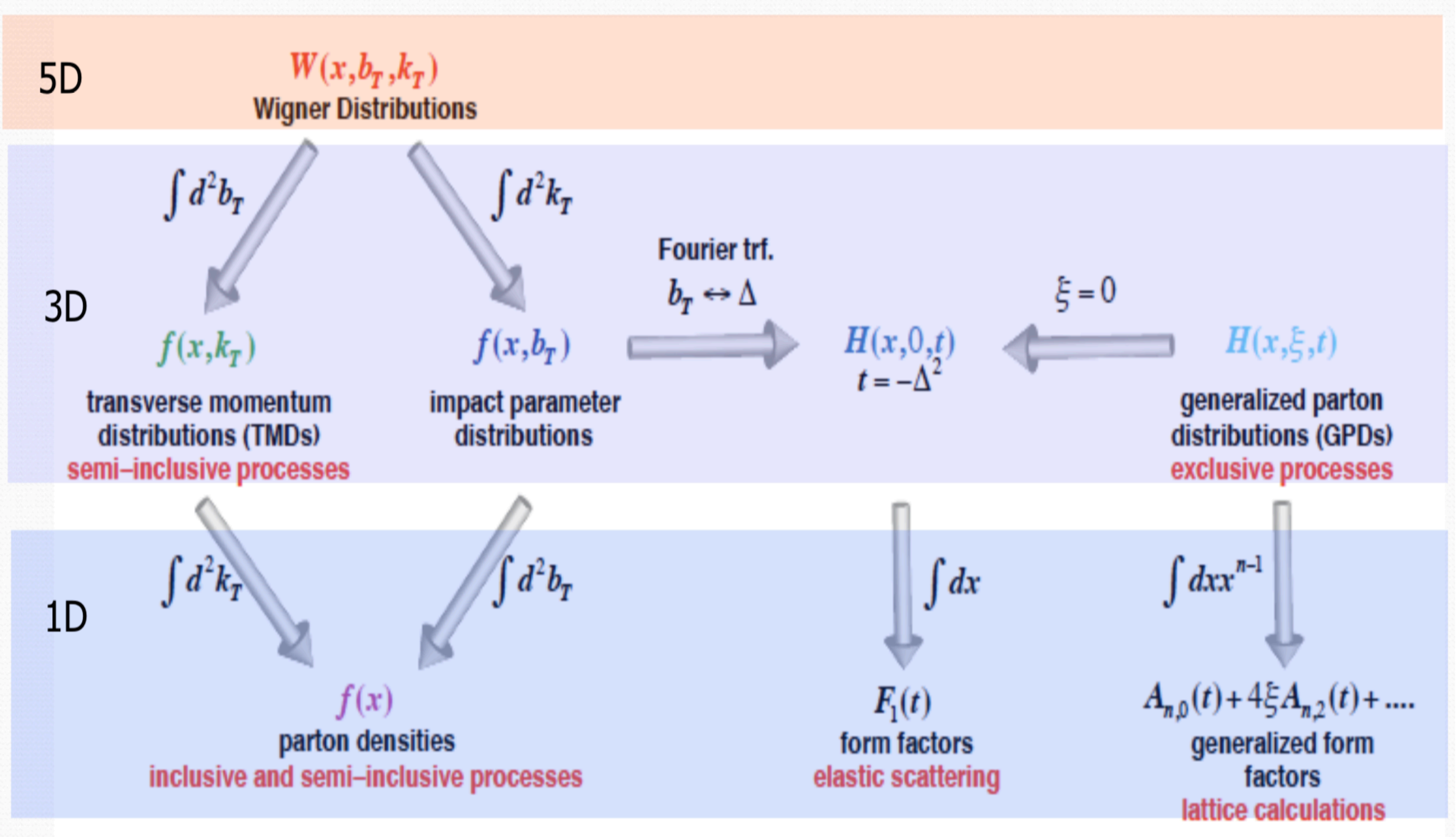
Towards precision calculations of partonic structure of hadrons

Jianhui Zhang
Beijing Normal University

Lattice 2021, Jul. 28, 2021

Partonic structure of hadrons



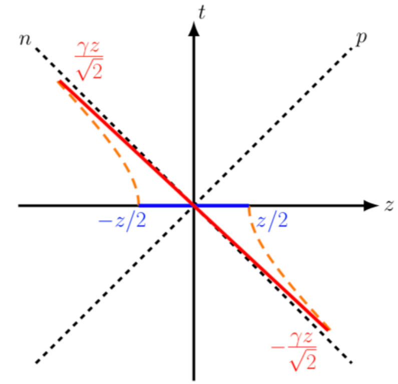
- It is naturally described by light-front correlations, e.g.

$$f(x) = \int \frac{d\lambda}{4\pi} e^{ix\lambda} \langle P | \bar{\psi}(0) n \cdot \Gamma W(0,\lambda n) \psi(\lambda n) | P \rangle$$

