NNLO Matching for Quark Quasi Distribution Functions

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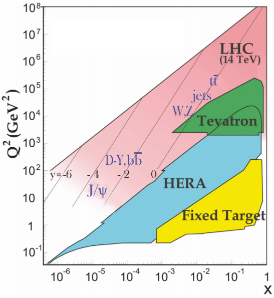
Based on recent works

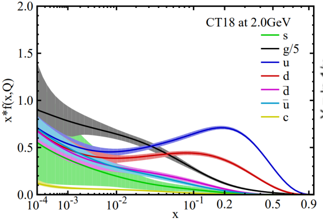
- Next-to-next-to-leading order corrections to non-singlet quark Quasi distribution functions
  L.-B. Chen, W. Wang, R. Zhu, Phys.Rev.Lett.126,072002(2021).
- Master Integrals for two-loop QCD corrections to Quasi PDFs
  L.-B. Chen, W. Wang, R. Zhu, JHEP10,079(2020).
- Quasi parton distribution functions at NNLO: flavor non-diagonal quark contributions
  L.-B. Chen, W. Wang, R. Zhu, Phys.Rev.D102,011503(2020).

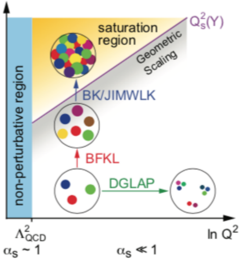
Outline

Available PDFs

PDG2018 and 1912.10053







- Parton distribution functions (PDFs) are fundamental inputs for hadron colliding processes

  Current available PDFs from the global fit

- We know some (more on perturbative aspects) of the PDFs at many different facilities over 50 years effort, however, we understand less from first principle of QCD

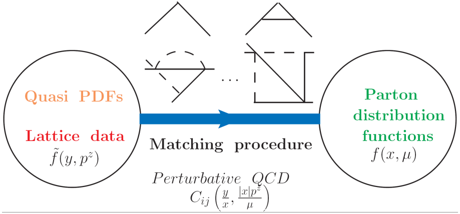
Large Momentum Effective Theory (LaMET)

- LaMET factorization formula

$$\tilde{f}_{i/H}(y,p^z)=\int_{-1}^{1}\frac{dx}{|x|}\left[C_{ij}\left(\frac{y}{x},\frac{|x|p^z}{\mu}\right)f_{j/H}(x,\mu)\right]+\mathcal{O}\left(\frac{m_h^2}{p^{z\,2}},\frac{\Lambda_{\rm QCD}^2}{p^{z\,2}}\right)$$

$x\in[-1,1]$, $y\in[-\infty,\infty]$

X. Ji, PRL110,262002 (2013), ... See previous talk by Jian-Hui Zhang



- other approaches such as pseudo-PDFs, Good lattice cross-section,

Radyushkin,1705.01488; Ma-Qiu,1709.03018

Perturbative calculation of $C_{ij}^{(0)}$, $C_{ij}^{(1)}$, $C_{ij}^{(2)}$, ...

- $C_{ij} = C_{ij}^{(0)} + \frac{\alpha_s}{2\pi} C_{ij}^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2 C_{ij}^{(2)} + \ldots$, independent from hadron
- Leading order(LO): $C_{ij}^{(0)}(y) = \delta(1-y)$
- Higher-order matching, renormalization scheme dependent.
- Next-to-leading order (NLO) $C_{ij}^{(1)}(y, \frac{p^z}{\mu})$

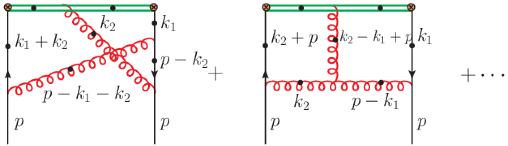
  $\overline{MS}$: Izubuchi,Ji,Jin,Stewart,Zhao,1801.03917;
  $M\overline{MS}$: Alexandrou,Cichy,Constantinou,Jansen,Scapellato,Steffens, 1803.02685;
  RI/MOM: Stewart,Zhao,1709.04933;Wang,Zhang,Zhao,Zhu,1904.00978;
  Others: Ji,Xiong,Zhang,Zhao,1310.7471; Ma,Qiu,1404.6860, Wang,Zhao,Zhu, 1708.02458,...

