Valence structure of pion: physical mass, chiral quarks

Xiang Gao
BNL & CCNU

In Collaboration with N. Karthik, S. Mukherjee,
P. Petreczky, S. Syritsyn and Y. Zhao

LATTICE 21
JULY 26-30 2021, ZOOM/GATHER@MIT

Pion valence quark PDF

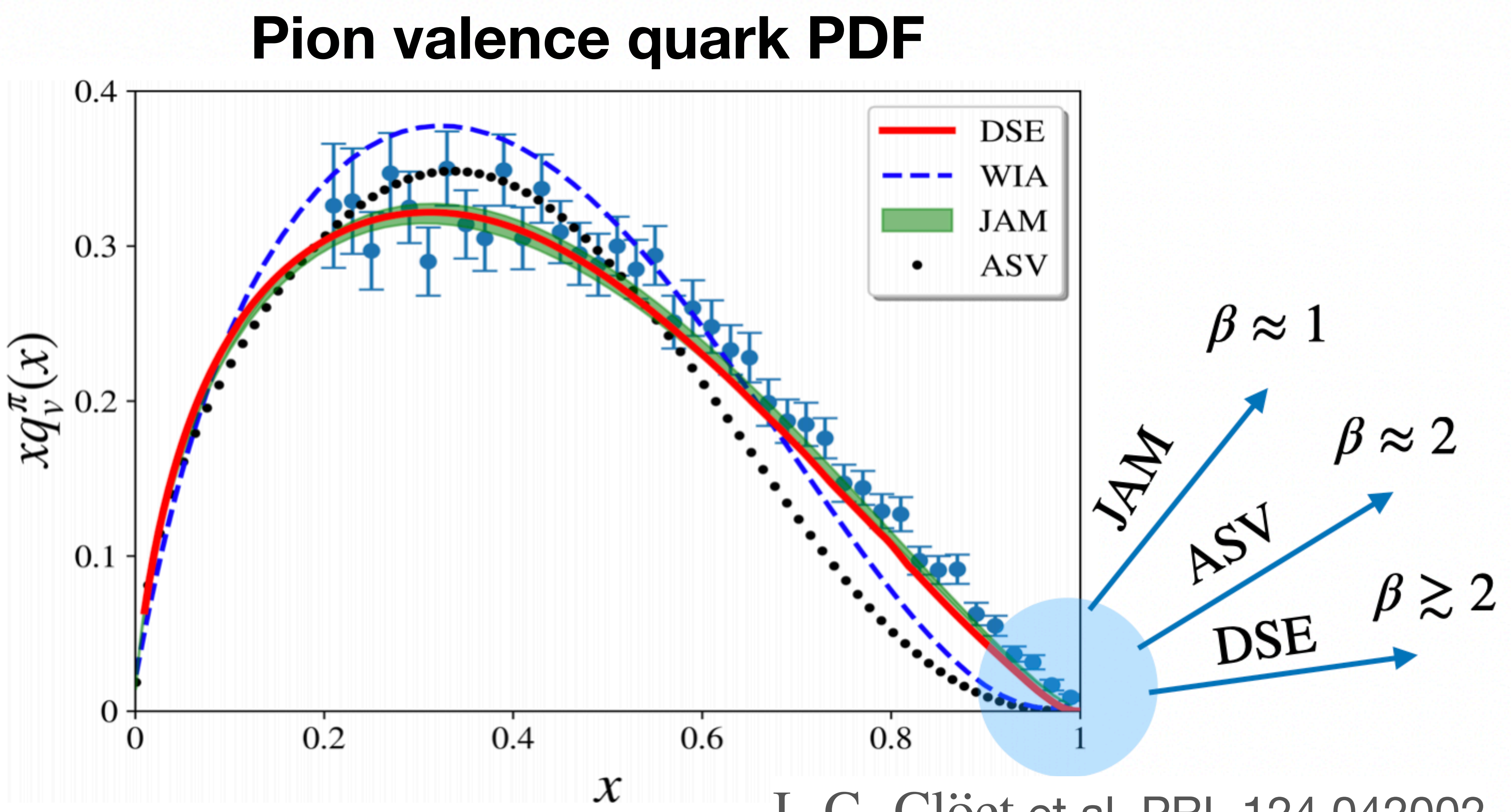
Pion play a central role in the study of the strong interactions.

$$m_\pi \approx 140 \text{ MeV} \xrightarrow[\text{m}_\text{q}=0]{\text{chiral limit}} 0$$

- Critical ingredient for understanding the dynamical chiral symmetry breaking in QCD.
- Quarks and gluons in massless NG bosons.
- …

However, the absence of fixed pion targets has made it difficult to determine the pion's structure experimentally. One of the key physics issue is x=1 behavior:

