Gravitational form factors of the proton

Lattice 2021

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Outline

Gravitational form factors

- Physics motivation
- Experimental accessibility

(Progress on) GFFs on the lattice

- Quark and glue contributions
- ~~Walk~~ Run through calculation & analysis

(Very) preliminary results

Gravitational form factors (GFFs)

$\overleftrightarrow{D} = (\overrightarrow{D} - \overleftarrow{D})/2$

For (symmetric, traceless) EMT, $T^{\{\mu\nu\}} = T_g^{\{\mu\nu\}} + \sum_q T_q^{\{\mu\nu\}}$

Gluons $T_g^{\{\mu\nu\}} = 2\,\mathrm{Tr}[G^{\alpha\{\mu}G^{\nu\}\alpha}]$ Quarks $T_q^{\{\mu\nu\}} = \bar{\psi}\gamma^{\{\mu} i\overleftrightarrow{D}^{\nu\}}\psi$

$\{\ \} \equiv$ symmetrize and subtract trace e.g. $a^{\{\mu}b^{\nu\}} \equiv \frac{1}{2}(a^\mu b^\nu + \mathrm{a}^\nu \mathrm{b}^\mu) - \mathrm{a}^\mu b^\nu$

GFFs decompose hadronic matrix elements of T, e.g. for proton:

$$\left\langle p', s' \middle| T_{g,q}^{\{\mu\nu\}} \middle| p, s \right\rangle = \bar{u}(p', s') \left[A_{g,q}(t)\,\gamma^{\{\mu}P^{\nu\}} + B_{g,q}(t)\,\frac{i\,P^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M} + D_{g,q}(t)\,\frac{\Delta^{\{\mu}\Delta^{\nu\}}}{4M} \right] u(p, s)$$

$u, \bar{u}$ = Dirac spinors $\quad P = (p' + p)/2 \quad \Delta = p' - p \quad t = \Delta^2$

Physics:

$A_{q,g}(t) \sim$ momentum of constituents

→ Momentum fraction $A_{q,g}(0) = \langle x \rangle_{q,g}$

$J_{q,g}(t) = \frac{1}{2}(A_{q,g}(t) + B_{q,g}(t)) \sim$ angular momentum

→ Total $J(0) = \frac{1}{2}$

$D_{q,g}(t) \sim$ pressure and shear forces

$D(0)$: "the last global unknown"

[from Polyakov & Schweitzer 1805.06596]

em:	$\partial_\mu J^\mu_{\mathrm{em}} = 0$	$\langle N' \vert J^\mu_{\mathbf{em}} \vert N \rangle$	$\longrightarrow$	$Q = 1.602176487(40) \times 10^{-19}\mathrm{C}$ $\mu = 2.792847356(23)\mu_N$
weak:	PCAC	$\langle N' \vert J^\mu_{\mathbf{weak}} \vert N \rangle$	$\longrightarrow$	$g_A = 1.2694(28)$ $g_p = 8.06(55)$
gravity:	$\partial_\mu T^{\mu\nu}_{\mathbf{grav}} = 0$	$\langle N' \vert T^{\mu\nu}_{\mathbf{grav}} \vert N \rangle$	$\longrightarrow$	$m = 938.272013(23)\,\mathrm{MeV}/c^2$ $J = \frac{1}{2}$ $\boldsymbol{D} = \boldsymbol{?}$

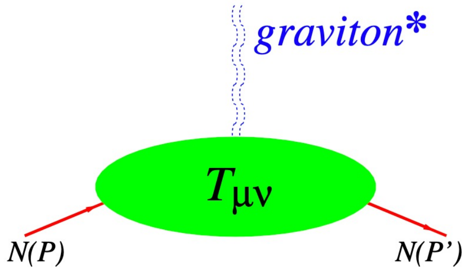
Table I. The global properties of the proton defined in terms of matrix elements of the conserved currents associated with respectively electromagnetic, weak, and gravitational interaction. Notice the weak currents include the partially conserved axial current, and g_A or g_p are strictly speaking defined in terms of transition matrix elements in the neutron β-decay or muon-capture. The values of the properties are from the particle data book [107] and [108] (for g_p) except for the unknown D-term.

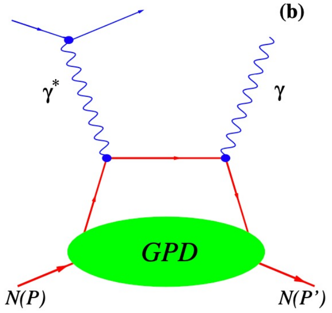
Experimentally accessible?

