Towards precision calculations of partonic structure of hadrons

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Partonic structure of hadrons

It is naturally described by light-front correlations, e.g.

\[
f(x) = \int \frac{d\lambda}{4\pi} e^{ix\lambda} \langle P | \bar{\psi}(0)n \cdot \Gamma W(0,\lambda n)\psi(\lambda n) | P \rangle
\]
Partons from Euclidean correlations

- **Large-Momentum Effective Theory**
  Ji, PRL 13’ & SCPMA 14’,
  Ji, Liu, Liu, JHZ, Zhao, arXiv:2004.03543,
  to appear in Rev. Mod. Phys.

- **Other options**
  Braun and Mueller, EPJC 08’, Chambers et al, PRL 17’, Detmold and Lin, PRD 06’,
  Liu and Dong, PRL 94’, Ma and Qiu, 14’ & PRL 17’, Radyushkin, PRD 17’ ...

- **A factorization similar to OCD factorization exists**

\[
\tilde{q}(y, P^z) = C(y/x, \mu/xP^z) \otimes q(x, \mu) + \mathcal{O}(\Lambda_{QCD}^2/(yP^z)^2, \Lambda_{QCD}^2/((1 - y)P^z)^2) \\
\text{or} \\
q(x, \mu) = \tilde{C}(x/y, \mu/yP^z) \otimes \tilde{q}(y, \mu) + \mathcal{O}(\Lambda_{QCD}^2/(xP^z)^2, \Lambda_{QCD}^2/((1 - x)P^z)^2)
\]

- Can be accessed on a Euclidean lattice
Partons from lattice QCD

- Existing studies (many are exploratory)
- Leading-twist quark/gluon distributions
  - LP3, ETMC, BNL, JLab, LPC…
- Distribution amplitudes/light-front wave functions
  - LP3, LPC
- Generalized parton distributions
  - LP3, MSU, ETMC
- Transverse-momentum-dependent parton distributions
  - MIT, LPC, ETMC
- Higher-twist distributions
  - ETMC, Regensburg

We are entering a stage calling for precision calculations
Towards precision calculations

- Appropriate non-perturbative renormalization
- Control of Fourier transform
- Perturbative corrections
- Physical limit
- Control of higher-twist contamination
Towards precision calculations

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NPR of quasi-LF correlations

Example: \( \tilde{h}(z, P^z) = \langle P | \bar{\psi}(z) \gamma^t W(z,0) \psi(0) | P \rangle \)

The quasi-LF correlation operator is multiplicatively renormalized Ji, JHZ, Zhao, PRL 18’, Ishikawa et al, PRD 17’, Green et al, PRL 18’

\[
[\tilde{\psi}(z) \Gamma W(z,0) \psi(0)]_B = e^{\delta m |z|} Z [\tilde{\psi}(z) \Gamma W(z,0) \psi(0)]_R
\]

\( \langle P | \bar{\psi}(z) \Gamma W(z,0) \psi(0) | P \rangle / \langle X | \bar{\psi}(z) \Gamma W(z,0) \psi(0) | X \rangle \) is UV finite

Previous proposals:

RI/MOM: Alexandrou et al, NPB 17’, Stewart, Zhao, PRD 18’

\(|X\rangle\) is chosen as a single off-shell quark state

Ratio: Radyushkin, PRD 17’

\(|X\rangle\) is chosen as a zero momentum hadron state

VEV: Braun, Vladimirov, JHZ, PRD 19’, Li, Ma, Qiu, PRL 21’

\(|X\rangle\) is chosen as the vacuum
NPR of quasi-LF correlations

Application to lattice calculations

Alexandrou et al, PRD 21’

Lin et al, 20’
Non-perturbative renormalization

**Problem**: these schemes introduce undesired IR effects at large distances

**Example**: LPC (Huo, Su et al, NPB 21’
Non-perturbative renormalization

- **Problem**: these schemes introduce undesired IR effects at large distances

- **Solution 1**: hybrid renormalization Ji, JHZ et al, NPB 21’

  Separate the short and long distance quasi-LF correlations and renormalize them differently

  \[
  \tilde{h}^R(z, a, P_z) = \frac{\tilde{h}(z, a, P_z)}{Z_X(z, a)} \theta(z_S - |z|) + \tilde{h}(z, a, P_z)e^{-\delta m|z|}Z_{\text{hybrid}}(z_S, a)\theta(|z| - z_S)
  \]

  \[
  \delta m = m_{-1}/a + m_0 \text{ can be extracted from asymptotic behavior } \sim e^{-\delta m|z|} \text{ of hadron matrix element etc.}
  \]

  \[
  Z_{\text{hybrid}}(z_S, a) \text{ can be determined by continuity condition at } z = z_S
  \]

  \[
  \tilde{h}(z, a, P_z)e^{-\delta m|z|}Z_{\text{hybrid}}(z_S, a) = \frac{\tilde{h}(z, a, P_z)}{Z_X(z, a)}, \quad Z_{\text{hybrid}}(z_S, a) = e^{\delta m z_S}/Z_X(z_S, a)
  \]
Non-perturbative renormalization

- **Problem**: these schemes introduce undesired IR effects at large distances

- **Solution 2**: Self renormalization LPC (Huo, Su et al, 21’)