Partons from Euclidean correlations

- Large-Momentum Effective Theory

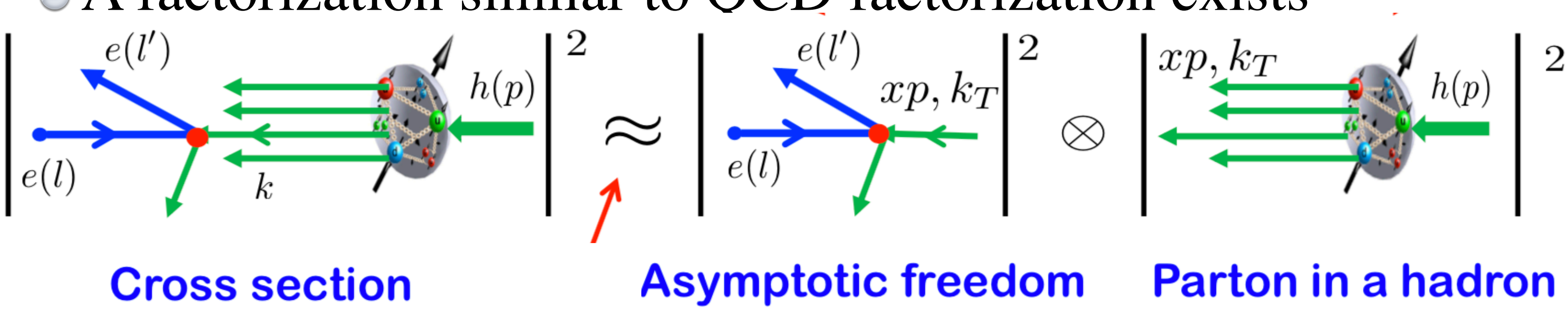
  Ji, PRL 13' & SCPMA 14',
  Ji, Liu, Liu, JHZ, Zhao, arXiv:2004.03543,
  to appear in Rev. Mod. Phys.



- Other options

  Braun and Mueller, EPJC 08', Chambers et al, PRL 17', Detmold and Lin, PRD 06', Liu and Dong, PRL 94', Ma and Qiu, 14' & PRL 17', Radyushkin, PRD 17' …

- A factorization similar to QCD factorization exists



$$\tilde{q}(y, P^z) = C(y/x, \mu/xP^z) \otimes q(x, \mu) + \mathcal{O}(\Lambda^2_{QCD}/(yP^z)^2, \Lambda^2_{QCD}/((1-y)P^z)^2)$$

or

$$q(x, \mu) = \bar{C}(x/y, \mu/yP^z) \otimes \tilde{q}(y, \mu) + \mathcal{O}(\Lambda^2_{QCD}/(xP^z)^2, \Lambda^2_{QCD}/((1-x)P^z)^2)$$

- Can be accessed on a Euclidean lattice

Partons from lattice QCD

- Existing studies (many are exploratory)
- Leading-twist quark/gluon distributions
  - LP3, ETMC, BNL, JLab, LPC…
- Distribution amplitudes/light-front wave functions
  - LP3, LPC
- Generalized parton distributions
  - LP3, MSU, ETMC
- Transverse-momentum-dependent parton distributions
  - MIT, LPC, ETMC
- Higher-twist distributions
  - ETMC, Regensburg
- We are entering a stage calling for precision calculations

Towards precision calculations

- Appropriate non-perturbative renormalization
- Control of Fourier transform
- Perturbative corrections
- Physical limit
- Control of higher-twist contamination

Towards precision calculations

- Appropriate non-perturbative renormalization
- Control of Fourier transform
- Perturbative corrections
- Physical limit
- Control of higher-twist contamination

NPR of quasi-LF correlations

- Example: $\tilde{h}(z, P^z) = \langle P | \bar{\psi}(z)\gamma^t W(z,0)\psi(0) | P \rangle$
- The quasi-LF correlation operator is multiplicatively renormalized Ji, JHZ, Zhao, PRL 18', Ishikawa et al, PRD 17', Green et al, PRL 18'

$$[\bar{\psi}(z)\Gamma W(z,0)\psi(0)]_B = e^{\delta m|z|} Z\, [\bar{\psi}(z)\Gamma W(z,0)\psi(0)]_R$$

- $\langle P | \bar{\psi}(z)\Gamma W(z,0)\psi(0) | P \rangle / \langle X | \bar{\psi}(z)\Gamma W(z,0)\psi(0) | X \rangle$ is UV finite
- Previous proposals:
- RI/MOM: Alexandrou et al, NPB 17', Stewart, Zhao, PRD 18'