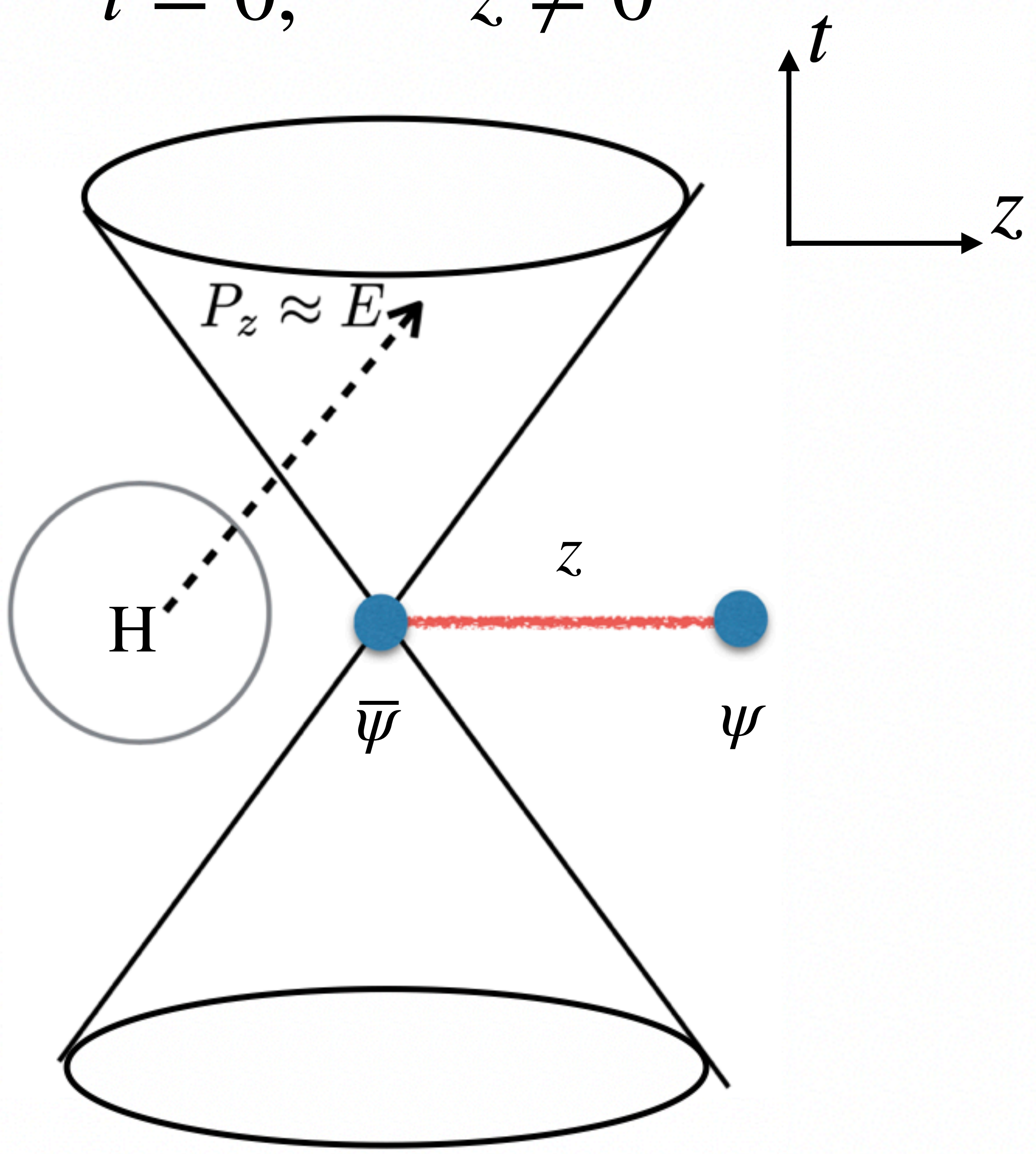
$$\lim_{x\to 1} f_v^\pi(x) \sim (1-x)^\beta$$



I . C . Clöet et al. PRL.124.042002

Equal-time correlators and QCD factorization

$t = 0, \qquad z \neq 0$



Quasi PDF:

- X. Ji, PRL 110 (2013); SCPMA57 (2014);

$$\tilde{q}(x) \equiv \int \frac{dz}{4\pi} e^{-ixP_z z} \langle P | \tilde{O}_\Gamma(z,\epsilon) | P \rangle,$$

$$\tilde{O}_\Gamma(z,\epsilon) = \overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)$$

Quasi-PDFs Factorization of Large-momentum effective theory:

- X. Ji, PRL 110 (2013); SCPMA57 (2014);
- X. Ji, Y. Zhao, et al, arXiv:2004.03543

Quasi-PDF

$$\tilde{q}(x, P_z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{x^2 P_z^2}, \frac{\Lambda^2_{QCD}}{(1-x)^2 P_z^2}\right)$$

PDF

Perturbative kernel

Large P_z is essential

Short distance Factorization in coordinate space:

- V. Braun et al., EPJC 55 (2008)
- A. V. Radyushkin et al., PRD 96 (2017)
- Y. Ma et al., PRL 120 (2018)
- T. Izubuchi et al., PRD 98 (2018)

$$\langle P | \tilde{O}_\Gamma(z,\mu) | P \rangle = \sum_n C_n(\mu^2 z^2) \frac{(-izP_z)^n}{n!} \int_{-1}^{1} dy\, y^n q(y,\mu) + \mathcal{O}(z^2 \Lambda^2_{QCD})$$

Moments of PDF

Wilson coefficients

Small z and large P_z is essential

Pion valence quark PDF: Renormalization

The operator can be multiplicatively renormalized

- X. Ji, J. H. Zhang and Y. Zhao, PRL120.112001
- J. Green, K. Jansen and F. Steffens, PRL.121.022004

$$[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B$$

$$= e^{-\delta m(a)|z|}Z(a)[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

$$\delta m = m_{-1}/a + m_0$$

Ratio scheme renormalization

- A. V. Radyushkin et al., PRD 96 (2017)
- BNL, PRD 102 (2020)

$$h_B(z, P_z, a) = \langle P_z | [\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B | P_z \rangle$$

$$\frac{h_B(z, P_z, a)}{h_B(z, P_z^0, a)} = \frac{\sum_n C_n(\mu^2 z^2)\frac{(-izP_z)^n}{n!}\int_{-1}^{1} dy y^n q(y,\mu) + \mathcal{O}(z^2\Lambda_{QCD}^2)}{\sum_n C_n(\mu^2 z^2)\frac{(-izP_z^0)^n}{n!}\int_{-1}^{1} dy y^n q(y,\mu) + \mathcal{O}(z^2\Lambda_{QCD}^2)}$$

Hybrid renormalization

- X. Ji, Y. Liu, A. Schafer, W. Wang, Y.-B. Yang, J.-H. Zhang, and **Y. Zhao**, NPB 964, 115311

- Short distance $z \leq z_s$:

Ratio scheme

$$\frac{h_B(z, P_z, a)}{h_B(z, P_z^0 = 0, a)}$$

- Long distance $z > z_s$:

$$\frac{h_B(z, P_z, a)}{h_B(z_s, P_z^0 = 0, a)} e^{\delta m(a)|z - z_s|}$$

$$\tilde{q}^{\text{Hybrid}}(x, P_z) = \int \frac{dy}{|y|} C^{\text{Hybrid}}\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2 P_z^2}, \frac{\Lambda_{QCD}^2}{(1-x)^2 P_z^2}\right)$$

See Yong Zhao's talk for more details.