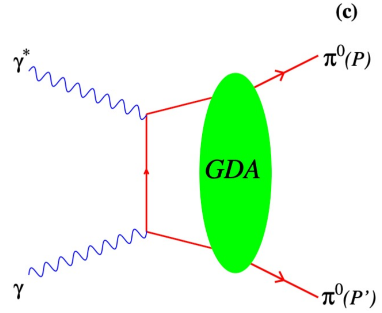
[from Polyakov Schweitzer 1805.06596]

Graviton colliders not presently feasible

but: GFFs ~ moments of (unpolarized) generalized parton distributions (GPDs), constrained by hard exclusive processes







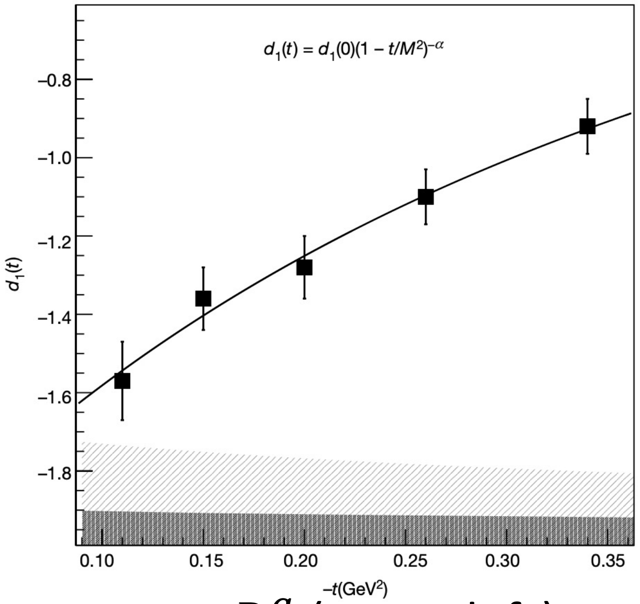
Some extractions of quark GFFs from experiments:

JLAB: proton D term extracted from DVCS

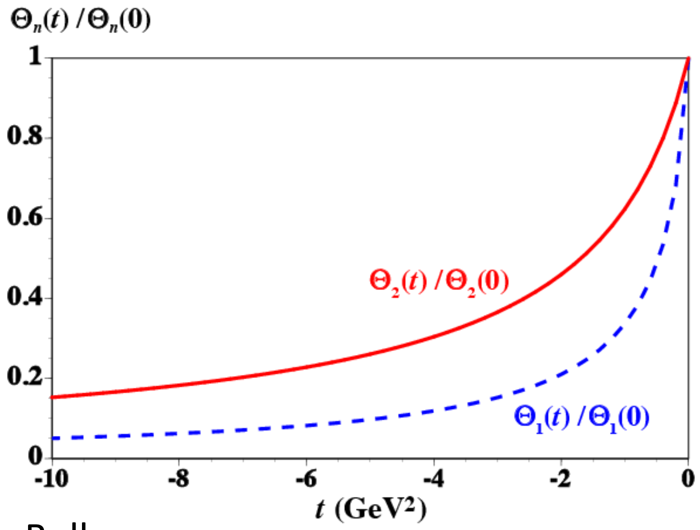
[Burkert Elouadrhiri Girod 2018]

Belle: pion GFFs extracted from $\gamma^*\gamma \to \pi^0\pi^0$

[Kumano Song Teryaev 1711.08088]



Proton D^q (up to defs)



Belle

Pion D^q (up to defs)

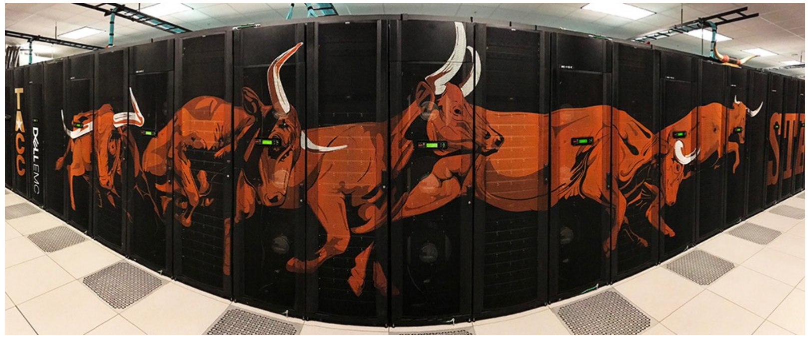
GFFs on the lattice

Ensemble ["a091m170"]

- Gauge action: Tree-level tadpole-improved Symanzik
- Fermion action: 2+1 Wilson clover, stout links
- $M_\pi = 170\ \mathrm{MeV}$
- $a = 0.091\ \mathrm{fm}$ (from w_0)
- $48^3 \times 96$

Sketch of calculation:

- Compute hadronic three-point functions
- Construct ratios of 3pts/2pts to isolate matrix element
- Bin ratios together to improve stats/reduce data volume
- Fit ratios to extract constraints on GFFs
- Fit system of equations to extract GFFs
- [Analysis as in new preprint 2107.10368]



Lattice EMT operators

Quark: $T_q^{\{\mu\nu\}} = \bar{\psi}\gamma^{\{\mu} i\overleftrightarrow{D}^{\nu\}}\psi$

$\overleftrightarrow{D} = (\overrightarrow{D} - \overleftarrow{D})/2$

Discretize covariant derivative:

$$(\overrightarrow{D}_\mu \psi)(x) = \frac{1}{2}\left[U_\mu(x)\psi(x+\mu) - U_\mu^\dagger(x-\mu)\psi(x-\mu)\right]$$

$$(\bar{\psi}\overleftarrow{D}_\mu)(x) = \frac{1}{2}\left[\bar{\psi}(x+\mu)U_\mu^\dagger(x) - \bar{\psi}(x-\mu)U_\mu(x-\mu)\right]$$

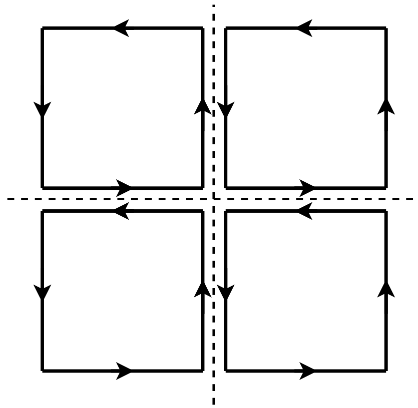
Glue: $T_g^{\{\mu\nu\}} = 2\,\mathrm{Tr}[G^{\alpha\{\mu}G^{\nu\}\alpha}]$

Use clover definition of field strength tensor

$G_{\mu\nu} \propto \left(Q_{\mu\nu} - Q_{\mu\nu}^\dagger\right)$

Gluons are noisy → use flow-smeared links

$t/a^2 = 2$



Hypercubic Irreps

Lorentz symmetry broken → project operators $O^{\mu\nu}$ to hypercubic irreps

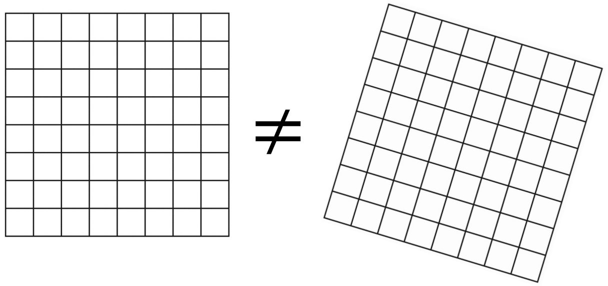
$\tau_1^{(3)}$: $\frac{1}{2}(O^{11} + O^{22} - O^{33} + O^{00})$, $\frac{1}{\sqrt{2}}(O_{33} + O_{00})$, $\frac{1}{\sqrt{2}}(O_{11} - O_{22})$

$\tau_3^{(6)}$: $\left\{\frac{i^{\delta_{\mu 0}}}{\sqrt{2}}(O^{\mu\nu} + O^{\nu\mu}),\ 0 \le \mu \le \nu \le 3\right\}$

Irreps renormalize differently, don't mix → Different sets of Z factors

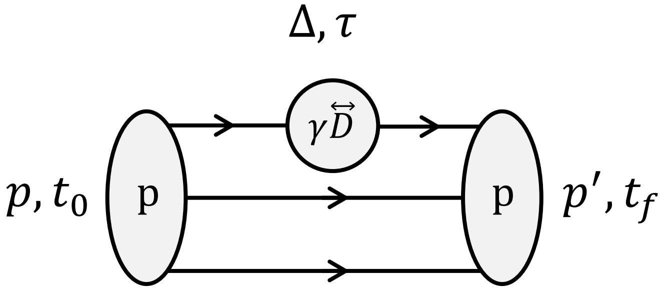
$\tau_1^{(3)}$: $Z_{1qq}\ Z_{1gg}\ Z_{1qg}\ Z_{1gq}$

$\tau_2^{(6)}$: $Z_{2gg}\ Z_{2qq}\ Z_{2qg}\ Z_{2gq}$



Hadronic three-point functions

Quark (connected)



Compute w/ sequential source method: invert through sink

Stats:

16 sources on ~ 1000 configs

3 sink momenta $\vec{p}'$

$t_f = 8, 10, 12, 14$

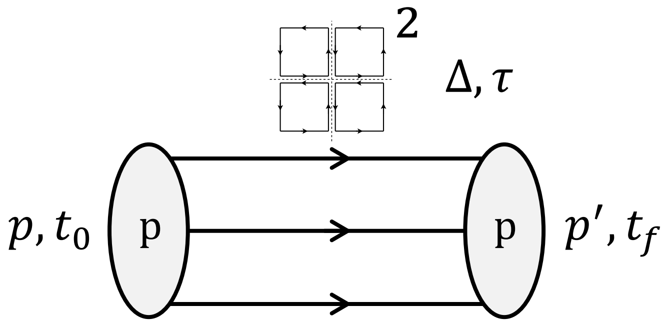
1 spin channel

1 source/sink smearing

All τ, operators

All $\vec{\Delta}^2 \leq 25 \left(\frac{2\pi}{L}\right)^2 \approx 2\ \mathrm{GeV}^2$

Glue (disconnected)



Compute by correlating two-point functions and glue EMT measurements

$C^{3pt}_{s's}(t, \tau; \mathrm{p}, \Delta, p') \sim T^g(\tau, \Delta)\, C^{2pt}_{s's}(t_f, p)$

Vacuum subtract to reduce noise

Stats:

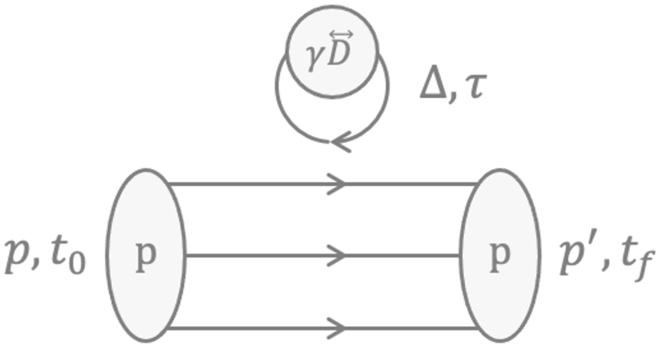
512 sources on ~ 1000 configs

1 source smearing, 2 sink smearings

All $t_f, \tau, \vec{p}'$, operators, spin channels

All $\vec{\Delta}^2 \leq 25 \left(\frac{2\pi}{L}\right)^2 \approx 2\ \mathrm{GeV}^2$

Quark (disconnected)



Similar to glue, but with quark loops instead of glue EMT

Requires stochastic estimators

In progress

Notation:

Δ ~ operator momentum

$p' = p + \Delta$ ~ sink momentum

τ ~ operator insertion time

t_f ~ sink time

s, s' ~ source, sink spins

Ratios

$$R_{s's;\mathcal{R},i}(p,p';\tau,t_f) = \frac{C^{3pt}_{s's;\mathcal{R},i}(p,p';t_f,\tau)}{C^{2pt}_{s's'}(p',t_f)} \sqrt{\frac{C^{2pt}_{ss}(p,t_f-\tau)}{C^{2pt}_{s's'}(p',t_f-\tau)} \frac{C^{2pt}_{s's'}(p',t_f)}{C^{2pt}_{ss}(p,t_f)} \frac{C^{2pt}_{s's'}(p',\tau)}{C^{2pt}_{ss}(p,\tau)}} = \bar{R}_{s's;\mathcal{R},i}(p,p') + (\text{excited states})$$

$$\bar{R}_{s's;\mathcal{R},i}(p,p') = \frac{\mathrm{Tr}\left[\Gamma_{s's}\,(\not{p}'+M)\,\mathcal{O}_{\mathcal{R},i}[A_\mathcal{R},B_\mathcal{R},D_\mathcal{R}]\,(\not{p}+M)\right]}{4\sqrt{E\,E'(E+M)(E'+M)}}$$

$$\mathcal{O}_{\mathcal{R},i}[A_\mathcal{R},B_\mathcal{R},D_\mathcal{R}] = A_\mathcal{R}(t)\,\gamma^{\{\mu}P^{\nu\}} + B_\mathcal{R}(t)\,\frac{i\,P^{\{\mu}\sigma^{\nu\}\rho}\Delta_\rho}{2M} + D_\mathcal{R}(t)\,\frac{\Delta^{\{\mu}\Delta^{\nu\}}}{4M}$$

Extract $\bar{R}_{\mathcal{R},i} \to$ (linear) constraints on GFFs for irrep $\mathcal{R}$

$$\bar{R}_{s's;\mathcal{R},i}(p,p') = K_{s's;\mathcal{R},i}(p,p') \cdot [A_\mathcal{R}(t), B_\mathcal{R}(t), D_\mathcal{R}(t)]$$

Binning:

- Ratios associated with (many) values of $t = \Delta^2$
  - → Bin nearby t together, estimate $A(t), B(t), D(t)$ for bins
- For many ratios, same K up to an overall sign
  - → Average together (inside t bins)
  - Keep different irreps, source/sink smearings separate