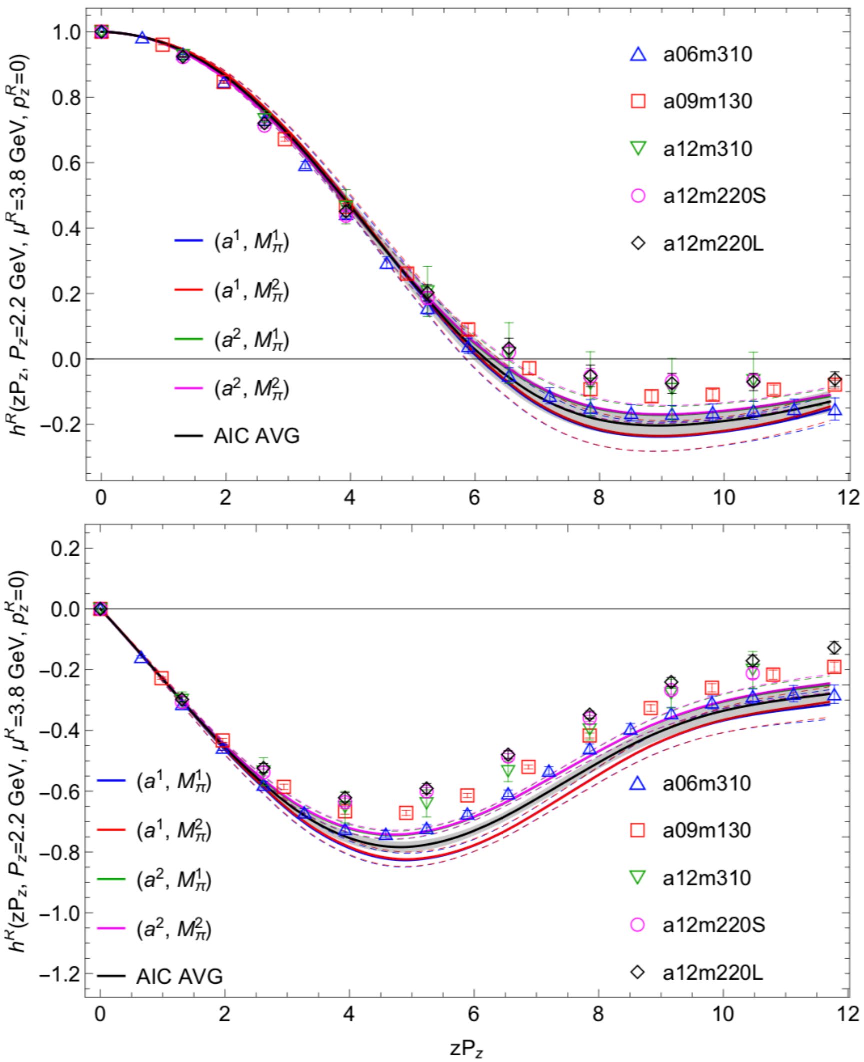
  $|X\rangle$ is chosen as a single off-shell quark state
- Ratio: Radyushkin, PRD 17'

  $|X\rangle$ is chosen as a zero momentum hadron state
- VEV: Braun, Vladimirov, JHZ, PRD 19', Li, Ma, Qiu, PRL 21'

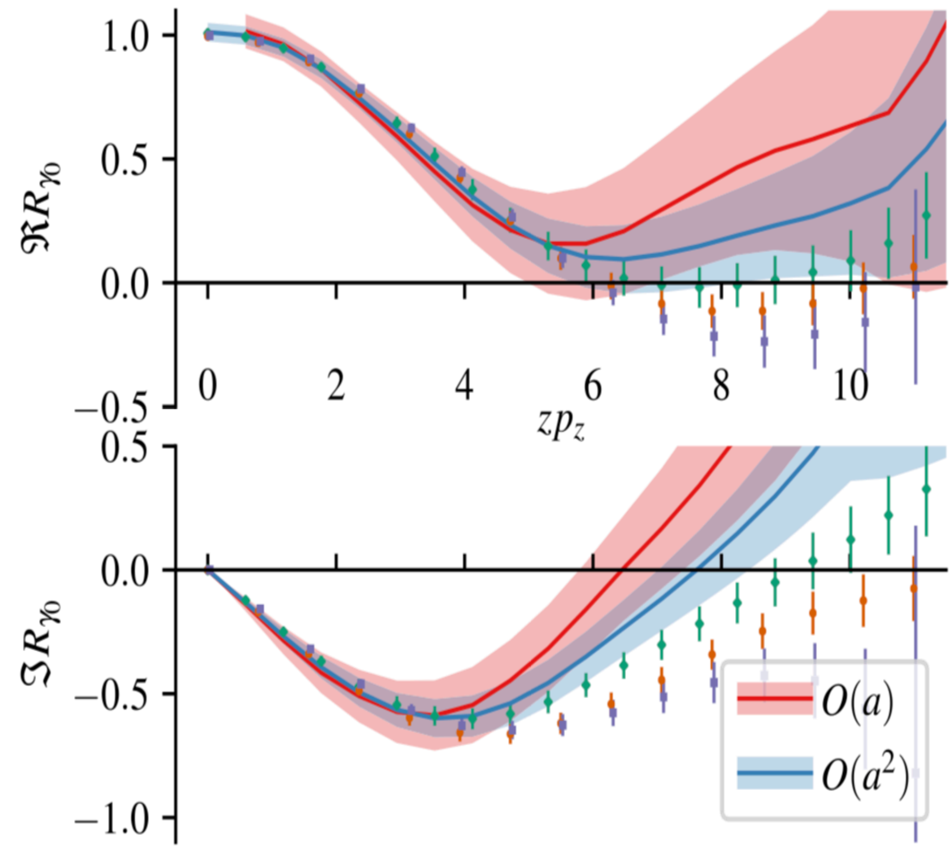
  $|X\rangle$ is chosen as the vacuum

NPR of quasi-LF correlations

- Application to lattice calculations



Lin et al, 20'