- Fitting the bare matrix elements at multi-lattice-spacings to

\[
\ln \mathcal{M}(z, a) = \frac{kz}{a \ln[a \Lambda_{\text{QCD}}]} + m_0 z + g'(z) + f(z) a + \frac{3 C_F}{b_0} \ln \left[ \frac{\ln[1/(a \Lambda_{\text{QCD}})]}{\ln[\mu/\Lambda_{\text{QCD}}]} \right] + \ln \left[ 1 + \frac{d}{\ln(a \Lambda_{\text{QCD}})} \right]
\]

- The pieces other than \( g'(z) \) are renormalization factors

- \( m_0 \) can be determined by matching the renormalized matrix element to the continuum \( \overline{\text{MS}} \) result at short distance

- Such renormalized matrix element can then be matched to the PDF using the \( \overline{\text{MS}} \) matching
Non-perturbative renormalization

- **Problem:** these schemes introduce undesired IR effects at large distances

- **Solution 2: Self renormalization** LPC (Huo, Su et al, 21’)

![Graphs showing Z_{MS}, \Lambda_{MS}, overlap_Pion, \delta_{sys}, \Lambda_{QCD}, d, m_0, clover_Pion, clover_Nucleon, clover_Nucleon_hyp1, clover_Nucleon_hyp0, delta_{sys}, d, m_0, k, \Lambda_{QCD}, delta_{sys}, d, m_0, k, \Lambda_{QCD}, delta_{sys}, d, m_0, k, \Lambda_{QCD}, delta_{sys}, d, m_0]
Fourier transform to momentum space

- Truncated FT introduces additional systematic uncertainty

- Supplemented with sophisticated reconstruction techniques

\[ \tilde{q}(x) \]

\[ \Delta \tilde{q}(x) \]

\[ z_{\text{max}}p_z = 3.5 \]
\[ z_{\text{max}}p_z = 4.7 \]
\[ z_{\text{max}}p_z = 5.9 \]

- e.g. Backus–Gilbert method & Bayes–Gauss–FT

Alexandrou et al, PRD 21'
**Alternative strategy**

- **Extrapolation to asymptotic distance**
- **Lattice data** (available up to limited $z_{\text{max}}$ or $\lambda_{\text{max}}$) may be supplemented with **physics-based extrapolation**
- At finite momentum, quasi-LF correlation has a finite correlation length and exhibits an exponential decay behavior $e^{-\lambda/\xi_{\lambda}}$ (usually associated with algebraic terms) in $\lambda$ space

$$\xi_{\lambda} \sim P^z/\Lambda_{\text{QCD}}, \quad 0 < P^z_1 < P^z_2 < P^z_3$$

- Depending on hadron momentum, one can choose to do an exponential or an algebraic extrapolation

Ji, JHZ et al, NPB 21’
Alternative strategy

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- At finite momentum, quasi-LF correlation has a finite correlation length and exhibits an exponential decay behavior \( e^{-\frac{\lambda}{\xi}} \) (usually associated with algebraic terms) in \( \lambda \) space

Example: vector meson distribution amplitude

\[
\hat{H}(z, P_z) = \left[ \frac{c_1}{(-i\lambda)^a} + e^{i\lambda} \frac{c_2}{(i\lambda)^b} \right] e^{-\frac{\lambda}{\xi}},
\]

\[
\hat{H}(z, P_z) = \frac{c_1}{(-i\lambda)^a} + e^{i\lambda} \frac{c_2}{(i\lambda)^b},
\]

LPC (Hua, JHZ et al, 20')
Alternative strategy

- Extrapolation to asymptotic distance
- Lattice data (available up to limited $z_{\text{max}}$ or $\lambda_{\text{max}}$) may be supplemented with physics-based extrapolation
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Example: vector meson distribution amplitude

$$\tilde{H}(z, P_z) = \left[ \frac{c_1}{(-i\lambda)^a} + e^{i\lambda} \frac{c_2}{(i\lambda)^b} \right] e^{-\frac{\lambda}{\lambda_0}},$$

$$\tilde{H}(z, P_z) = \frac{c_1}{(-i\lambda)^a} + e^{i\lambda} \frac{c_2}{(i\lambda)^b},$$

LPC (Hua, JHZ et al, 20')
Perturbative corrections

- Development on perturbative calculations
- Anomalous dimension and renormalization factors to three-loop accuracy

Braun, Chetyrkin, Kniehl, JHEP 20’

- NNLO matching (\(\overline{\text{MS}}/\text{RIMOM}/\text{VEV} \) scheme)

Chen, Wang, Zhu, PRL 21’

Li, Ma, Qiu, PRL 21’
Perturbative corrections

- Development on perturbative calculations
- NNLO matching in hybrid scheme

No significant correction from NLO to NNLO

Broader shape than global fit, excited state contamination in lattice data?
Conclusions

- Theory developments allow to make a comparison between theoretical predictions and experimental measurements of various partonic quantities
  - PDFs, DAs, GPDs, TMDs, LFWFs……

- For certain quantities, precision calculations become important in order to match experimental accuracy

- A systematic strategy of doing such calculations will help to study other partonic quantities that are not known well experimentally