Fitting ratios

Ratios $R(\tau, t_f) = \bar{R} + (\text{excited states})(\tau, t_f)$

- when $t_f \gg \tau \gg 0$, excited state contamination is small
- ...but signal degrades as τ, t_f increase

Glue: try *all* triangular fit regions

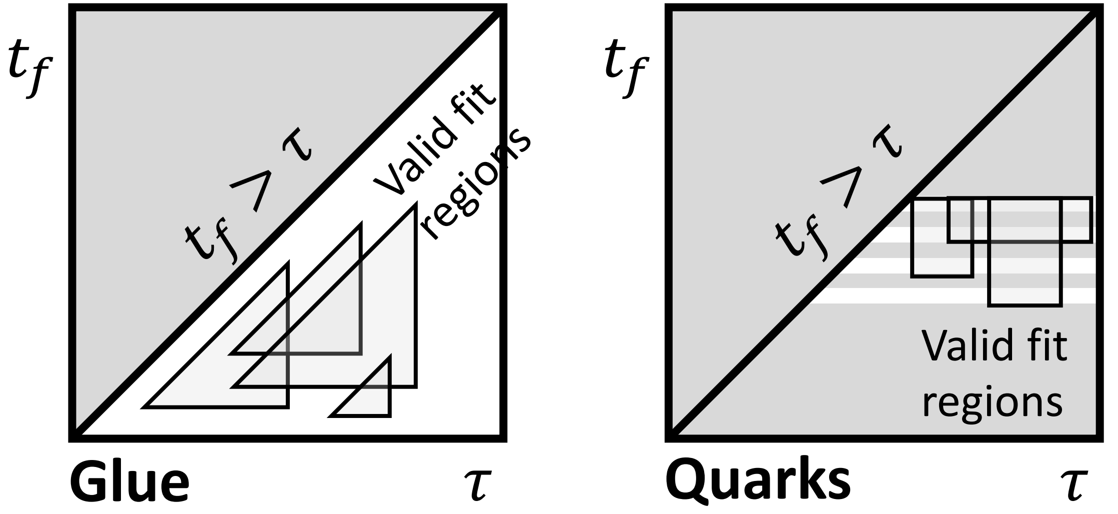
- $\tau > \tau_{\min} \quad t_f - \tau > \delta \quad t_f < t_{f\,\max}$