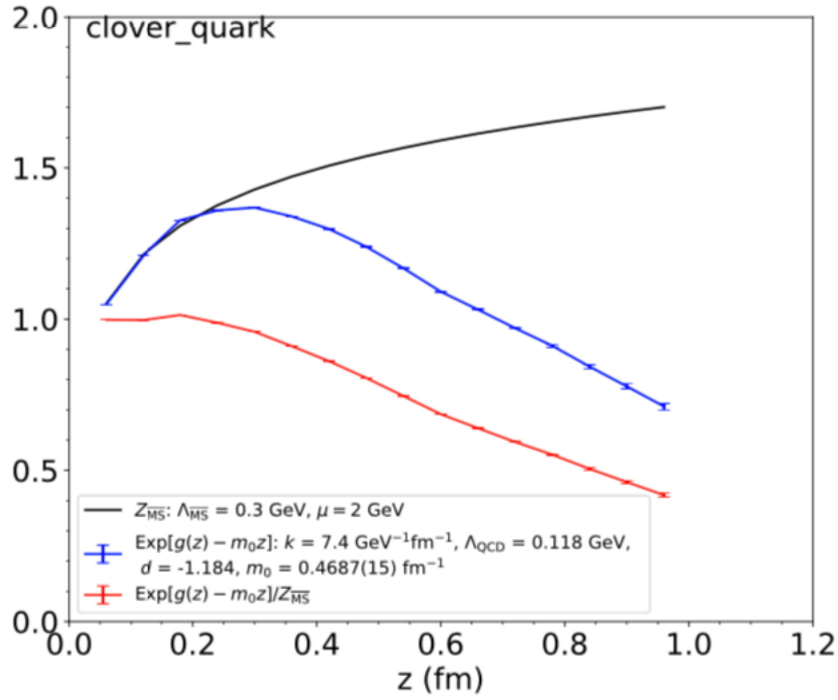


Alexandrou et al, PRD 21'

Non-perturbative renormalization

- Problem: these schemes introduce undesired IR effects at large distances

- Example:



LPC (Huo, Su et al, NPB 21')

Non-perturbative renormalization

- Problem: these schemes introduce undesired IR effects at large distances

- Solution 1: hybrid renormalization Ji, JHZ et al, NPB 21'
- Separate the short and long distance quasi-LF correlations and renormalize them differently

$$\tilde{h}^R(z,a,P_z) = \frac{\tilde{h}(z,a,P_z)}{Z_X(z,a)}\theta(z_S - |z|) + \tilde{h}(z,a,P_z)e^{-\delta m|z|}Z_{\text{hybrid}}(z_S,a)\theta(|z|-z_S)$$

- $\delta m = m_{-1}/a + m_0$ can be extracted from asymptotic behavior $\sim e^{-\delta m|z|}$ of hadron matrix element etc.

- $Z_{\text{hybrid}}(z_S,a)$ can be determined by continuity condition at $z = z_S$

$$\tilde{h}(z,a,P_z)e^{-\delta m|z|}Z_{\text{hybrid}}(z_S,a) = \frac{\tilde{h}(z,a,P_z)}{Z_X(z,a)}, \quad Z_{\text{hybrid}}(z_S,a) = e^{\delta m\, z_S}/Z_X(z_S,a)$$

Non-perturbative renormalization

- Problem: these schemes introduce undesired IR effects at large distances

- Solution 2: Self renormalization LPC (Huo, Su et al, 21')
- Fitting the bare matrix elements at multi-lattice-spacings to

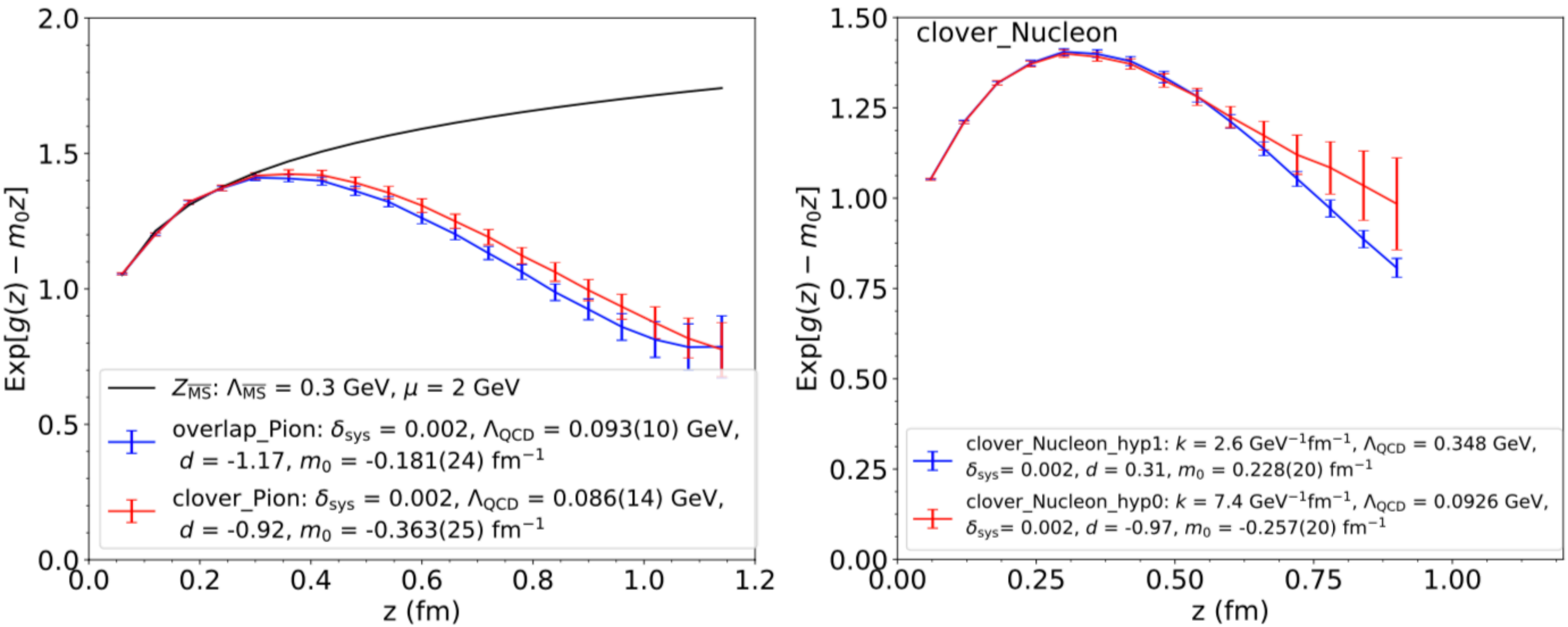
$$\ln \mathcal{M}(z,a) = \frac{kz}{a\ln[a\Lambda_{\rm QCD}]} + m_0 z + g'(z) + f(z)a + \frac{3C_F}{b_0}\ln\left[\frac{\ln[1/(a\Lambda_{\rm QCD})]}{\ln[\mu/\Lambda_{\rm QCD}]}\right] + \ln\left[1+\frac{d}{\ln(a\Lambda_{\rm QCD})}\right]$$

