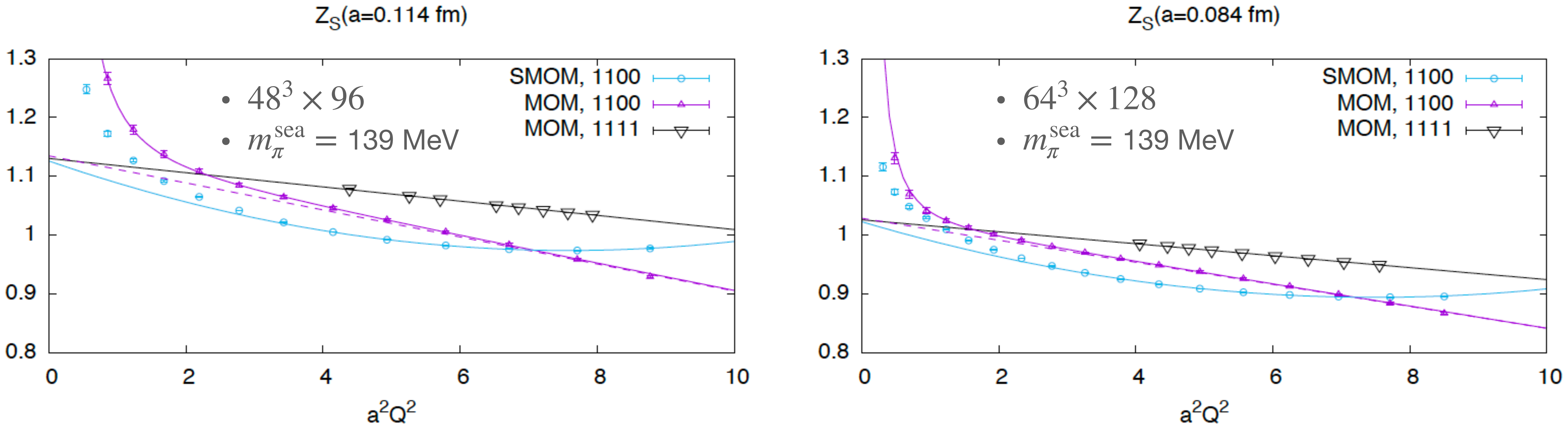
- a=0.116 fm
- 48^4
- Clover
- $m_\pi^{\mathrm{sea}} = 137$ MeV