Pion valence quark PDF: Higher-twist effect

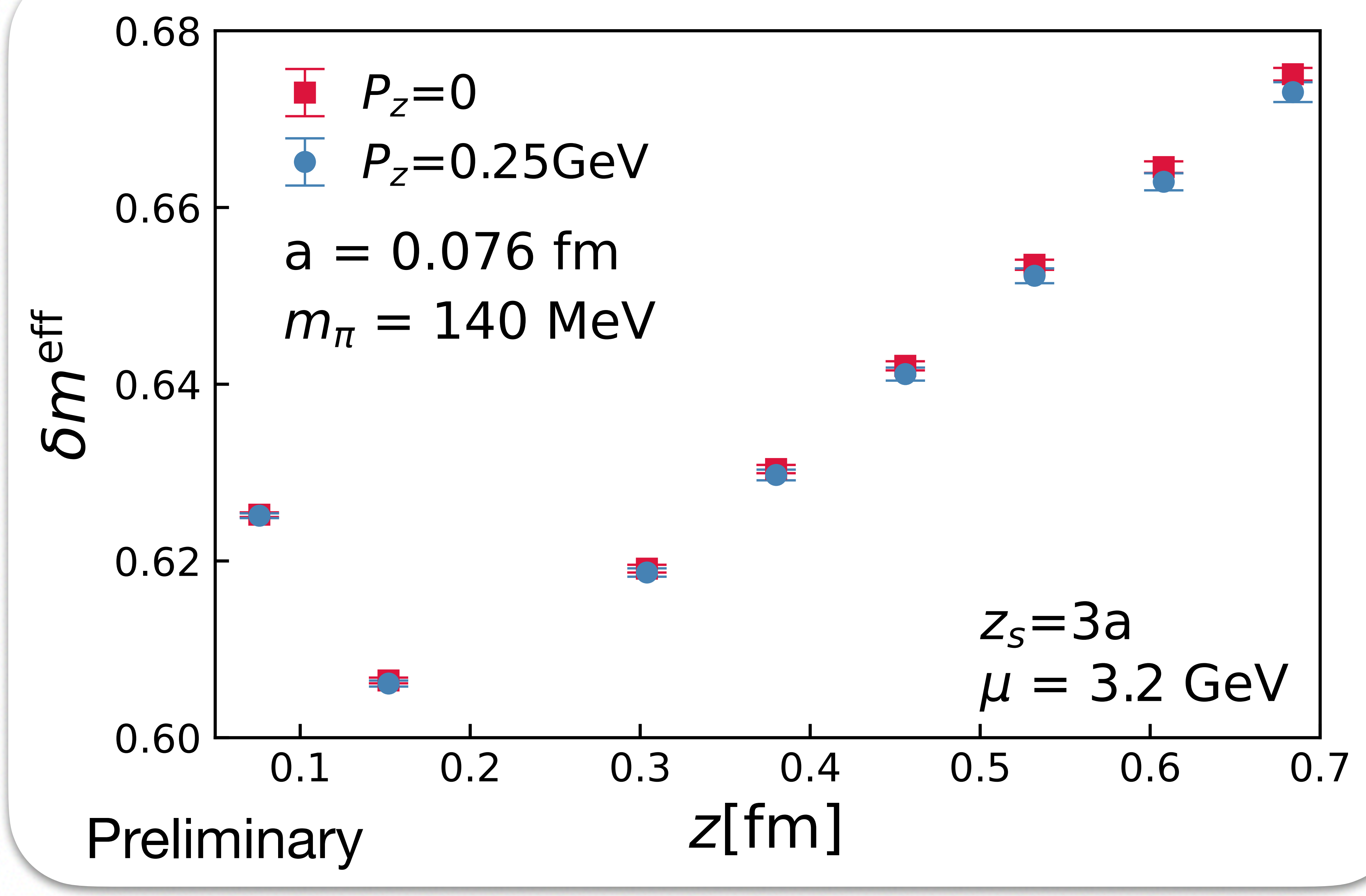
To estimate the **higher-twist/non-perturbative** effect as a function of z, one can define an effective δm^{eff}

$$-\delta m^{eff}|z-z_s|$$

$$= \ln \frac{h_B(z,P_z,a)}{h_B(z_s,P_z^0=0,a)} - \ln \frac{C_0(\mu^2z^2) - C_2(\mu^2z^2)\frac{(zP_z)^2}{2}\langle x^2\rangle_{z_s}}{C_0(\mu^2z_s^2)}$$

$$= -\delta m|z-z_s| + \mathcal{O}(z^2\Lambda_{QCD}^2)$$

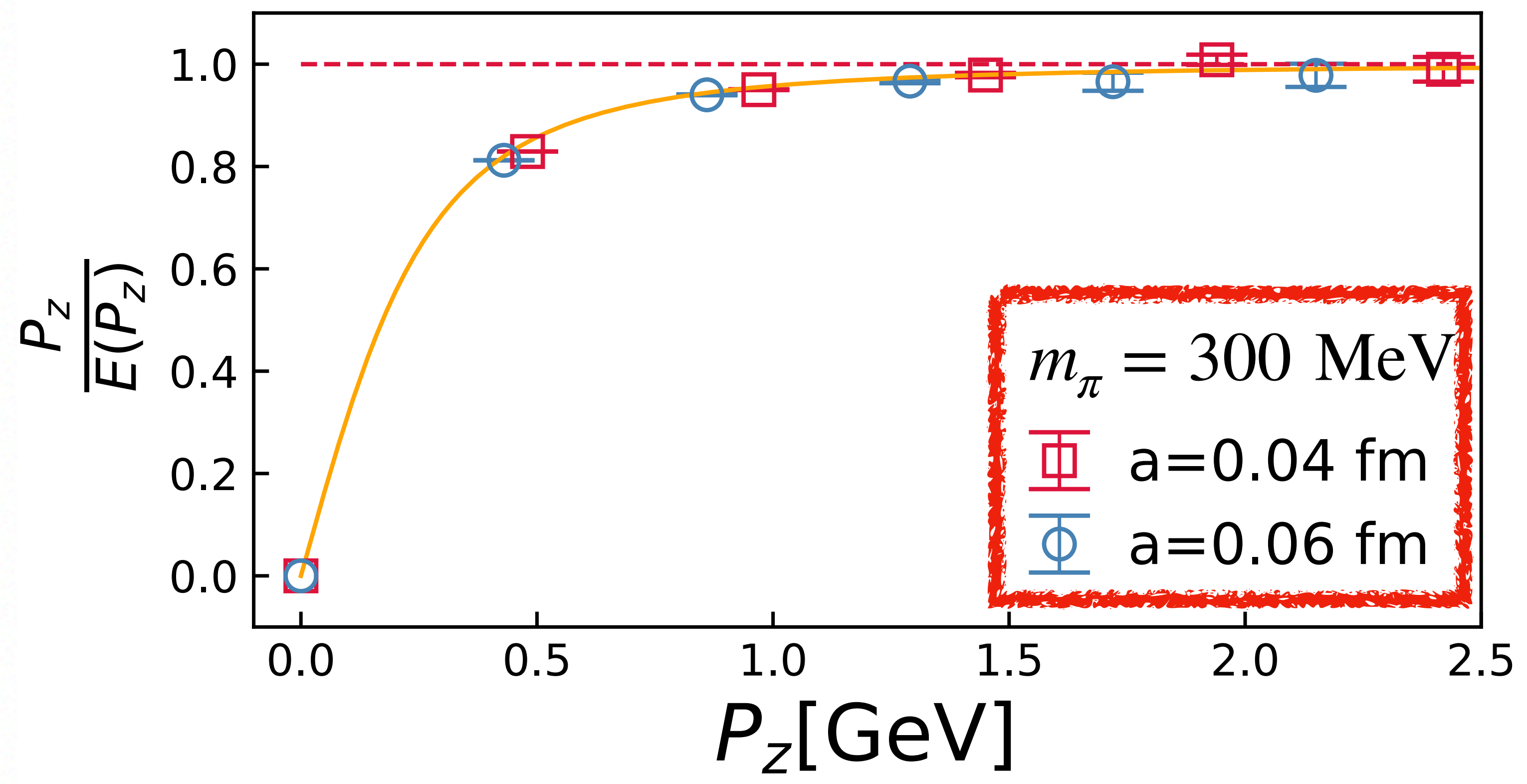
- Subtract the **twist-2** contribution from the matrix elements.
- Matrix elements of **non-zero** P_z contains information of the moments of the PDFs.
- Limit $zP_z < 1$ where the data is only sensitive to the 2nd moment $\langle x^2\rangle$, which can be extracted at **small** z.