- The pieces other than $g'(z)$ are renormalization factors
- m_0 can be determined by matching the renormalized matrix element to the continuum $\overline{\rm MS}$ result at short distance
- Such renormalized matrix element can then be matched to the PDF using the $\overline{\rm MS}$ matching

Non-perturbative renormalization

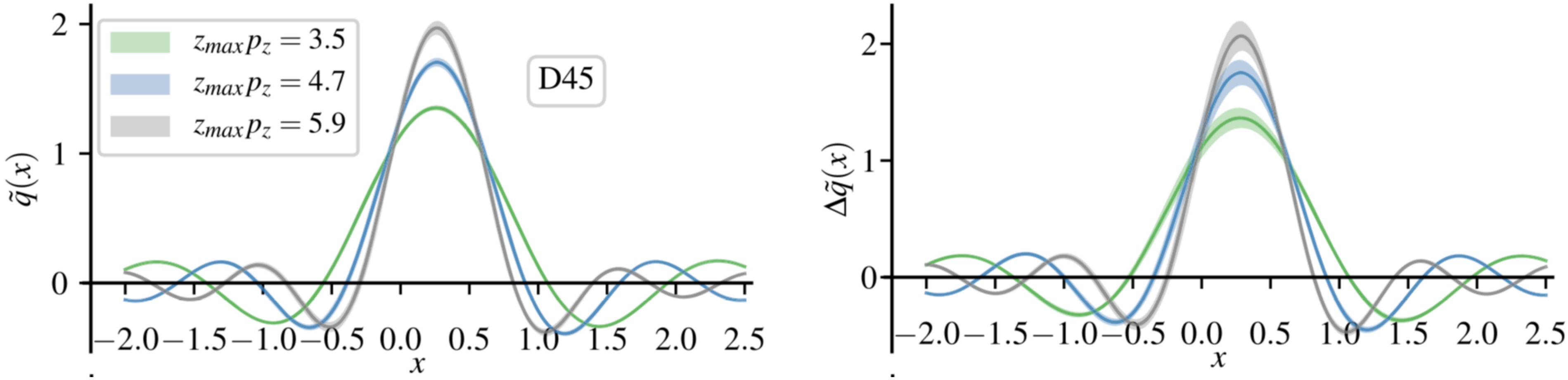
- Problem: these schemes introduce undesired IR effects at large distances

- Solution 2: Self renormalization LPC (Huo, Su et al, 21')