  3 regions for y: $[-\infty, 0], [0, 1], [1, +\infty]$, 1 color factor: C_F
- Next-to-next-to-leading order(NNLO) $C_{ij}^{(2)}$ (done only for quark case)
  - higher-order corrections are important in QCD
  - $\mu = 2GeV$, $\alpha_s(\mu = 2GeV) \sim 0.3$, α_s^2-correction is needed for a precision prediction
  - factorization proof at NNLO is nontrivial
  - Li,Ma,Qiu 2020; Chen,Wang,Zhu 2020

Calculation procedures

- FeynRules and FeynArts to auto produce the Feynman diagrams and amplitudes

Christensen et al,1310.1921, T. Hahn,0012260



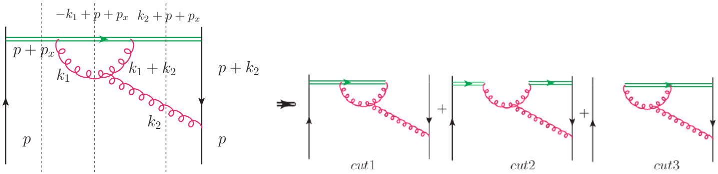
- Cutkosky rules, *J.Math.Phys.1,429(1960)*

$$\delta(k_z - xp_z) = \frac{1}{2\pi i}\left(\frac{1}{k_z - xp_z - i0} - \frac{1}{k_z - xp_z + i0}\right).$$

- Solve Master Integrals(MIs)

Differential equation, A.V.Kotikov, PLB254,158(1991); PLB267,123(1991)

An example in Feynman gauge



- From auxiliary field back to Wilson line, we need to do the cuts. For cut1, we have $p_x = -p - k_2$

$$\mathcal{M}|_{cut1} = \mu^{4\epsilon} \int\int \frac{d^{4-2\epsilon}k_1}{(2\pi)^{4-2\epsilon}} \frac{d^{4-2\epsilon}k_2}{(2\pi)^{4-2\epsilon}} \text{ampcut1} \times \delta\left(k_2^z + p^z - yp^z\right),$$

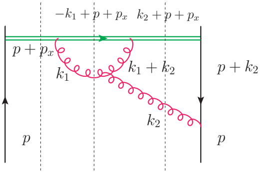
- For cut2, we have $p_x = -p + k_1$; both them give real contribution

$$\mathcal{M}|_{cut2} = \mu^{4\epsilon} \int\int \frac{d^{4-2\epsilon}k_1}{(2\pi)^{4-2\epsilon}} \frac{d^{4-2\epsilon}k_2}{(2\pi)^{4-2\epsilon}} \text{ampcut2} \times \delta\left(-k_1^z + p^z - yp^z\right),$$

- For cut3, we have $p_x = -p$ and it gives virtual contribution

$$\mathcal{M}|_{cut3} = \mu^{4\epsilon} \int\int \frac{d^{4-2\epsilon}k_1}{(2\pi)^{4-2\epsilon}} \frac{d^{4-2\epsilon}k_2}{(2\pi)^{4-2\epsilon}} \text{ampcut3} \times \delta\left(1 - y\right),$$

An example in Feynman gauge



- Use the identity

$$\frac{1}{k_1\cdot n k_2\cdot n} = \frac{1}{(k_1\cdot n + k_2\cdot n)k_2\cdot n} + \frac{1}{k_1\cdot n(k_1\cdot n + k_2\cdot n)},$$

- do the momentum transformation, then

$$\mathcal{M}|_{cut1+cut2+cut3} = \left[\mu^{4\epsilon}\int\int \frac{d^{3-2\epsilon}k_1}{(2\pi)^{4-2\epsilon}}\frac{d^{4-2\epsilon}k_2}{(2\pi)^{4-2\epsilon}}\mathrm{ampcut1}'|_{k_1^z=yp_z}\right]_+ + \left[\mu^{4\epsilon}\int\int \frac{d^{3-2\epsilon}k_1}{(2\pi)^{4-2\epsilon}}\frac{d^{4-2\epsilon}k_2}{(2\pi)^{4-2\epsilon}}\mathrm{ampcut2}'|_{k_1^z=yp_z}\right]_+$$