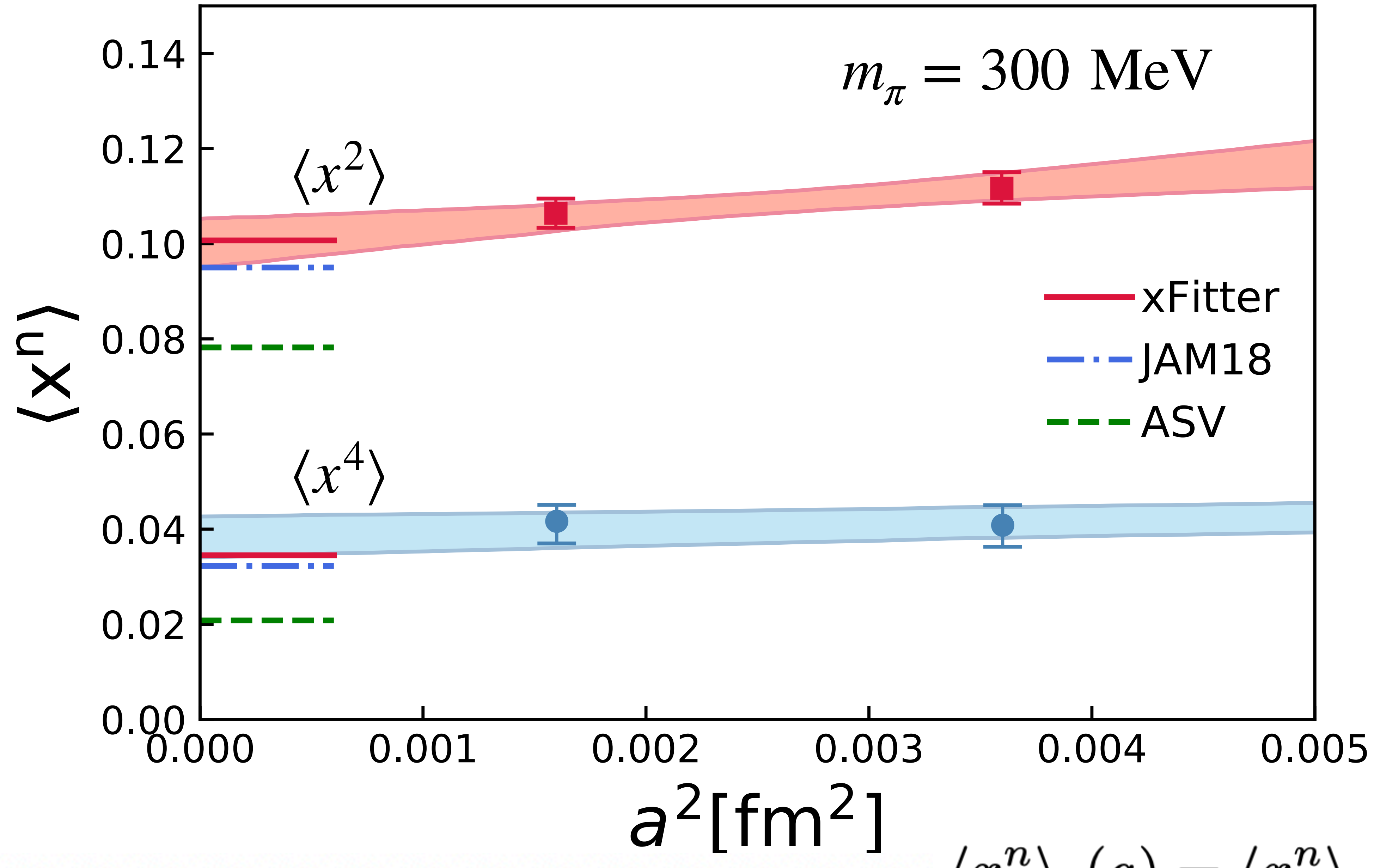
- We used $C_n(\mu^2z^2)$ at **NNLO** level. • Li, Ma and Qiu, PRL 126 (2021)
- δm^{eff} doesn't show a plateau, suggesting the **higher-twist/non-perturbative** effects as a function of z.
- Two different momentum produce consistent results at least up to 0.6 fm, where we can still apply the short distance factorization based on **ratio scheme renormalization**.