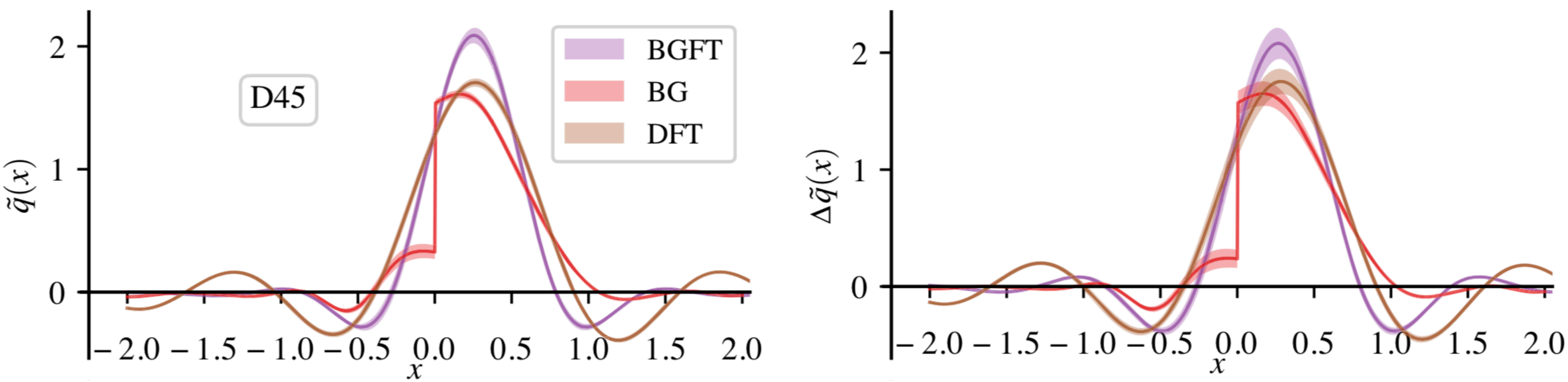


Fourier transform to momentum space

- Truncated FT introduces additional systematic uncertainty



- Supplemented with sophisticated reconstruction techniques



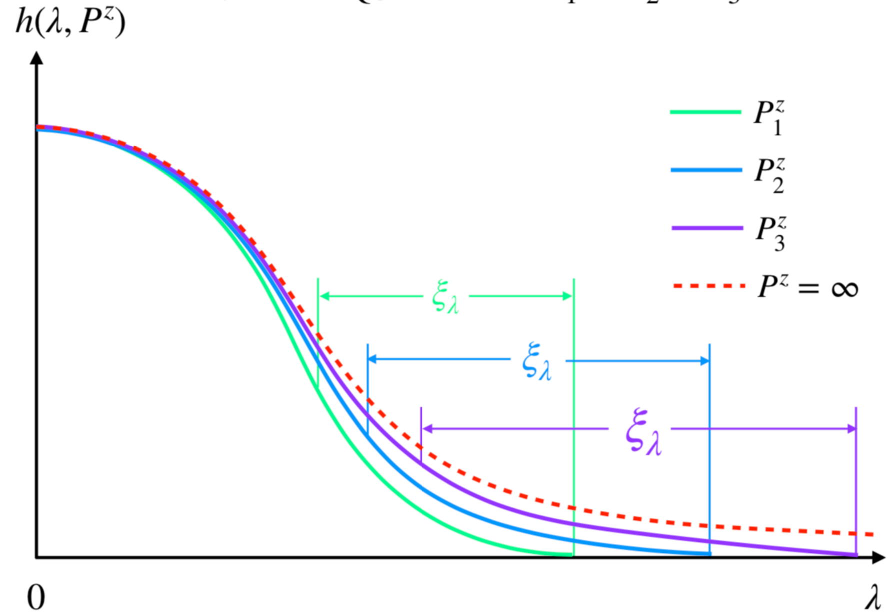
e.g. Backus – Gilbert method & Bayes – Gauss – FT   Alexandrou et al, PRD 21'

Alternative strategy

- Extrapolation to asymptotic distance
- Lattice data (available up to limited $z_{\rm max}$ or $\lambda_{\rm max}$) may be supplemented with physics-based extrapolation
- At finite momentum, quasi-LF correlation has a finite correlation length and exhibits an exponential decay behavior $e^{-\lambda/\xi_\lambda}$ (usually associated with algebraic terms) in λ space

$$\xi_\lambda \sim P^z/\Lambda_{\rm QCD}\,, \qquad 0 < P_1^z < P_2^z < P_3^z$$

- Depending on hadron momentum, one can choose to do an exponential or an algebraic extrapolation



Ji, JHZ et al, NPB 21’

Alternative strategy

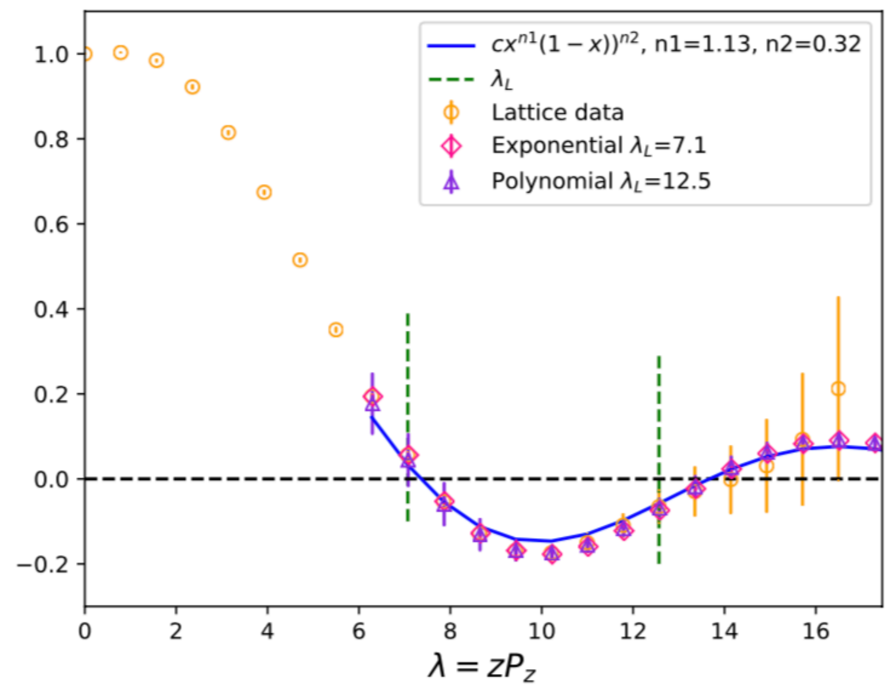
- Extrapolation to asymptotic distance
- Lattice data (available up to limited $z_{\max}$ or $\lambda_{\max}$) may be supplemented with physics-based extrapolation
- At finite momentum, quasi-LF correlation has a finite correlation length and exhibits an exponential decay behavior $e^{-\lambda/\xi_\lambda}$ (usually associated with algebraic terms) in λ space

