Electromagnetic effects in charged pion decay

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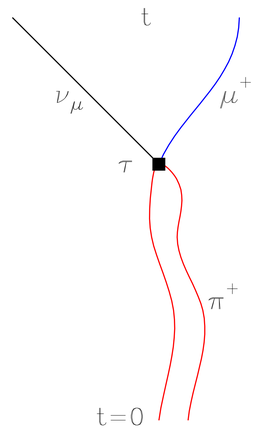
Three-point Function



Lattice QCD has advanced to the point where we can now investigate small effects, such as the isospin violation due to the mass difference between u and d quarks, and corrections due to QED. We calculate the decay

$$\pi^+ \to \mu^+ + \nu_\mu$$

by calculating a three-point function involving a pion and muon propagating through a simulated QCD+QED background field [2, 3].
We create a stationary pion at $t = 0$, which decays at time τ, producing a muon with momentum $\vec{p}$ which propagates to a sink at time t. When τ lies between 0 and t the three-point function will have a time dependence of the form

$$Q(t, \tau, \vec{p}) \propto e^{-m_\pi \tau} e^{-E_\mu(\vec{p})(t-\tau)}$$

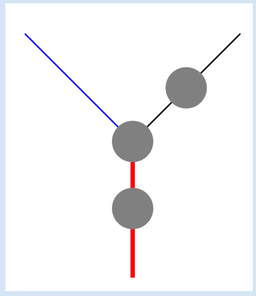
The Lattice approach

Calculate the three-point functions

$$Q_{ij;A_\mu/V_\mu}(t, \tau, \vec{p}) = \sum_x \left\langle e^{-i\vec{p}\cdot\vec{x}} S^\mu_{ij}(t, \vec{p}; \tau, \vec{x}) C_{A_\mu/V_\mu,\pi}(\tau, \vec{x}) \right\rangle$$

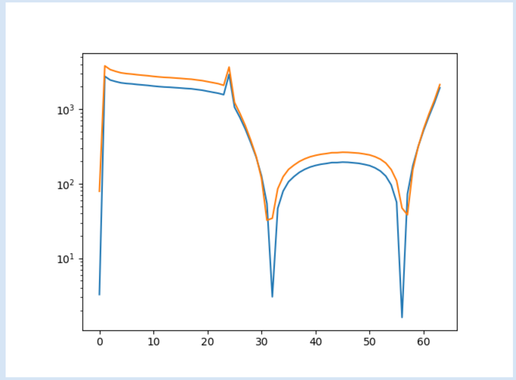
There are 16×8 of these, because the muon propagator is a 4×4 matrix, and we consider 8 possible operators at the meson sink. Despite the large number of observables, the number of inversions is very modest.

Amputation



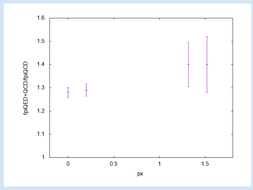
We have measurements of the three point function Q as well as the pion and fermion two-point functions C and S. This enables us to separate the QED effects on the decay vertex from the effects of dressing the propagators with photons.

Three-point Data



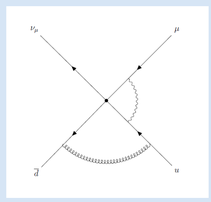
We compare the complete three-point function $\langle S(t,\tau)C(\tau)\rangle$ (orange curve) with the product of two-point functions $\langle S(t,\tau)\rangle\langle C(\tau)\rangle$ (blue curve). The ratio gives the amputated $\pi\mu\nu$ vertex.

Three-point Data



Ratio between the photon dressed pion vertex, and the bare vertex. The calculations are made with a large QED coupling, $\alpha_{QED} \approx 0.1$.

Renormalisation

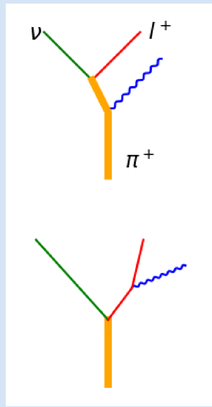


Renormalisation of the weak vertex is no longer the same as renormalisation of the axial current, because there are additional photon graphs connecting to the muon.

References

[1] M. Di Carlo *et al.*, Phys Rev D 100, 034514 (2019).
[2] R. Horsley *et al.*, JHEP **04**, 093 (2016).
[3] R. Horsley *et al.*, J.Phys.G 43 (2016) 10

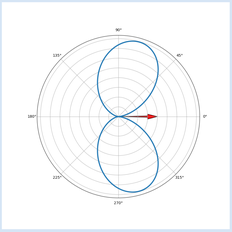
Soft Photons

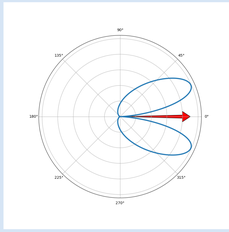


- Accelerating charges produce soft photons, giving infra-red logarithms in cross-sections.
- On a finite lattice we have a cut-off due to the lattice size.
- In the soft photon limit it is reasonable to treat the pion as a point particle, and estimate the infra-red contribution from tree-level perturbation theory.

Soft Photons

$\pi^+ \to \mu^+ + \nu_\mu + \gamma.$ $K^+ \to \mu^+ + \nu_\mu + \gamma.$





Angular distribution of soft photons in meson decay. θ is the angle between the muon and photon in the meson rest frame.

The angular distribution of soft photons depends on the velocity of the charged lepton. In π^+ decay the muon velocity is low, and most photons are emitted at right angles to the muon velocity. The pattern closely follows the non-relativistic Larmor formula for an accelerating charge.
In the kaon case the energy released is much larger, and we now see that a colinear singularity is starting to develop.