- a=0.093 fm
- 64^4
- Clover
- $m_\pi^{\mathrm{sea}} = 133$ MeV



Nesreen Hasan, et.al, PRD99(2019)114505, 1903.06487

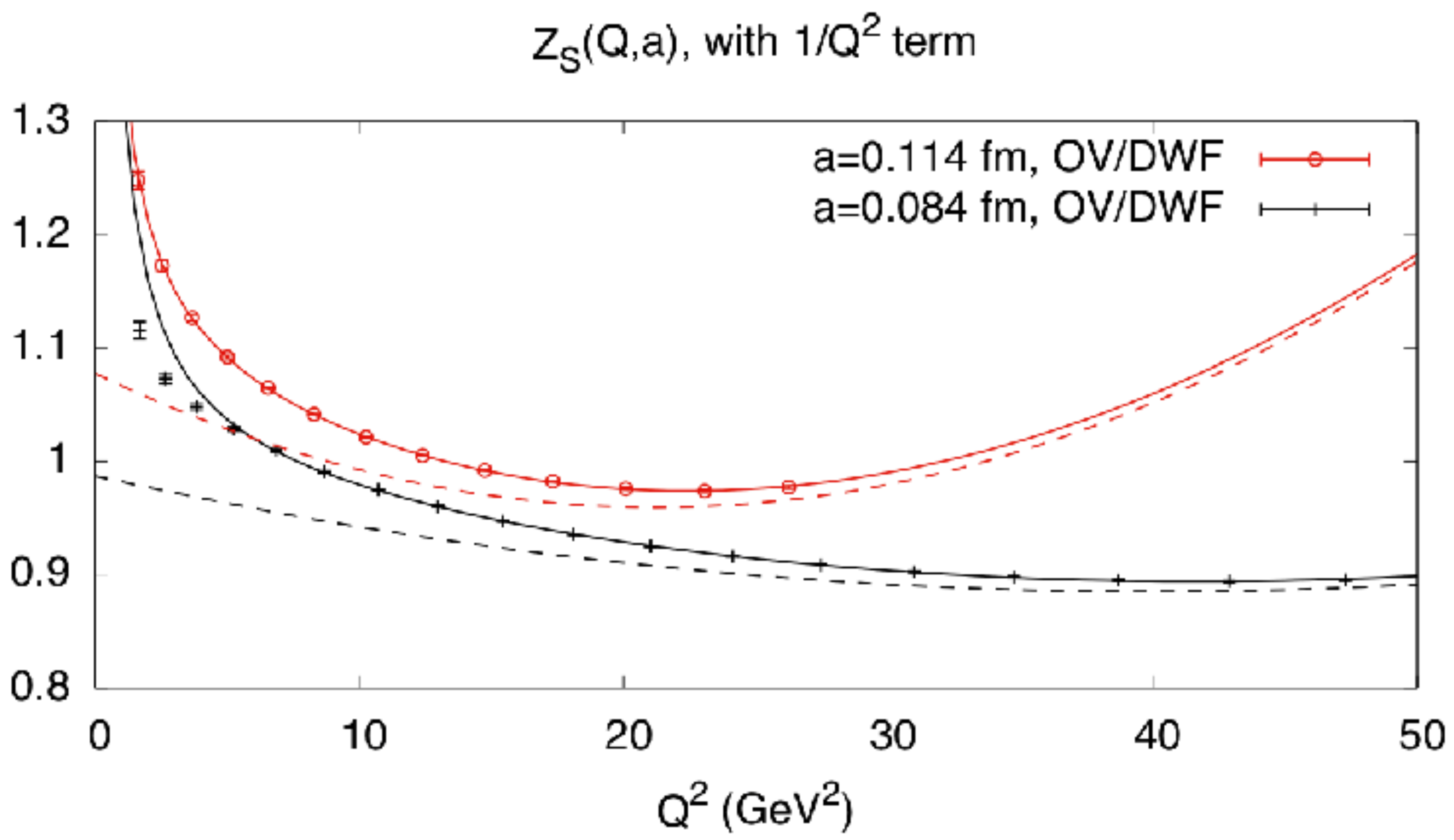
OV@DWF at different lattice spacing



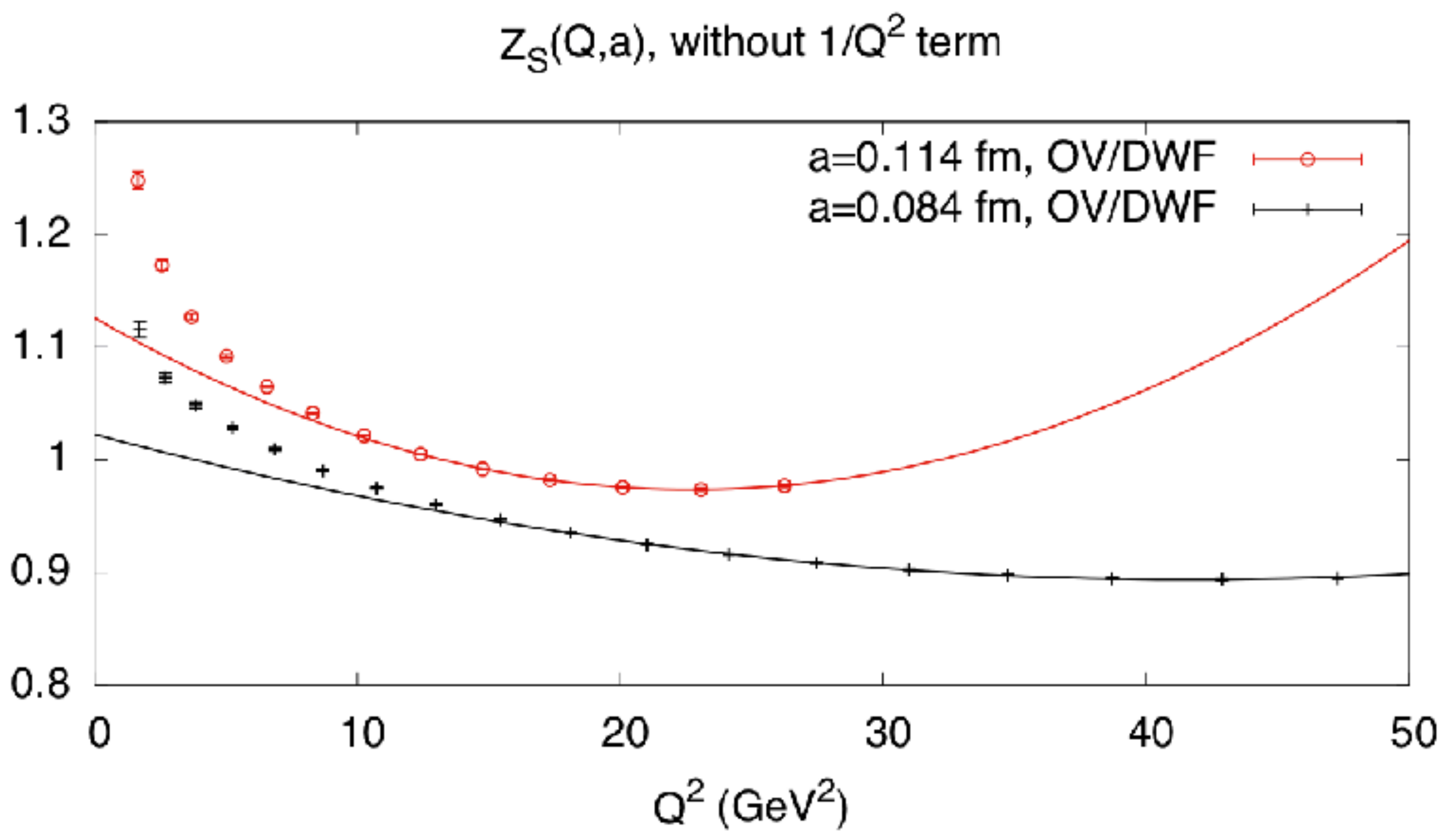
- Use the polynomial form in a range of $Q^2 \in [Q^2_{min}, 9/a^2]$ and tune the Q_{min} to make $\chi^2/\mathrm{d.o.f} \sim 1$ for the SMOM result;
- The results from different scheme and momentum setups seem to be consistent with each other;
- But……