- Example: vector meson distribution amplitude

$$\tilde{H}(z, P_z) = \left[\frac{c_1}{(-i\lambda)^a} + e^{i\lambda}\frac{c_2}{(i\lambda)^b}\right] e^{-\frac{\lambda}{\lambda_0}},$$

$$\tilde{H}(z, P_z) = \frac{c_1}{(-i\lambda)^a} + e^{i\lambda}\frac{c_2}{(i\lambda)^b},$$

LPC (Hua, JHZ et al, 20')



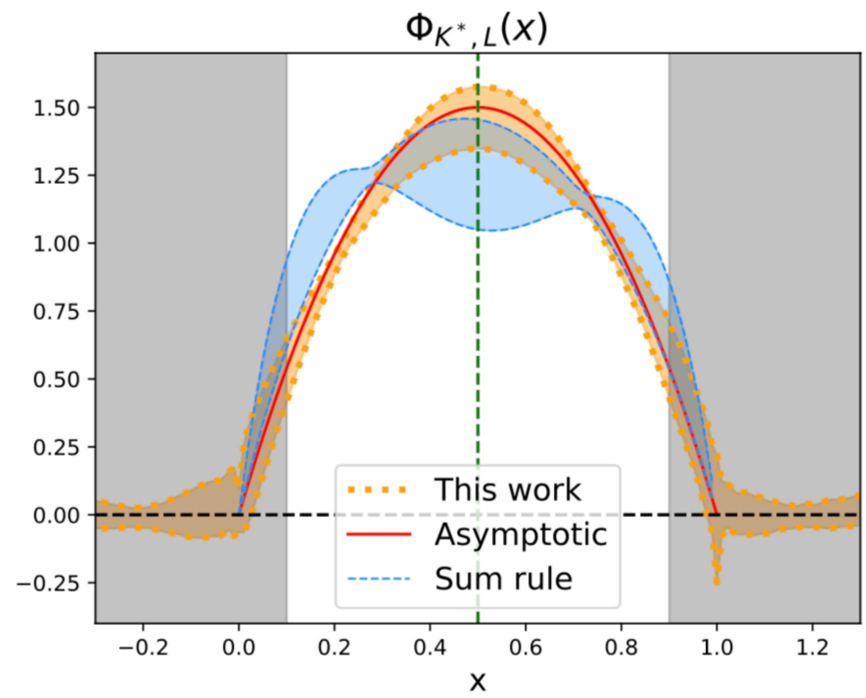
Alternative strategy

- Extrapolation to asymptotic distance
- Lattice data (available up to limited $z_{\max}$ or $\lambda_{\max}$) may be supplemented with physics-based extrapolation
- At finite momentum, quasi-LF correlation has a finite correlation length and exhibits an exponential decay behavior $e^{-\lambda/\xi_\lambda}$ (usually associated with algebraic terms) in λ space
- Example: vector meson distribution amplitude

$$\tilde{H}(z, P_z) = \left[\frac{c_1}{(-i\lambda)^a} + e^{i\lambda}\frac{c_2}{(i\lambda)^b}\right] e^{-\frac{\lambda}{\lambda_0}},$$

$$\tilde{H}(z, P_z) = \frac{c_1}{(-i\lambda)^a} + e^{i\lambda}\frac{c_2}{(i\lambda)^b},$$

LPC (Hua, JHZ et al, 20')



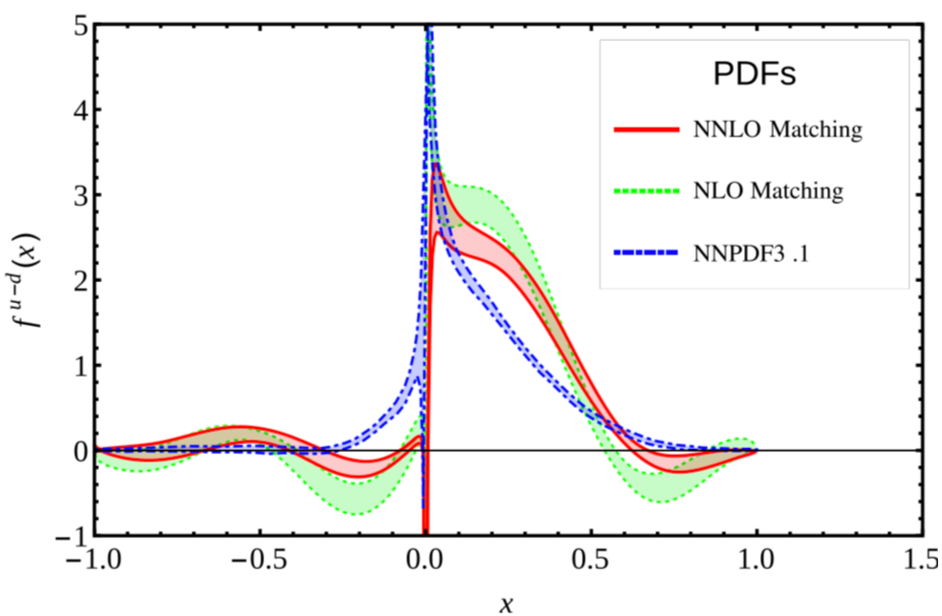
Perturbative corrections

- Development on perturbative calculations
- Anomalous dimension and renormalization factors to three-loop accuracy