- It includes both the virtual and real contributions

Master Integrals Calculation:Differential Equations

- To calculate MIs f_i, we can set up a differential equation with respect to Lorentz invariant kinematics z, for example $z = \frac{p^0}{p^z}$(or p^2)
- If the number of MIs is larger than 1, A is $n \times n$ coefficient matrix and depends on both z and ϵ

$$\frac{d}{dz}\begin{pmatrix} f_1(z,\epsilon) \\ \vdots \\ f_n(z,\epsilon) \end{pmatrix} = \begin{pmatrix} A_{11}(z,\epsilon) & \dots & A_{1n}(z,\epsilon) \\ \vdots & & \vdots \\ A_{n1}(z,\epsilon) & \dots & A_{nn}(z,\epsilon) \end{pmatrix} \begin{pmatrix} f_1(z,\epsilon) \\ \vdots \\ f_n(z,\epsilon) \end{pmatrix}$$

A.V.Kotikov, PLB254,158(1991); PLB267,123(1991)

- It is not easy to determine all the boundary condition for MIs f_i

A suitable choice of basis: Canonical basis

$$\frac{d}{dz}\begin{pmatrix} g_1(z;\epsilon) \\ \vdots \\ g_n(z;\epsilon) \end{pmatrix} = \epsilon \begin{pmatrix} B_{11}(z) & \ldots & B_{1n}(z) \\ \vdots & & \vdots \\ B_{n1}(z) & \ldots & B_{nn}(z) \end{pmatrix} \begin{pmatrix} g_1(z;\epsilon) \\ \vdots \\ g_n(z;\epsilon) \end{pmatrix}$$

where

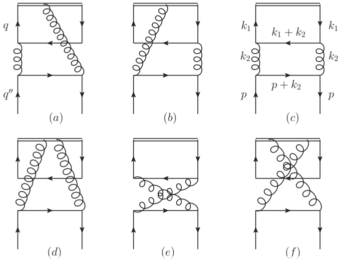
$$\vec{f} = T\vec{g}$$
$$B = T^{-1}AT - T^{-1}\partial_z T$$

- New strategy in dimensional regularization with $D = 4 - 2\epsilon$
- A linear transformation of MIs to the canonical basis
- The coefficient matrix B only depends on z

J.M.Henn,PRL110,251601(2013)

Outline

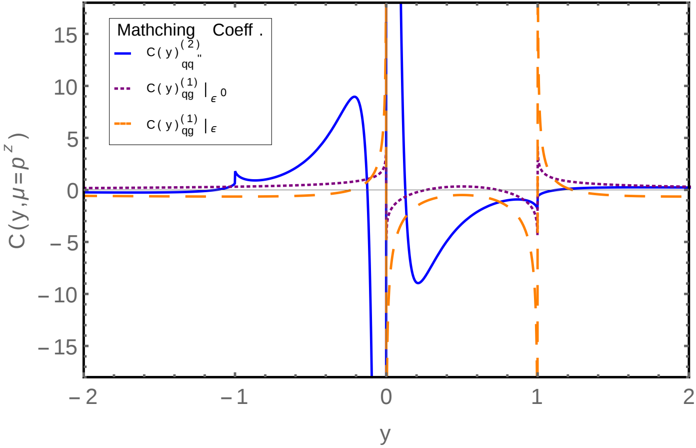
$q \to q'$ case



$q \to q'$ Feynman diagrams

$$\tilde{f}^{(2)}_{q/q'}\left(y, \frac{p^z}{\mu}, \epsilon_{\mathrm{IR}}\right) = C^{(2)}_{qq''}\left(\frac{y}{x}, \frac{|x|p^z}{\mu}\right) \otimes f^{(0)}_{q''/q'}(x, \epsilon_{\mathrm{IR}})$$
$$+ C^{(1)}_{qq''}\left(\frac{y}{x}, \frac{|x|p^z}{\mu}\right) \otimes f^{(1)}_{q''/q'}(x, \epsilon_{\mathrm{IR}})$$
$$+ C^{(0)}_{qq''}\left(\frac{y}{x}, \frac{|x|p^z}{\mu}\right) \otimes f^{(2)}_{q''/q'}(x, \epsilon_{\mathrm{IR}}).$$