Pion valence quark PDF: NLO results

Boosted pion state on the lattice

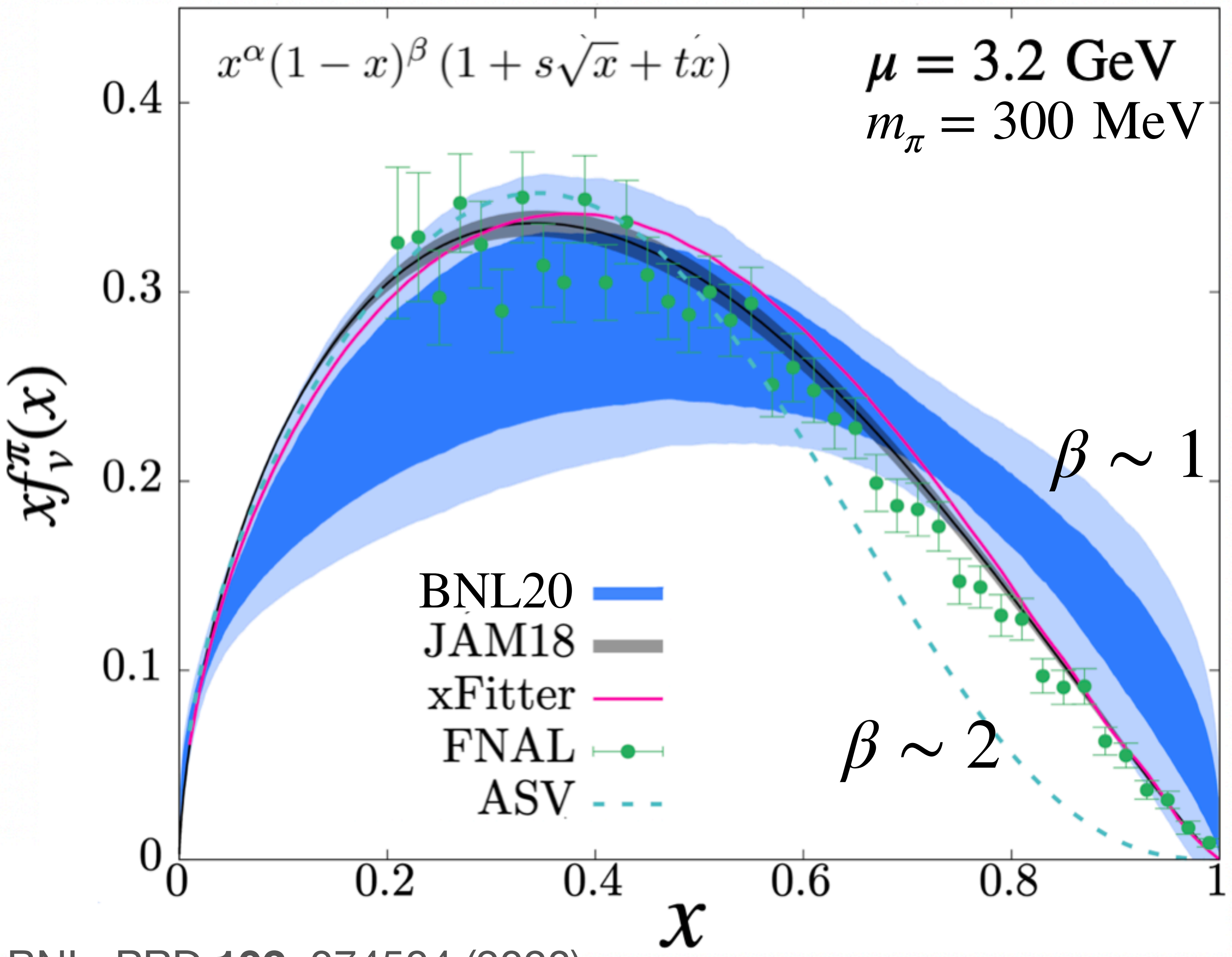


Moments of pion valence quark PDF



$\langle x^n \rangle_v(a) = \langle x^n \rangle_v + d_n a^2$

Pion valence quark PDF



BNL, PRD **102**, 074504 (2020)

Improvement:

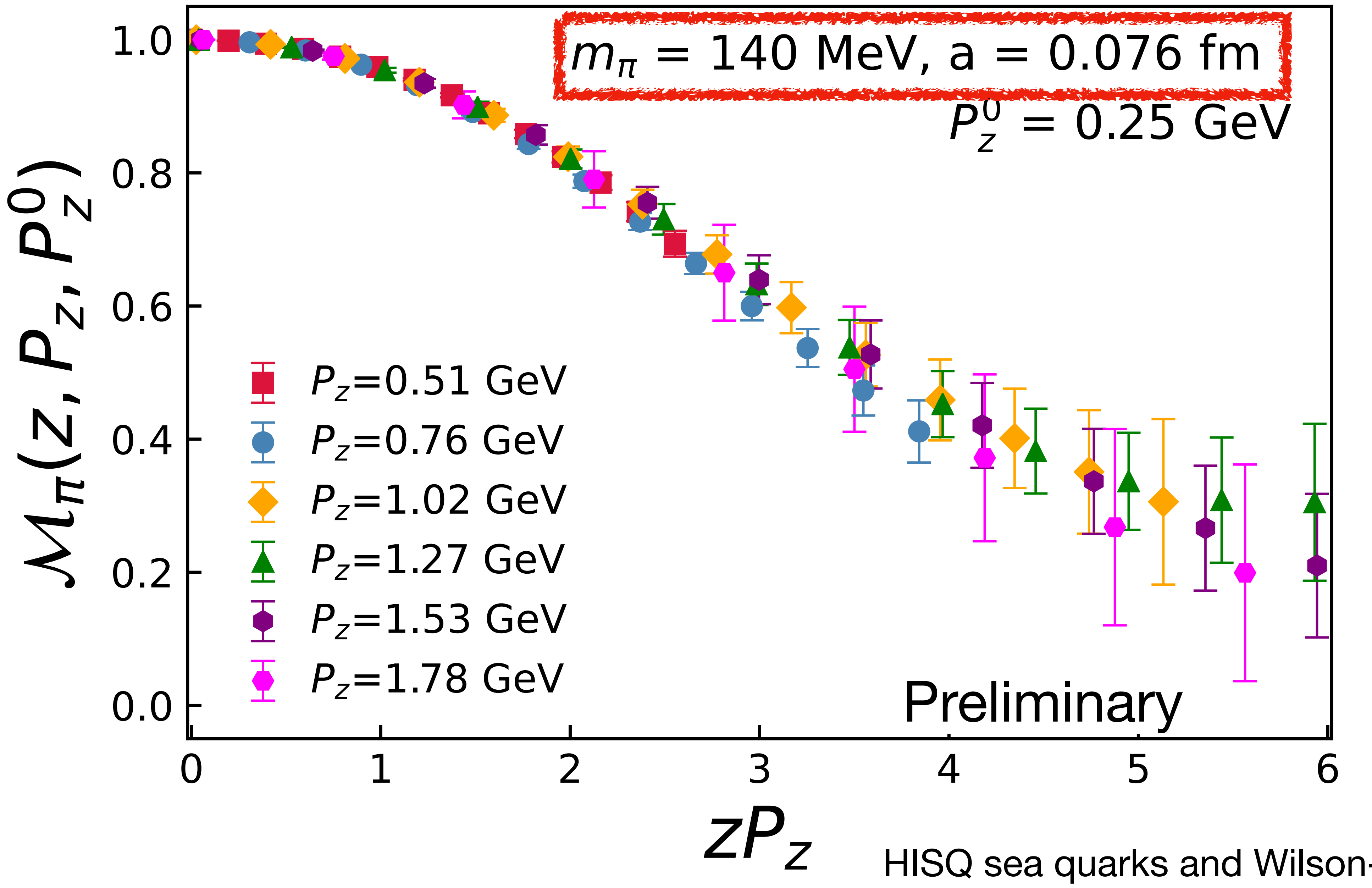
- Matching formula beyond one-loop.
- Computation with physical pion mass.
- Extract PDFs information from chiral fermions.