Jian Liang, et.al, χQCD, 2102.05380

Fitting Ansatz Dependence



$$Z_S^{\mathrm{SMOM,b}}(Q^2) = \frac{c_{-1}^{\mathrm{SMOM}}}{Q^2} + Z_S + c_1^{\mathrm{SMOM}} a^2 Q^2 + c_2^{\mathrm{SMOM}} a^4 Q^4$$



$$Z_S^{\mathrm{SMOM,a}}(Q^2) = Z_S + c_1^{\mathrm{SMOM}} a^2 Q^2 + c_2^{\mathrm{SMOM}} a^4 Q^4$$

- Fit in a range of $Q^2 \in [Q^2_{min}, 9/a^2]$ and tune the Q_{min} to make $\chi^2/\mathrm{d.o.f} \sim 1$;
- The discretization errors are close to each other in different cases;
- The difference between Z_S with different fitting ansatz become closer at smaller lattice spacing.

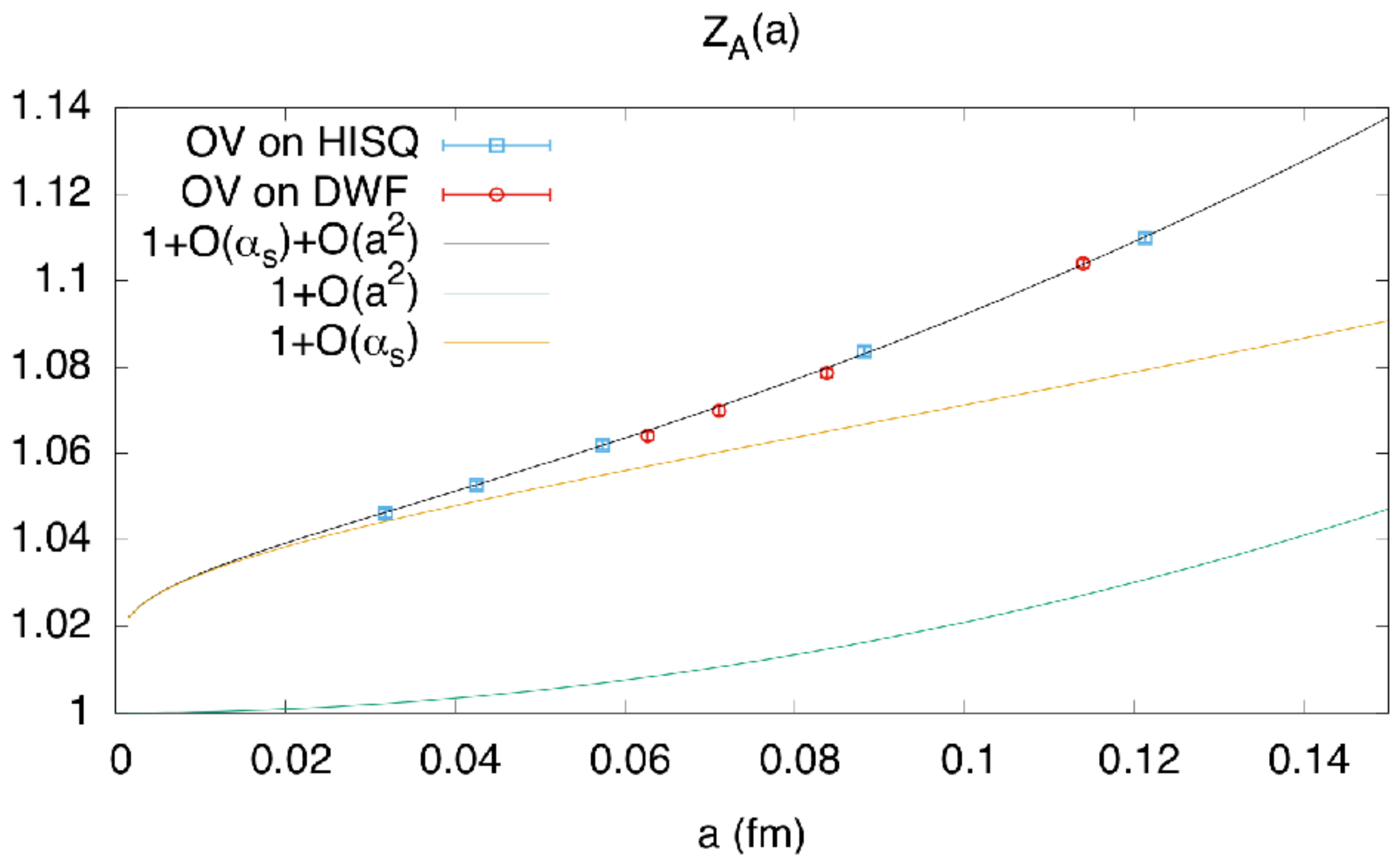
a (fm)	$\lvert Q_{min}\rvert$ (GeV)	c_{-1}^{SMOM} (GeV^2)	Z_S	c_1^{SMOM}	c_2^{SMOM}
0.114 fm	3.2		1.126(2)	-0.040(1)	0.0026(1)
	1.2	0.32(1)	1.078(3)	-0.033(1)	0.0024(1)
0.084 fm	4.1	–	1.023(2)	-0.034(1)	0.0023(1)
	2.0	0.37(2)	0.987(3)	-0.028(1)	0.0020(1)