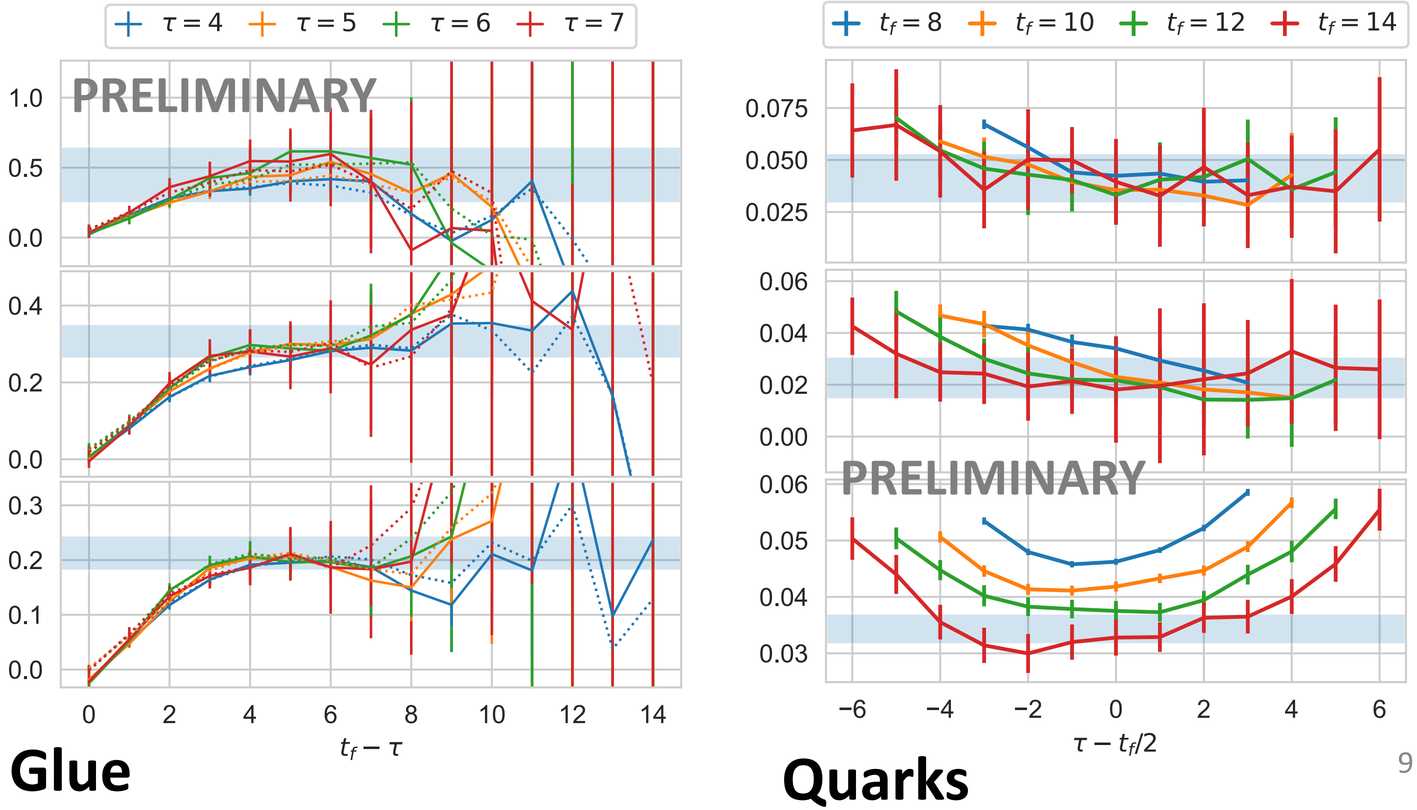
Quarks: rectangles including max t_f

- $\tau_{\min} < \tau < \tau_{\max} \quad t_{f\,\min} < t_f \leq 14$

Average fit results together

- Bayesian model averaging/AIC weights
  - [Jay, Neil 2008.01069]
- p-value/err^2 weights





Extracting the GFFs (constraint fits)

In each t bin, ratio fit results $\bar{R}(t)$ times Z factor ~ constraints on the renormalized GFFs

$$K_1^g(t) \cdot [A^g(t), B^g(t), D^g(t)] = Z_{1,gg}\bar{R}_1^g(t)$$

$$K_2^g(t) \cdot [A^g(t), B^g(t), D^g(t)] = Z_{2,gg}\bar{R}_2^g(t)$$

(similar for quarks w/ $g \to q$)

where K are matrices of kinematic coefficients

Solve by doing correlated χ^2 fits

Problem: d'Agostini bias

due to non-Gaussianity of product $Z\ \bar{R}$

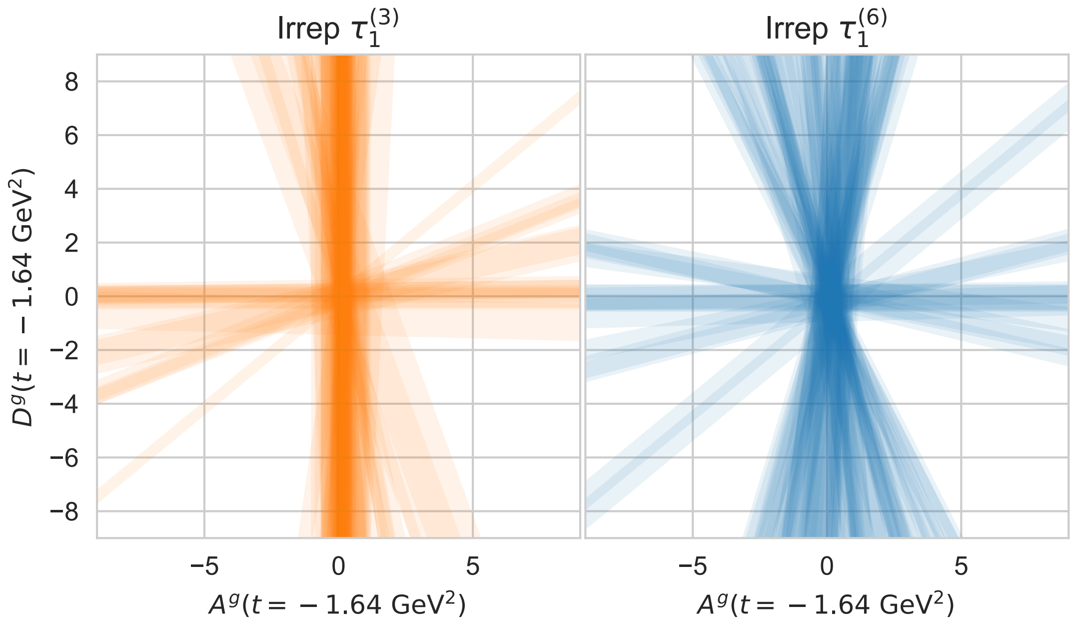
Solution: (Bayesian) penalty trick

Measured $\bar{R}$ can constrain Z_1/Z_2

Measured Z factors enter as priors, get updated

Extracts $A_1(t), B_1(t), D_1(t)$ and updated Z_1

[See 2107.10368 for details]