Pion valence quark PDF: Improvement

Improvement:
- Matching formula beyond one-loop.
- Computation with physical pion mass.
- Extract PDFs information from chiral fermions.

- **Physical pion mass**

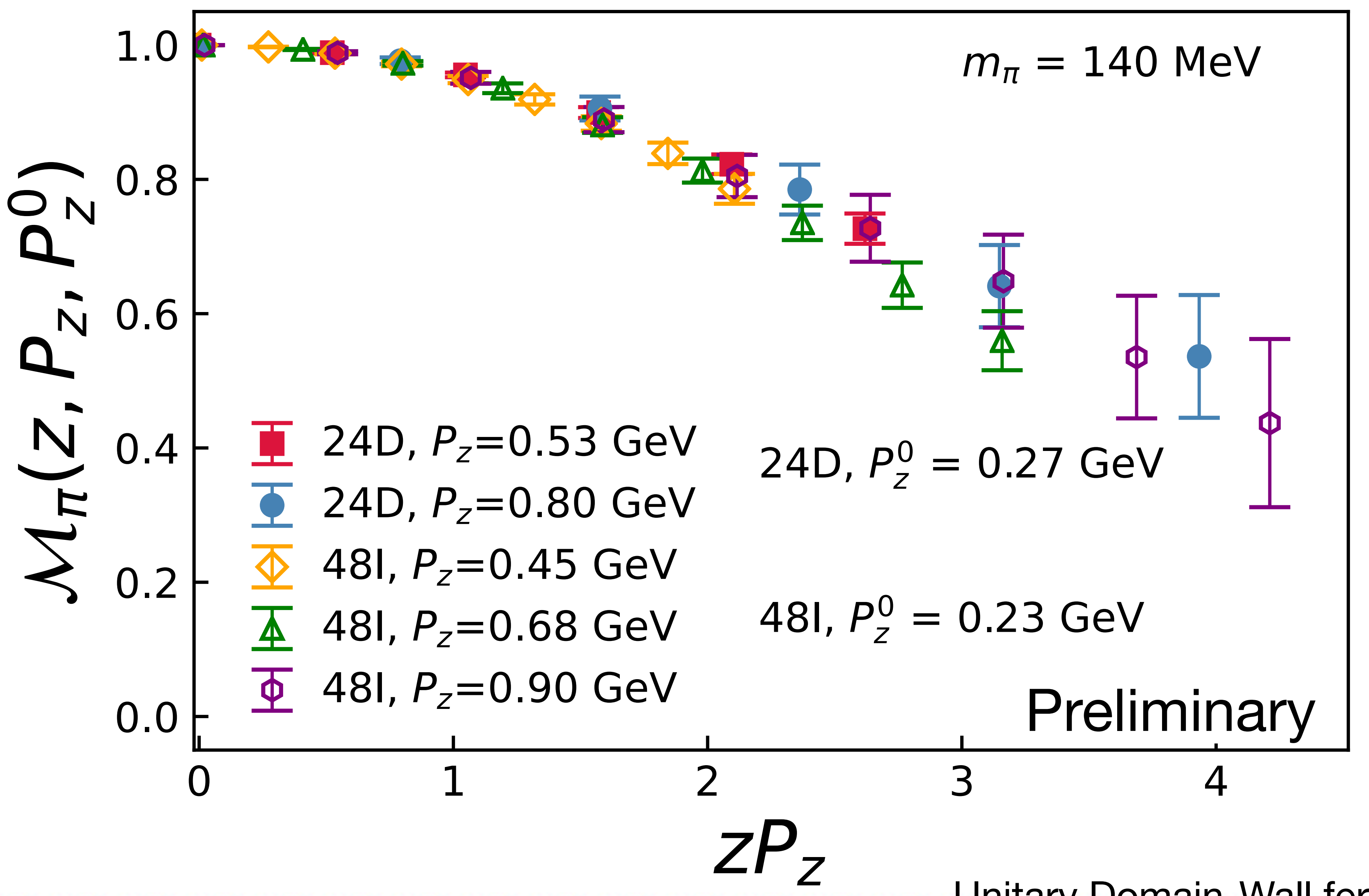
Ratio scheme renormalized matrix elements



HISQ sea quarks and Wilson-Clover valence quarks.

- **Chiral fermion**

Ratio scheme renormalized matrix elements



Unitary Domain-Wall fermion calculation.

Pion valence quark PDF: NNLO

Improvement:

- Matching formula beyond one-loop.
- Computation with physical pion mass.
- Extract PDFs information from chiral fermions.

- **NNLO matching**

- Li, Ma and Qiu, PRL 126 (2021)

$$C_n(z^2\mu^2) = 1 + \alpha_s(\mu)C_n^{(1)}(z^2\mu^2) + \alpha_s^2(\mu)C_n^{(2)}(z^2\mu^2) + \mathcal{O}(\alpha_s^3)$$

$$= 1 + \frac{\alpha_s(\mu)C_F}{2\pi}[(\frac{3+2n}{2+3n+n^2} + 2H_n)\ln(z_0^2\mu^2) + \ldots] + \ldots$$

