# Quark spin-orbit correlations in the proton

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# Direct evaluation of quark spin-orbit correlations

$$2L_3 S_3 \;=\; \int dx \int d^2 k_T \int d^2 r_T \,(r_T \times k_T)_3 \,\Sigma\; \mathcal{W}_\Sigma^{\mathcal{U}}(x, k_T, r_T) \qquad \text{Wigner distribution}$$

$$\frac{2L_3 S_3}{n} = \frac{\epsilon_{ij}\frac{\partial}{\partial z_{T,i}}\frac{\partial}{\partial \Delta_{T,j}} \langle P + \Delta_T/2 \,|\, \overline{\psi}(-z/2)\gamma^+\gamma^5 \,\mathcal{U}[-z/2, z/2]\psi(z/2) \,|\, P - \Delta_T/2 \,\rangle\big|_{z^+=z^-=0,\,\Delta_T=0,\,z_T\to0}}{\langle P + \Delta_T/2 \,|\, \overline{\psi}(-z/2)\gamma^+\mathcal{U}[-z/2, z/2]\psi(z/2) \,|\, P - \Delta_T/2 \,\rangle\big|_{z^+=z^-=0,\,\Delta_T=0,\,z_T\to0}}$$

$P$ in 3-direction, $P \to \infty$

Renormalization: Form ratio with number of valence quarks $n$ – note: are using Domain Wall Fermions!

Connection to GTMDs: $\qquad 2L_3 S_3 = \int dx \int d^2 k_T \, \frac{k_T^2}{M^2} \, G_{11} \bigg|_{\Delta_T=0}$
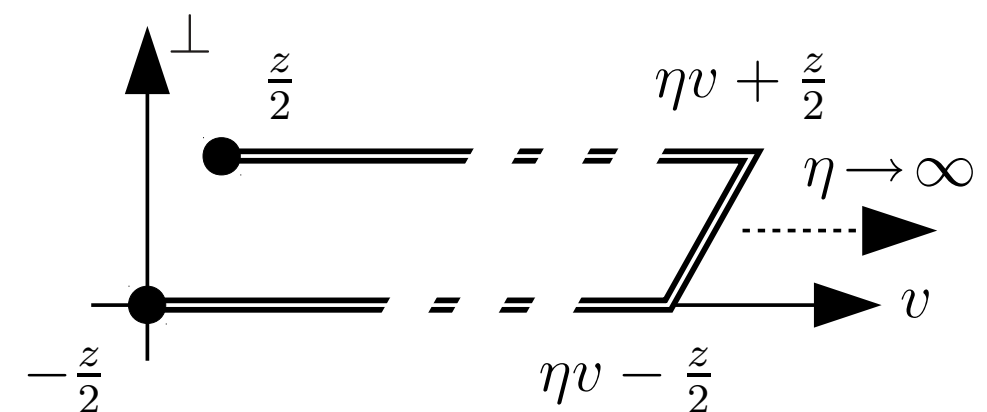
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Role of the gauge link $\mathcal{U}$:

- Straight $\mathcal{U}[-z/2, z/2] \longrightarrow$ Ji OAM
- Staple-shaped $\mathcal{U}[-z/2, z/2] \longrightarrow$ Jaffe-Manohar OAM



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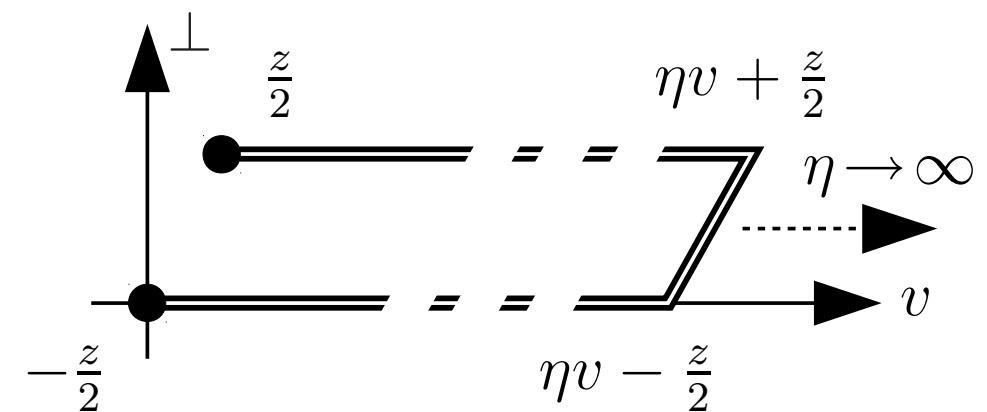
## Role of the gauge link $\mathcal{U}$:

Direction of staple taken off light cone (rapidity divergences)

Characterized by Collins-Soper parameter $\qquad \hat{\zeta} \equiv \dfrac{P \cdot v}{|P||v|}$

Are interested in $\hat{\zeta} \longrightarrow \infty$; synonymous with $P \longrightarrow \infty$ in the frame of the lattice calculation ($v = e_3$)

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# Direct evaluation of quark spin-orbit correlations