$q \to q'$ case



$q \to q'$ two loop matching coefficients

Outline

Renormalization

- Renormalization procedure

$$\tilde{f}(y, \frac{p^z}{\mu}, \epsilon_{\mathrm{IR}}) = \int \frac{dy_1}{|y_1|} \left[Z_q \tilde{Z} \left(\frac{y}{y_1} \right) \right] \left[Z_q^{-1} \tilde{f} \left(y_1, \frac{p^z}{\mu}, \epsilon \right) \right].$$

Z_q is quark renormalization constant, $\tilde{Z}$ is quasi distribution renormalization factor

$$\tilde{Z}(\xi) = \delta(1-\xi) \left(1 + \frac{\alpha_s}{2\pi} \frac{\tilde{Z}^{(1)}}{\epsilon_{UV}} + \left(\frac{\alpha_s}{2\pi} \right)^2 \frac{\tilde{Z}^{(2)}}{\epsilon_{UV}^2} \right),$$

$$\tilde{Z}^{(1)} = -\frac{3 C_F S_\epsilon}{2}, \quad \tilde{Z}^{(2)} = S_\epsilon^2 \left(\frac{a + 9 C_F^2}{4} + \frac{b}{4} \epsilon \right)$$

X. Ji and J.H. Zhang,1505.07699;

Braun, Chetyrkin and Kniehl,2004.01043

IR behavior in Quasi PDF

- Soft divergences are cancelled
- Reducible collinear divergences

$$\tilde{f}^{(2)}_{q/q}(y, \frac{p^z}{\mu}, \epsilon_{\rm IR})|_{div.part.1} = C^{(1)}_{qq}\left(\frac{y}{x}, \frac{|x|p^z}{\mu}\right) \otimes \left[-\frac{(1+x^2)}{(1-x)}\right]_+ \frac{1}{\epsilon_{\rm IR}}.$$

- "Irreducible" collinear divergences

the same as light cone PDFs, including both $\frac{1}{\epsilon_{\rm IR}}$ and $\left(\frac{1}{\epsilon_{\rm IR}}\right)^2$

$$\tilde{f}^{(2)}_{i/j}(y, \frac{p^z}{\mu}, \epsilon_{\rm IR})|_{div.part.2} = f^{(2)}_{i/j}(x, \epsilon_{\rm IR}).$$

$$f^{(2)}_{i/j}(x) = \frac{1}{2\epsilon^2_{\rm IR}}\left[\sum_k P^{(0)}_{ik}(z) \otimes P^{(0)}_{kj}(x) + \beta_0 P^{(0)}_{ij}(z)\right] - \frac{P^{(1)}_{ij}(x)}{\epsilon_{\rm IR}}$$

Factorization formula at NNLO

- Matching procedure between renormalized quasi and light-cone PDFs:

$$\tilde{f}^{(0)}_{i/k}(y, \frac{p^z}{\mu}) = C^{(0)}_{ij}\left(\frac{y}{x}, \frac{|x|p^z}{\mu}\right) \otimes f^{(0)}_{j/k}(x),$$

$$\tilde{f}^{(1)}_{i/k}(y, \frac{p^z}{\mu}, \epsilon_{\mathrm{IR}}) = C^{(1)}_{ij}\left(\frac{y}{x}, \frac{|x|p^z}{\mu}\right) \otimes f^{(0)}_{j/k}(x) + C^{(0)}_{ij}\left(\frac{y}{x}, \frac{|x|p^z}{\mu}\right) \otimes f^{(1)}_{j/k}(x, \epsilon_{\mathrm{IR}}),$$