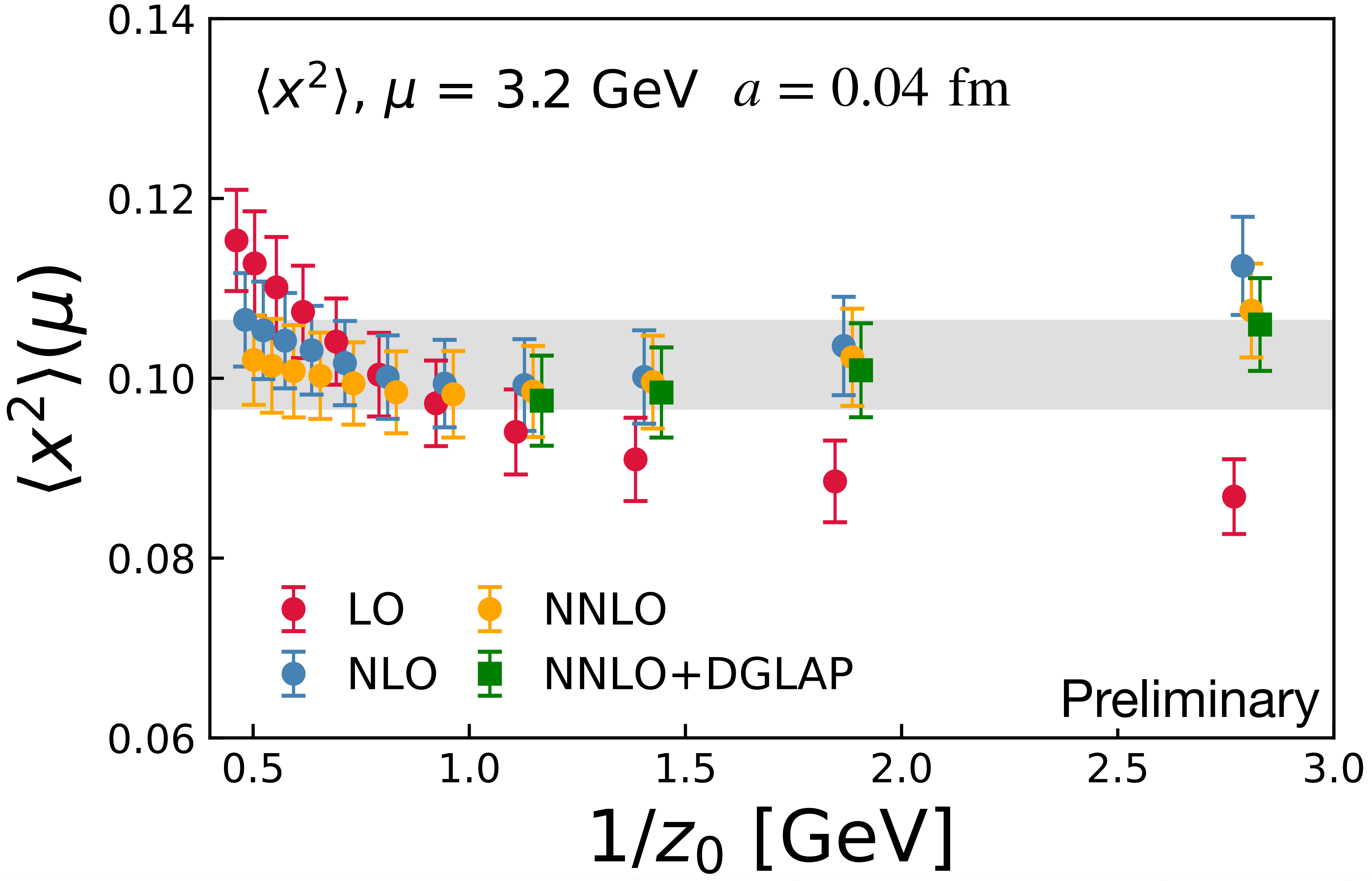
$$z_0^2 = z^2 e^{2\gamma_E}/4$$

When $\ln(z_0^2\mu^2)$ become large, one may need to include the DGLAP evolution:

$$\left[\frac{\partial}{\partial \ln \mu^2} + \beta(\alpha_s(\mu))\frac{\partial}{\partial \alpha_s} - \gamma_n\right] C_n^{evo} = 0$$

- A. V. Radyushkin, PLB 781 (2018)
- BNL, ANL, arXiv: 2102.01101

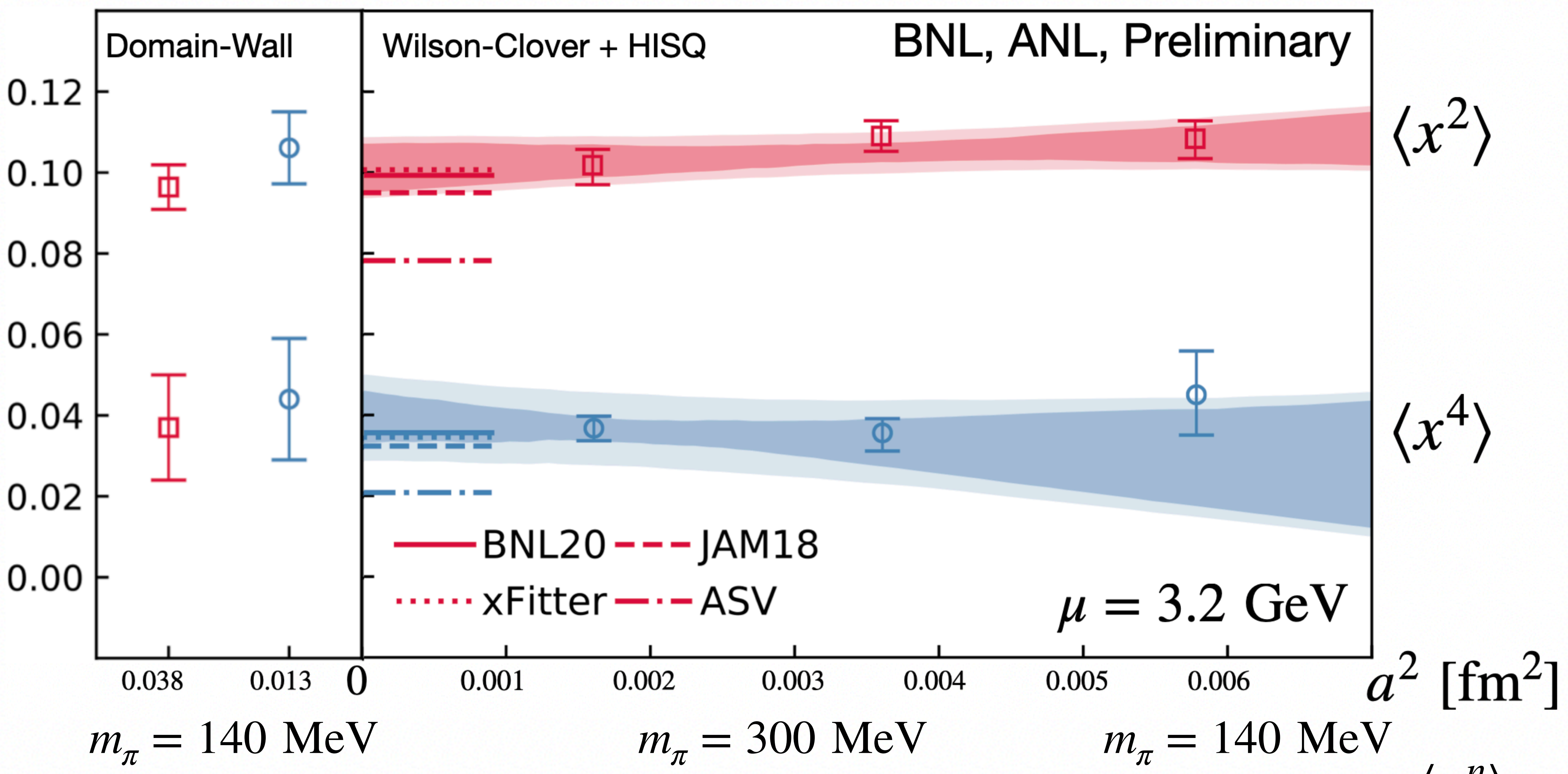
Moments extracted at fixed z by varying P_z