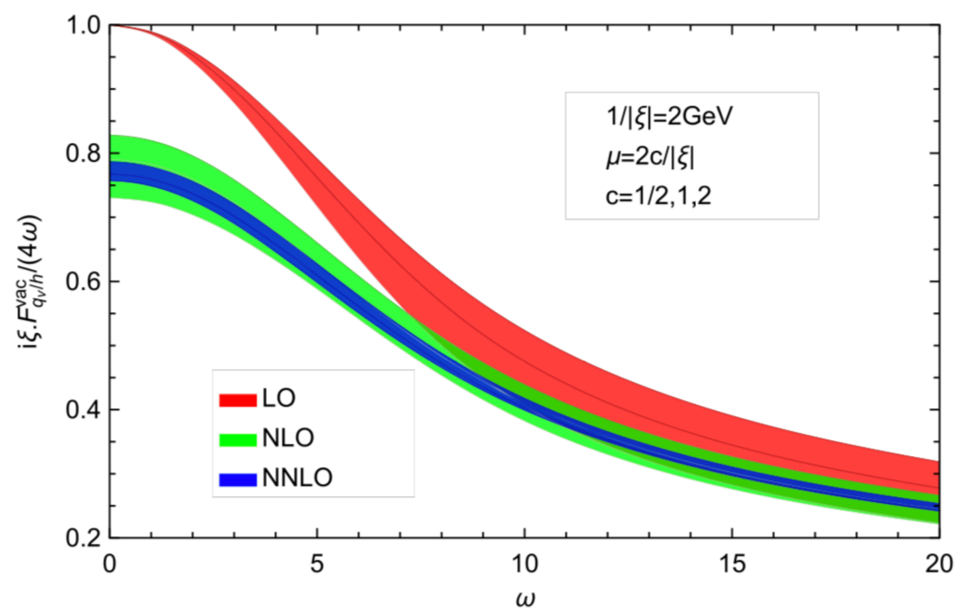
Braun, Chetyrkin, Kniehl, JHEP 20'

- NNLO matching ($\overline{\mathrm{MS}}$/RIMOM/VEV scheme)

Chen, Wang, Zhu, PRL 21'

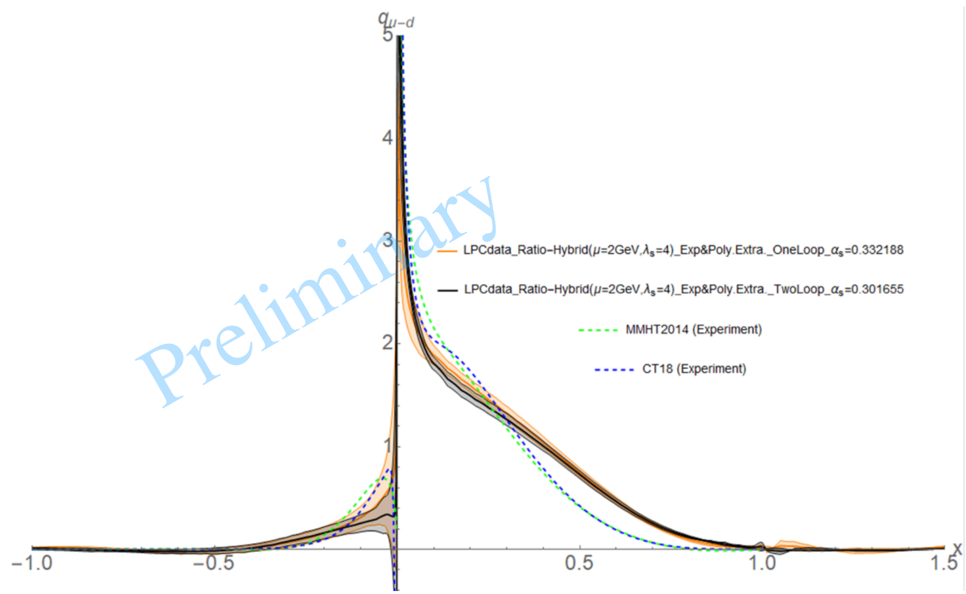


Li, Ma, Qiu, PRL 21'



Perturbative corrections

- Development on perturbative calculations
- NNLO matching in hybrid scheme



- No significant correction from NLO to NNLO
- Broader shape than global fit, excited state contamination in lattice data?

Conclusions

- Theory developments allow to make a comparison between theoretical predictions and experimental measurements of various partonic quantities
  - PDFs, DAs, GPDs, TMDs, LFWFs……
- For certain quantities, precision calculations become important in order to match experimental accuracy
- A systematic strategy of doing such calculations will help to study other partonic quantities that are not known well experimentally