Jian Liang, et.al, χQCD, 2102.05380

(Axial-)Vector current normalization

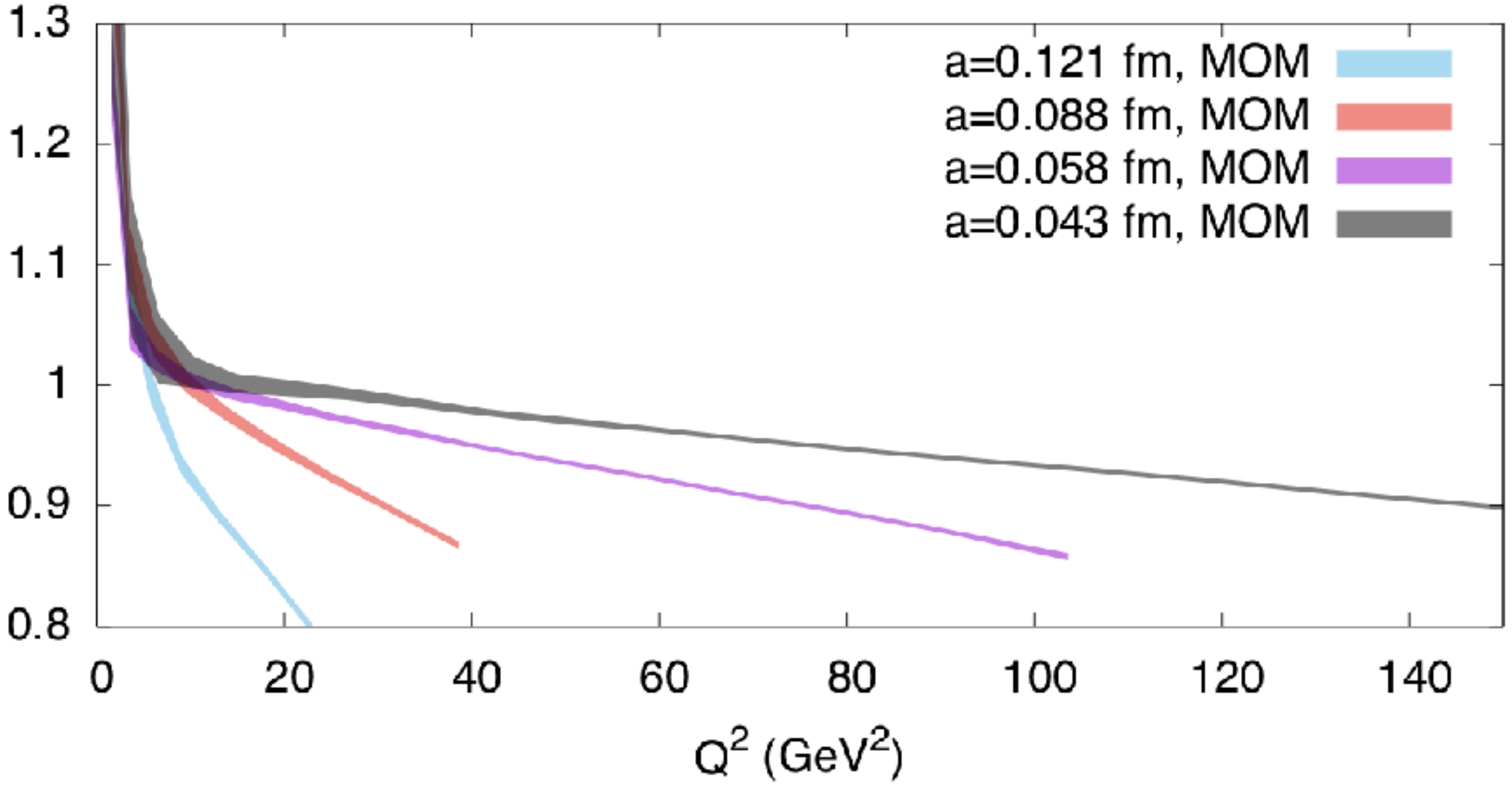
- The axial-vector current normalization constants using the overlap fermion seems to be insensitive to the fermion and gauge action in sea;