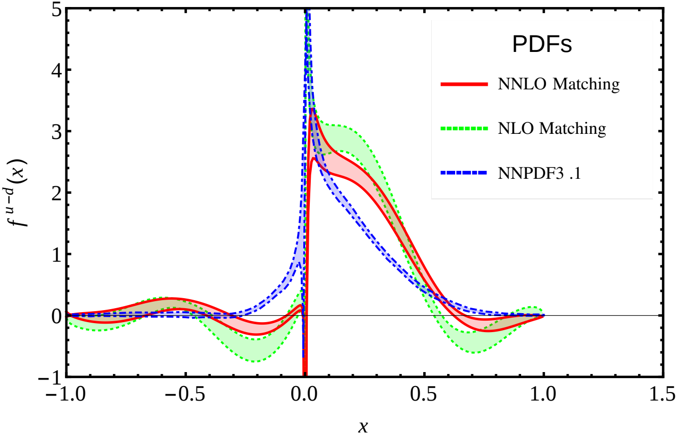
$$\tilde{f}^{(2)}_{i/k}(y, \frac{p^z}{\mu}, \epsilon_{\mathrm{IR}}) = C^{(2)}_{ij}\left(\frac{y}{x}, \frac{|x|p^z}{\mu}\right) \otimes f^{(0)}_{j/k}(x) + C^{(1)}_{ij}\left(\frac{y}{x}, \frac{|x|p^z}{\mu}\right) \otimes f^{(1)}_{j/k}(x, \epsilon_{\mathrm{IR}}) + C^{(0)}_{ij}\left(\frac{y}{x}, \frac{|x|p^z}{\mu}\right) \otimes f^{(2)}_{j/k}(x, \epsilon_{\mathrm{IR}}).$$

NNLO matching coefficients $C_{qq}^{(2)}$

- consistent results in $\overline{\mathrm{MS}}$ scheme by Li-Ma-Qiu *Phys.Rev.Lett. 126, 072001(2021)*
- We also obtained $C_{qq}^{(2)}(y, \frac{p^z}{\mu})$ in both RI/MOM and $\mathrm{M}\overline{\mathrm{MS}}$ scheme
- 4 regions for y and 3 color structures $(C_F,\ C_A,\ nf\ T_F)C_F$
- the final asymptotic behavior: $C_{qq}^{(2),\mathrm{M}\overline{\mathrm{MS}}}|_{y\to\infty} \propto \frac{1}{y^2}$

$$
\begin{aligned}
&C_{qq}^{(2),\mathrm{M}\overline{\mathrm{MS}}}(y, \frac{p^z}{\mu}) \\
=&[C_{qq}^{(2),\mathrm{M}\overline{\mathrm{MS}}}(y, \frac{p^z}{\mu})|_{y>1}]_+ + [C_{qq}^{(2),\mathrm{M}\overline{\mathrm{MS}}}(y, \frac{p^z}{\mu})|_{0<y<1}]_+ \\
&+ [C_{qq}^{(2),\mathrm{M}\overline{\mathrm{MS}}}(y, \frac{p^z}{\mu})|_{-1<y<0}]_+ + [C_{qq}^{(2),\mathrm{M}\overline{\mathrm{MS}}}(y, \frac{p^z}{\mu})|_{y<-1}]_+
\end{aligned}
$$

PDFs from NNLO Matching



using ETMC data with $z_{cut} = 10a$, $p^z = 2.3GeV$, $\mu = 2GeV$ and in modified $\overline{\mathrm{MS}}$ scheme; uncertainty is from lattice data: *C. Alexandrou et al., Phys. Rev. D 99, 114504 (2019)*

Summary

- NNLO correction is important
- NNLO matching coefficients of quark PDF are obtained
- Complete cancellation of IR divergence is confirmed, which nontrivially validates the LaMET factorization at NNLO

- Outlook
  - *Gluon quasi distribution functions at NNLO*
  - *Pion quasi distribution amplitudes at NNLO*
  - *A new stage of lattice calculation of PDFs with NNLO matching*

Thank you a lot!