Note: Results in following slides use priors* for renormalization factors,

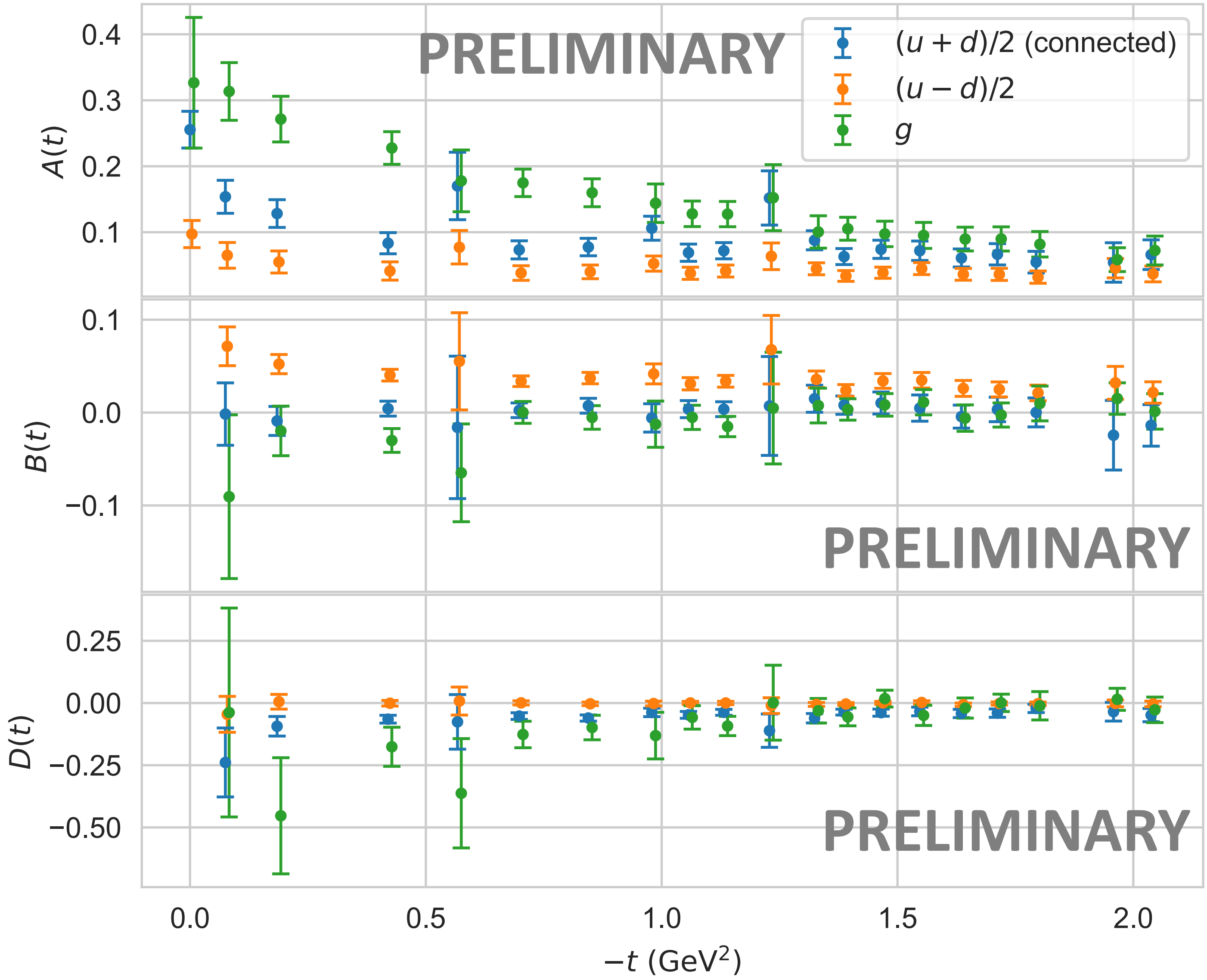
$$Z_{1,gg} = Z_{2,gg} = Z_{1,qq} = Z_{2,qq} = 1.0(1)$$