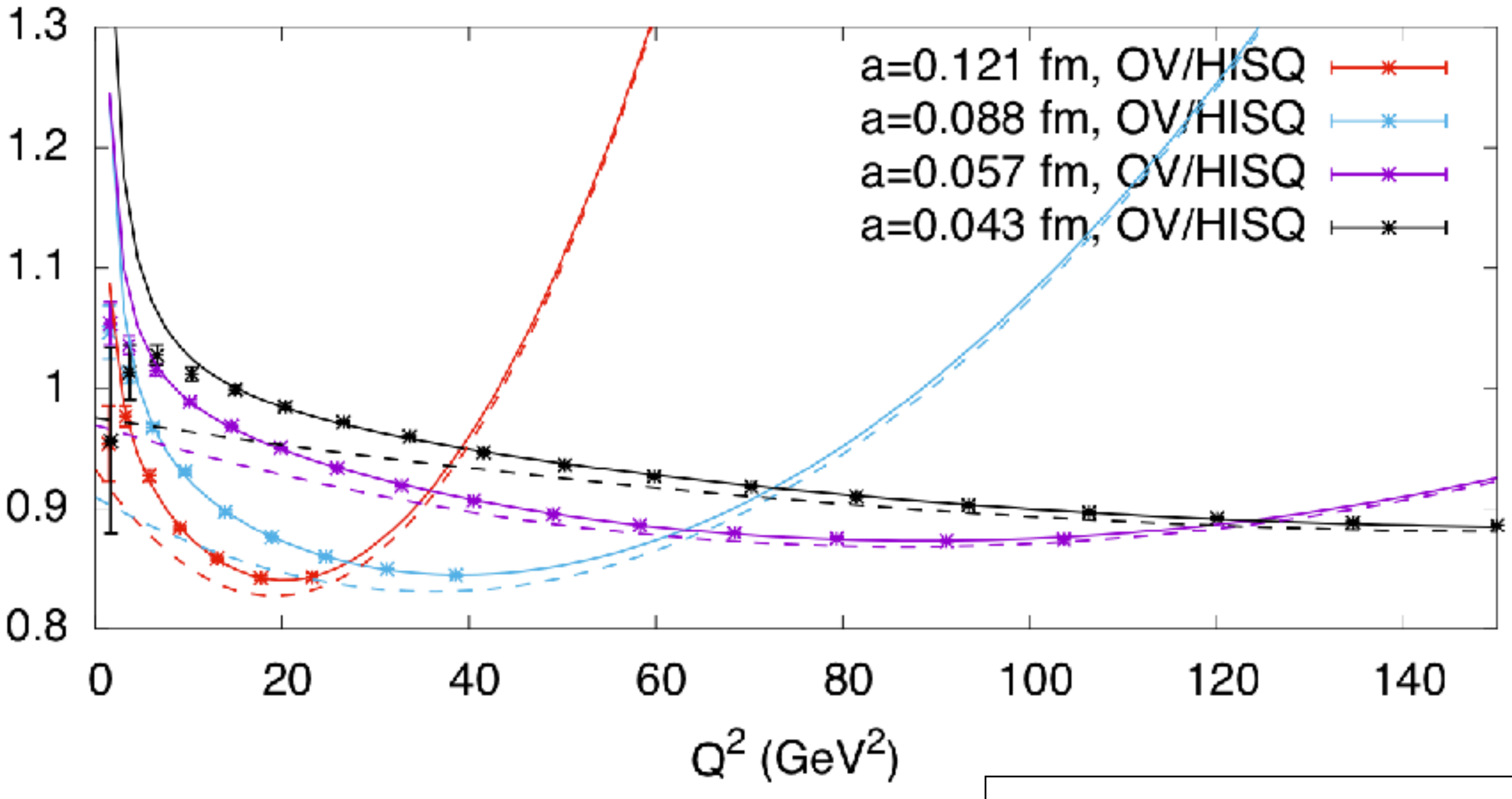
- We can consider the OV@HISQ cases to further investigate the lattice spacing dependence.