- Clear z_0 dependence can be observed at **LO**.
- Moments evolved from $1/z_0$ to μ from **NNLO** are consistent with **NLO** with current statistics but more flat, and agree with the **DGLAP** improved case.

Pion valence quark PDF: Moments

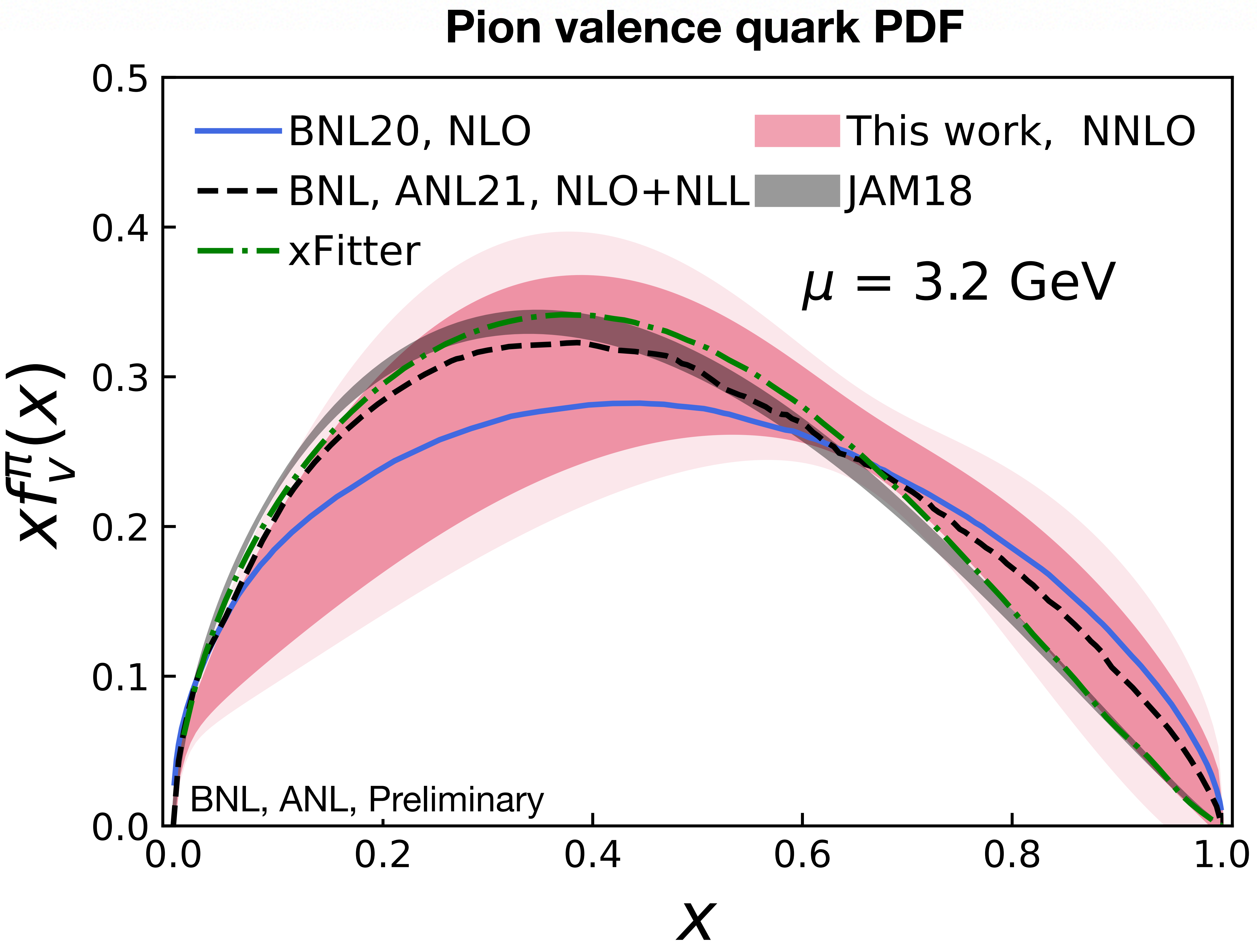
Moments: NNLO matching, physical point, chiral fermion



$$\langle x^n \rangle = \langle x^n \rangle_{\text{Ext}} + d_n a^2$$

- The mass dependence is mild for pion valence PDF.
- Chiral fermion shows good agreement with Wilson-Clover + HISQ fermion with fine lattice spacings.

Pion valence quark PDF



Preliminary results of the large-x behavior from model $x^{\alpha}(1-x)^{\beta}(1+t\sqrt{x}+sx)$:

$$\beta = 1.07(37)(29),$$

which shows good agreement with JAM18, xFitter.

More improvement:

- Resummation in perturbative matching. For example, NLO+NLL **threshold resummation** (BNL, ANL PRD 103 (2021)).
- More statistics and large momentum to extract **higher moments**.

Summary

- We studied pion valence quark PDF with NNLO matching, physical pion mass and chiral fermions.
- The mass dependence of the pion valence PDF is mild.
- Wilson-Clover fermion didn’t bring big trouble to the determination of pion valence structure.

Thanks for your attention