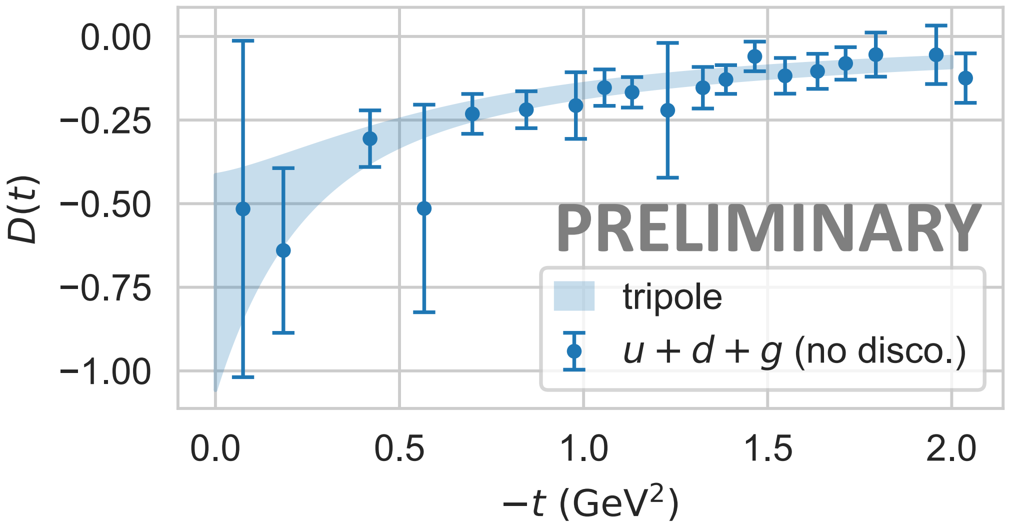
$$Z_{1,gq} = Z_{1,qg} = Z_{2,gq} = Z_{2,qg} = 0.0(1)$$

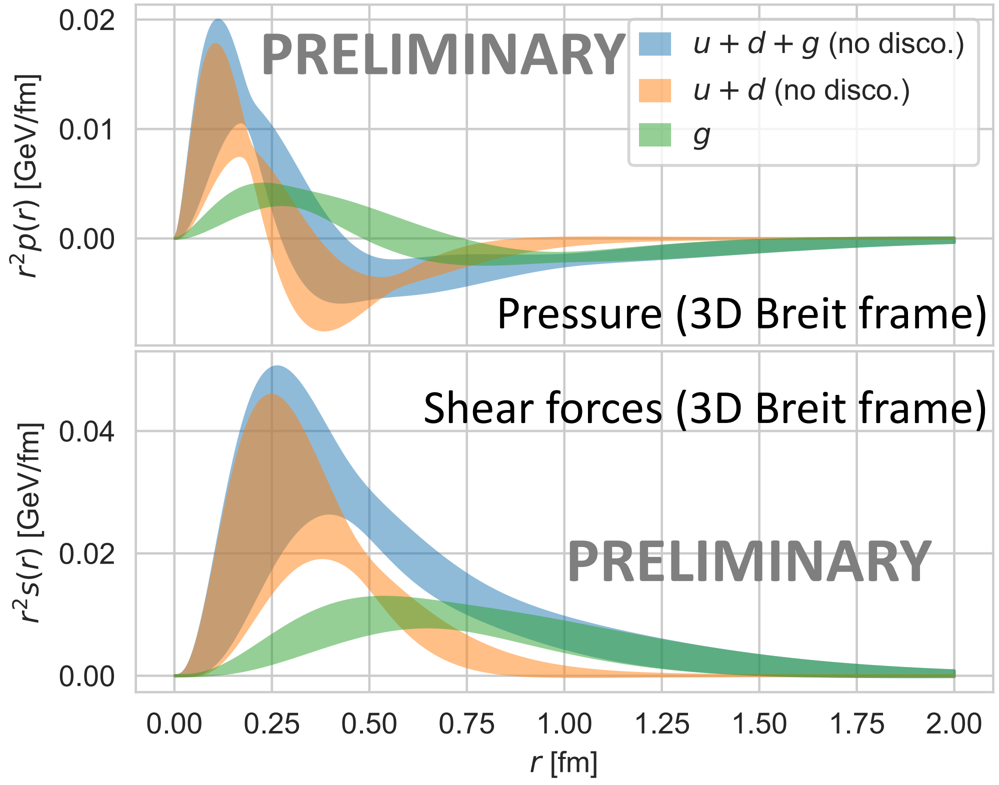
[* To be replaced with results of NPR calculation + perturbative matching to $\overline{\mathrm{MS}}$ in final results]

Interpretation: renormalized GFFs up to an overall constant ~ 1

(Very preliminary) results







Conclusions/upcoming:

GFFs encode fundamental, global properties of hadrons

…including some that are presently only poorly constrained

GPDs are targets for near-future experiments

→ Lattice results on GFFs can inform kinematic regimes to target

→ Lattice results are necessary to test against experimental results

Lattice calculation ongoing, early results promising. TODO:

Disconnected diagrams for quarks

Non-perturbative renormalization

Treatment of excited state contamination

Pion GFFs

Next talk:

gluon GFFs for pion, nucleon, rho, delta on a different ensemble

+ energy, pressure, shear force densities