Kuan Zhang, et.al, χQCD, 2012.05448

OV@HISQ at 310 MeV





Fangcheng He, et.al, χQCD, in preparation

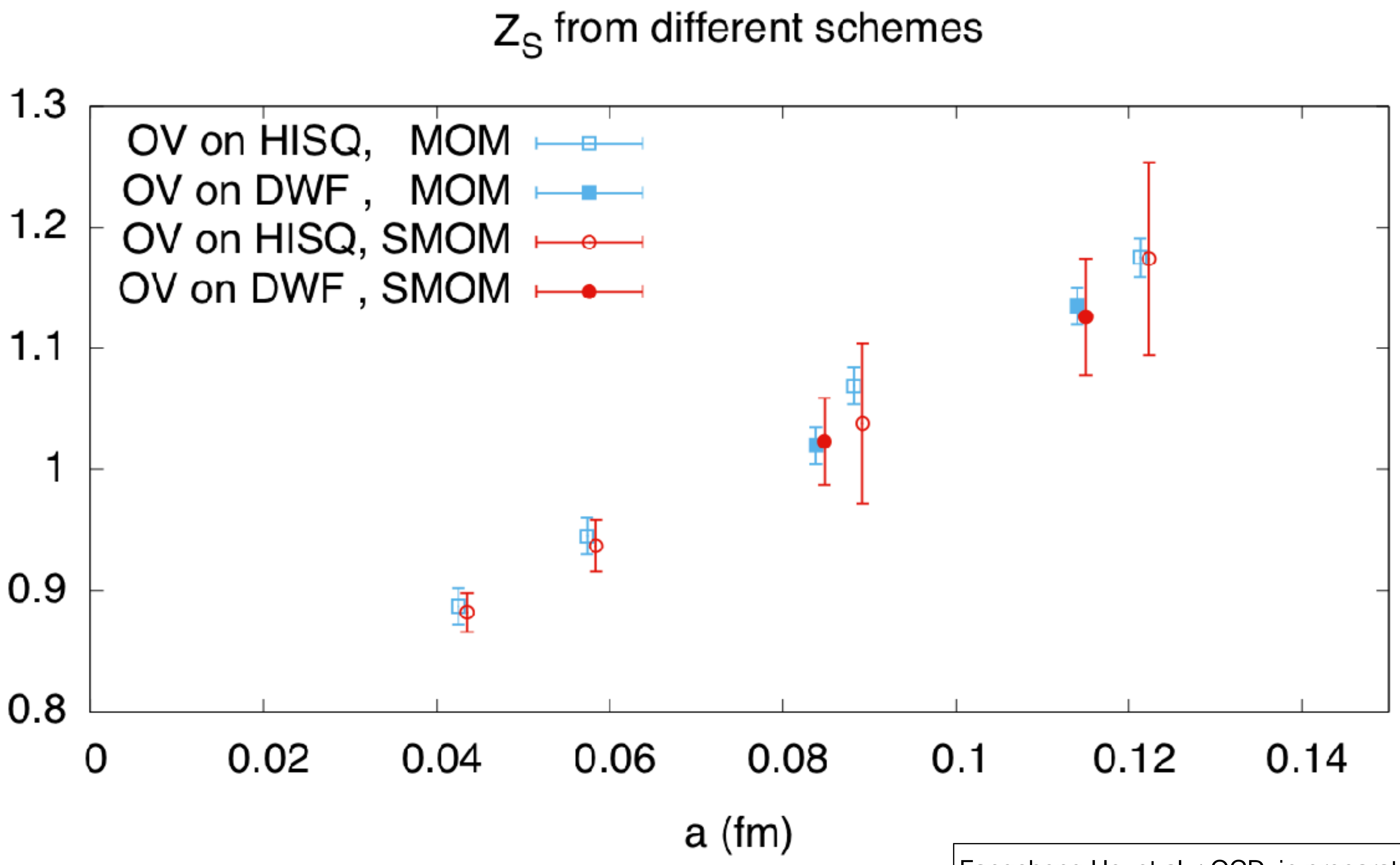
$Z_S^{\overline{\mathrm{MS}}}(2\mathrm{GeV})$ is normalized by its a^2Q^2 extrapolated value to show the residual Q^2 dependence:

- The residual Q^2 dependence becomes weaker at smaller lattice spacing.