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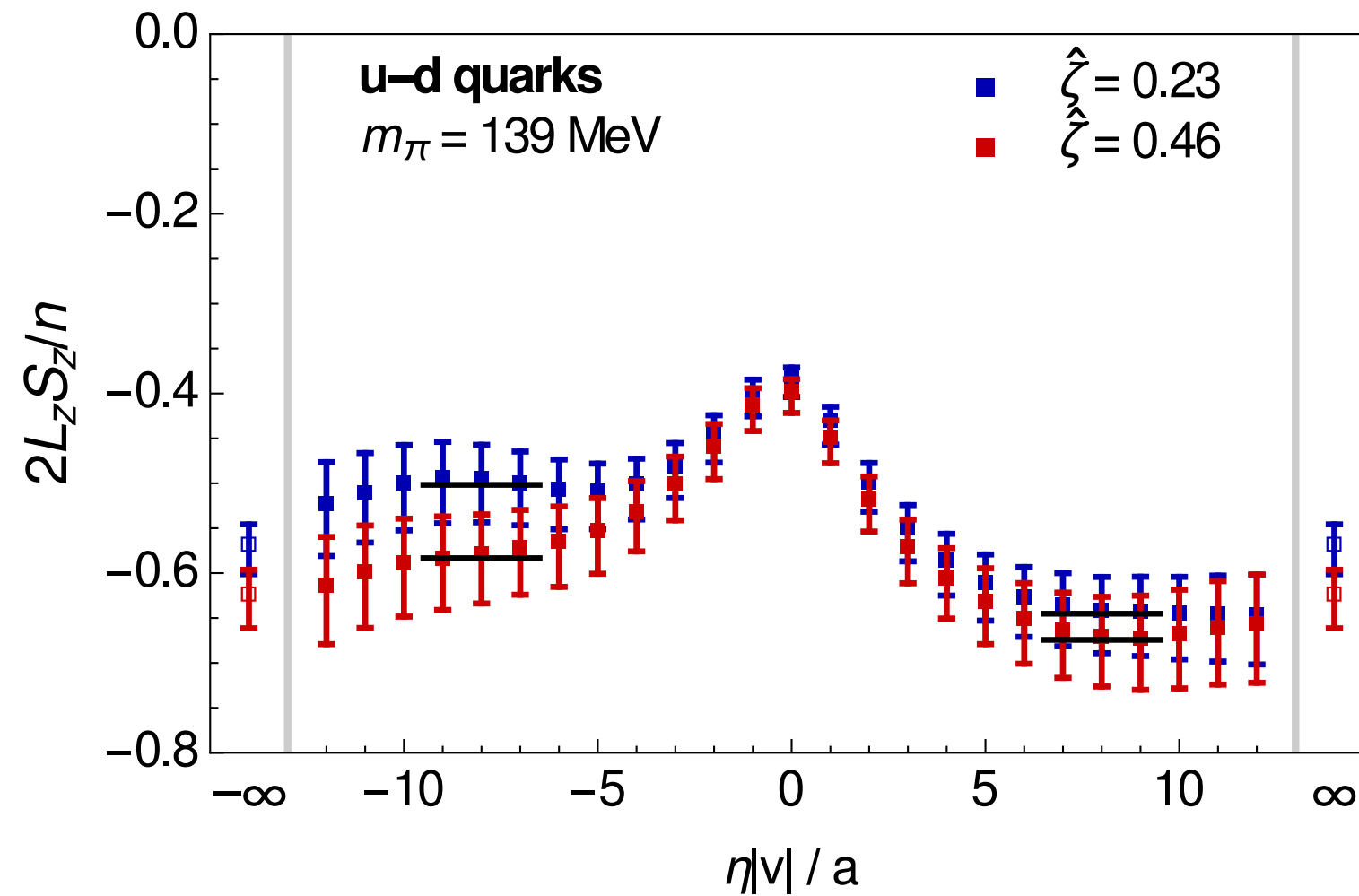
- Derivative w.r.t. $\Delta_T$ taken using direct derivative method (Rome method)

- Derivative w.r.t. $z_T$ taken as finite difference over lattice spacing $a$
  (cutoff on transverse momenta same as lattice cutoff)

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# Ensemble details

DWF ensemble provided by the RBC/UKQCD collaboration

| $L^3 \times T$ | $a$(fm) | $m_\pi$ (MeV) | #conf. | #meas. |
|---|---|---|---|---|
| $48^3 \times 96$ | 0.114 | 139 | 130 | (33280 sloppy + 520 exact) AMA |

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# Result



For comparison, in a polarized proton (from C. Alexandrou et al., PRD 101 (2020) 094513; 2003.08486):

$\langle L^u \rangle = -0.22(3) \, , \, \langle 2S^u \rangle = 0.86(2) \Rightarrow \langle L^u \rangle \langle 2S^u \rangle = -0.2$

$\langle L^d \rangle = 0.26(2) \, , \, \langle 2S^d \rangle = -0.42(2) \Rightarrow \langle L^d \rangle \langle 2S^d \rangle = -0.1$

$\Rightarrow \langle L^u \rangle \langle 2S^u \rangle - \langle L^d \rangle \langle 2S^d \rangle = -0.1$

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# Conclusion

- Strong spin-orbit coupling of quarks in the proton observed.

- Effect of final-state interactions even more pronounced than for orbital angular momentum alone.

- To date, results only for one source-sink separation, one lattice spacing - further systematics to be explored.

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