$Z_S^{\overline{\rm MS}}(2{\rm GeV})$ results

The uncertainty is majorly the systematic one from:

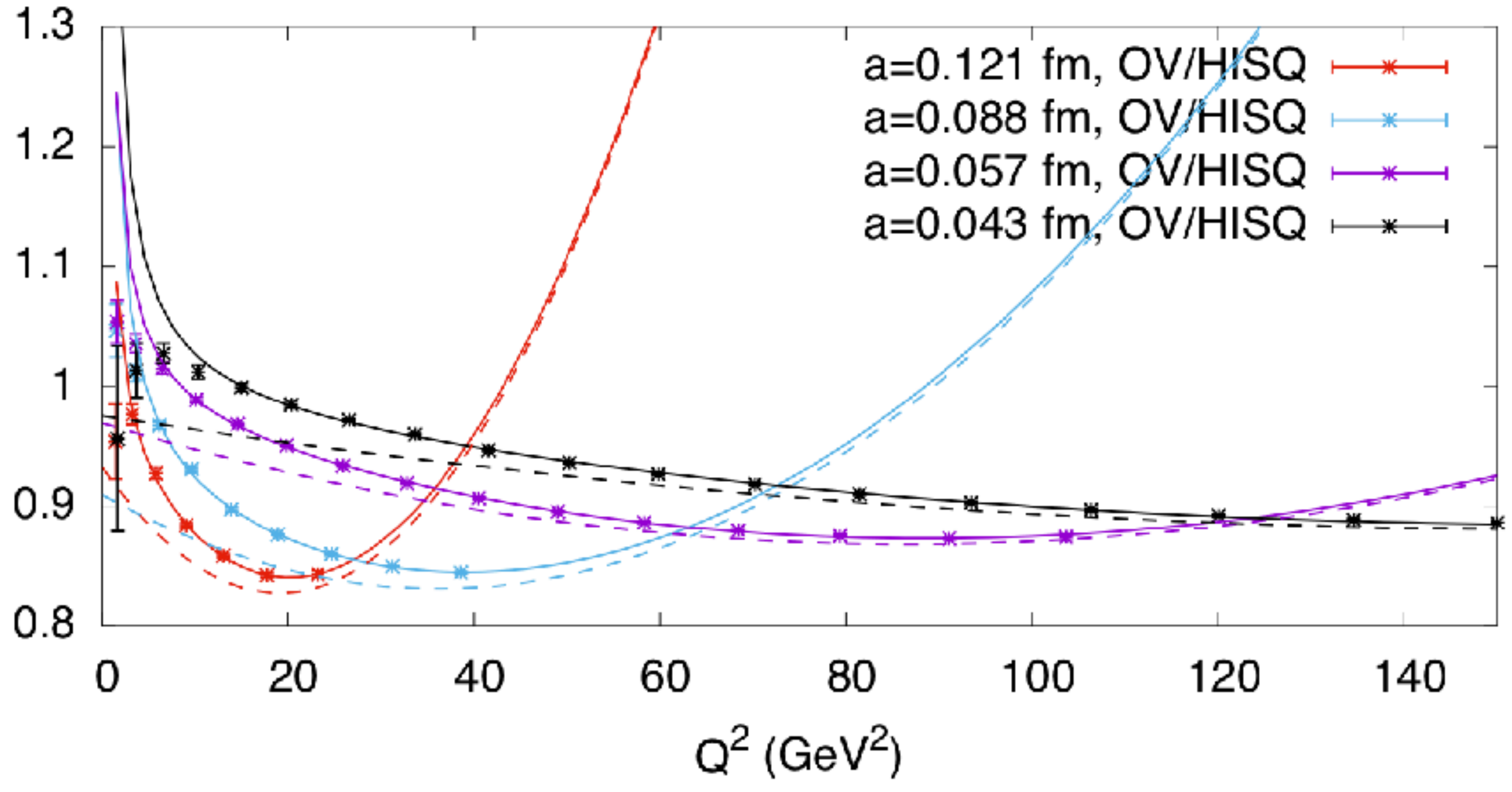
- Missing 4-loop matching (~1.5 % at 4 GeV) in the MOM case;
- Difference from two kinds of the fitting ansatz in the SMOM case.
- Such an uncertainty would vanishes for both the schemes in the continuum and provide consistent $Z_S^{\overline{\rm MS}}(2{\rm GeV})$.



Fangcheng He, et.al, χQCD, in preparation

Summary

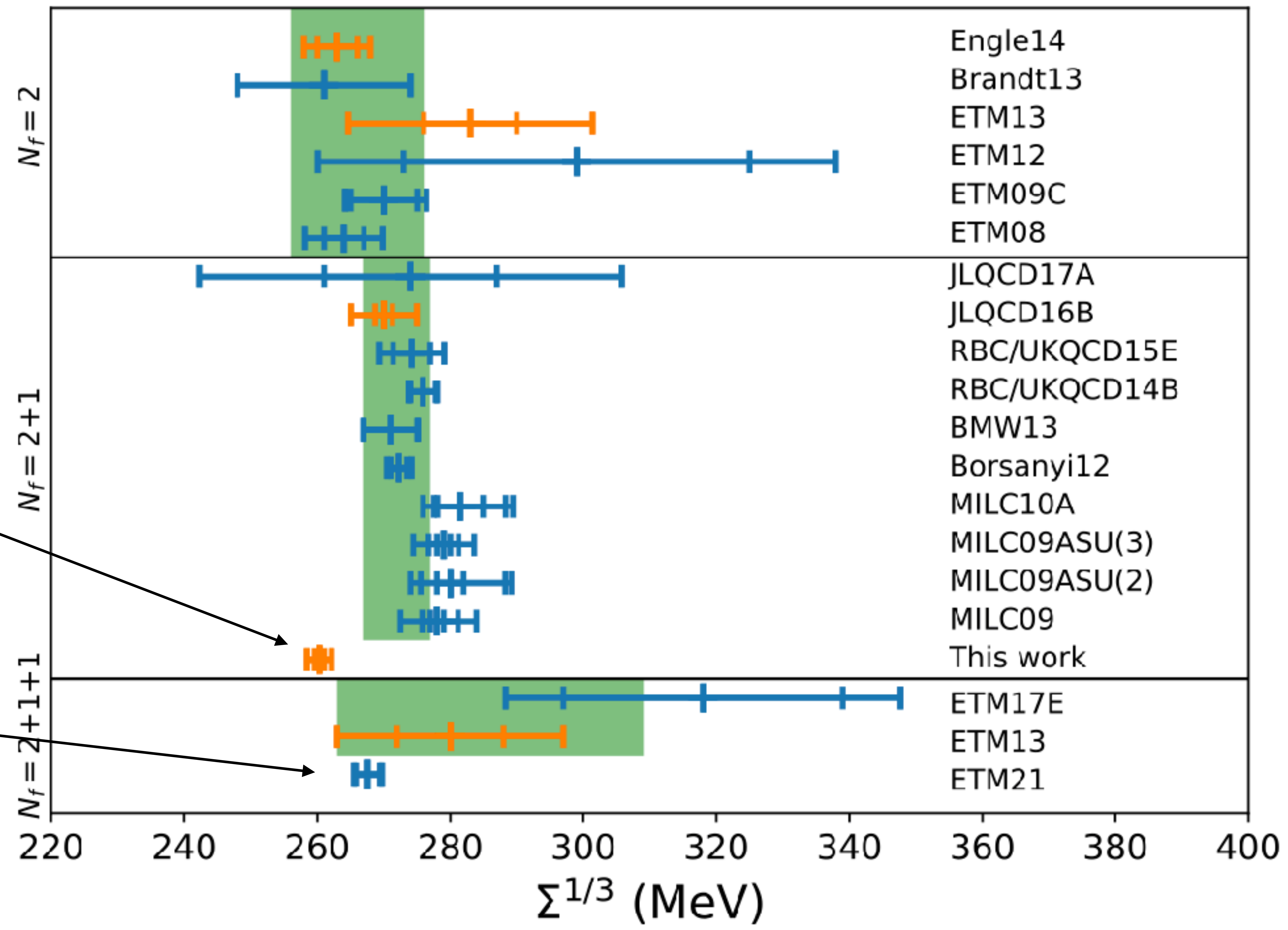
- The SMOM renormalization for the scalar current can have additional systematic uncertainty from the fitting ansatz;
- Such an uncertainty would vanishes in the continuum, if we drop the small Q and do the extrapolation from large Q.
- It would be essential to if anyone is using the SMOM scheme for the scalar current/quark mass renormalization.



Fangcheng He, et.al, χQCD, in preparation

Chiral condensate in the $N_f = 2$ chiral limit

- Blue for pion mass spectrum;
- Orange for the Dirac operator spectrum;
- Our result seems to be much smaller the previous results using the SMOM scheme;
- The recent ETM result using the MOM scheme is in the middle.



Jian Liang, et.al, χQCD, 2102.05380