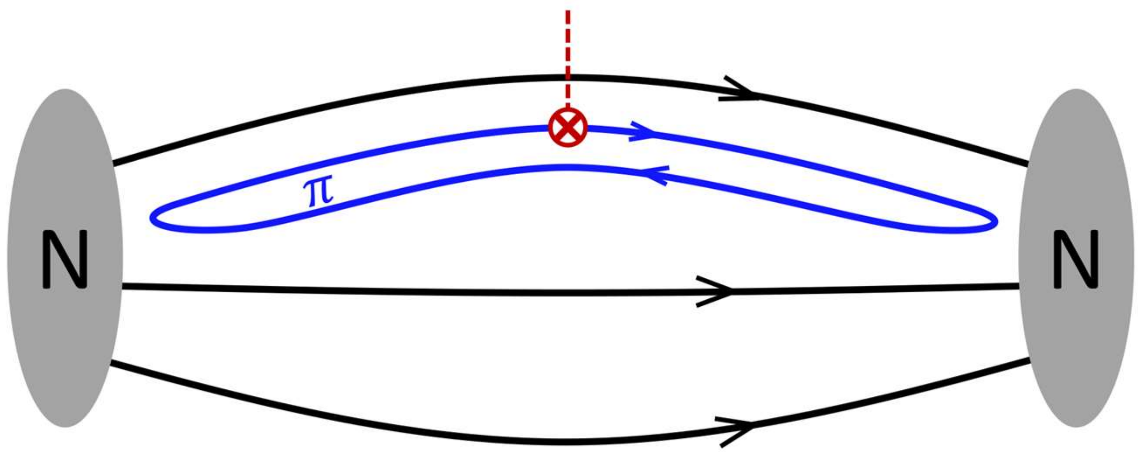
Flavor Diagonal Nucleon Charges

Sungwoo Park,
T. Bhattacharya, R. Gupta, S. Mondal, B. Yoon, H-W Lin, R. Zhang
LANL, MSU

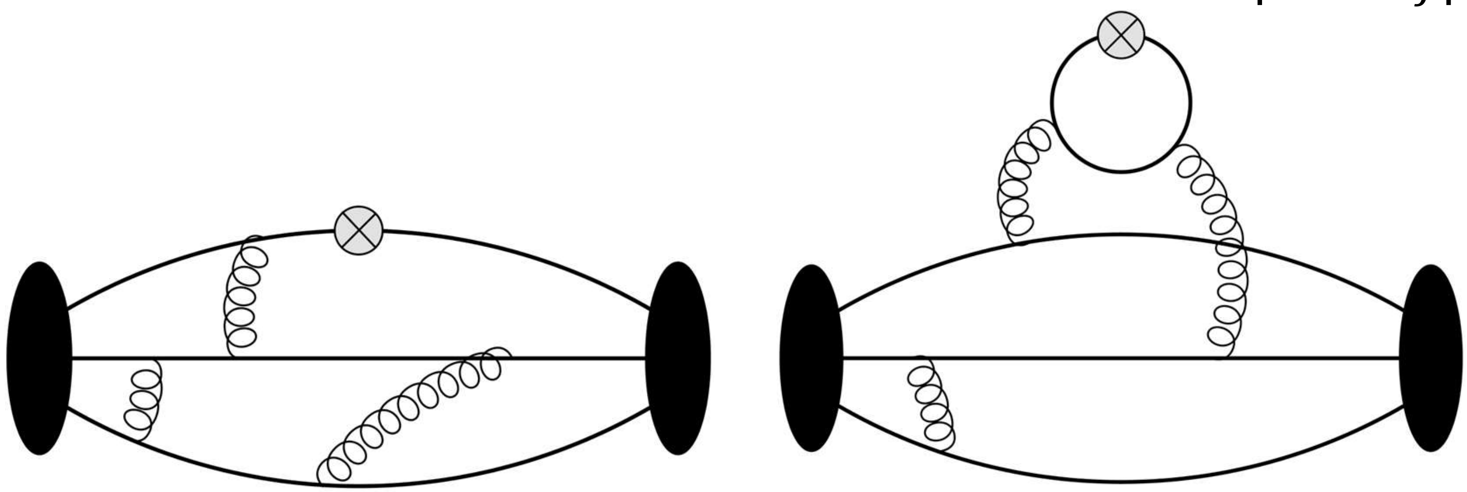


LA-UR 21-27171

Lattice 2021, July 2021

Flavor diagonal matrix elements require high precision measurements of quark bilinear operators within the nucleon state for both "connected" and "disconnected" 3-point correlation functions,

Nucleon charges g_A, g_T, and g_S obtained from ME $\langle N|\bar{q}_i\, \Gamma\, q_j|N\rangle$



Connected **Disconnected**

Outline of talk

- Methodology for calculation of 3-point functions very well established
  - Signal in connected versus disconnected
- Removing excited state contributions (ESC)
  - Fits using the spectral decomposition to connected plus disconnected contributions (for a scalar charge g_S^{u+d})
  - What states contribute? Do $N\pi$ / $N\pi\pi$ states contribute?
- Renormalization
  - Constructing the full mixing matrix
- Chiral-Continuum-Finite-Volume extrapolations
  - Chiral fits to $\sigma_{\pi N}$, the nucleon sigma term

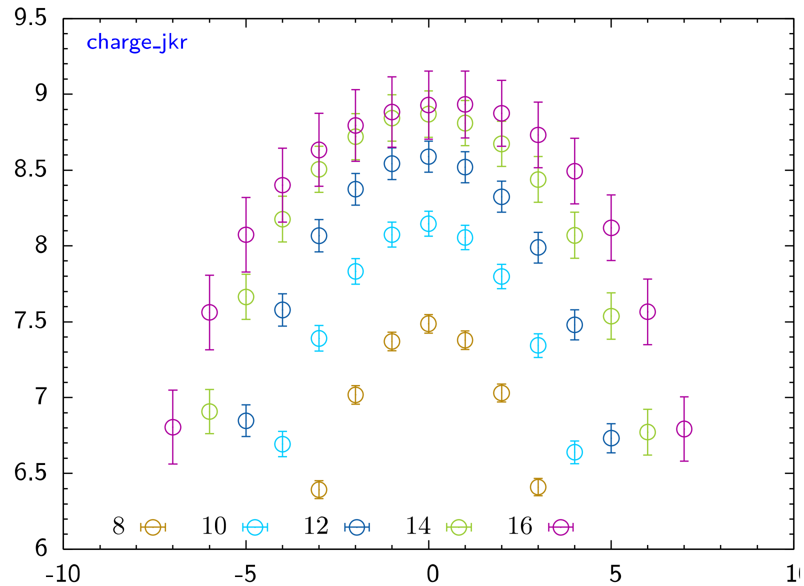
All discussion based on Clover-on-HISQ data

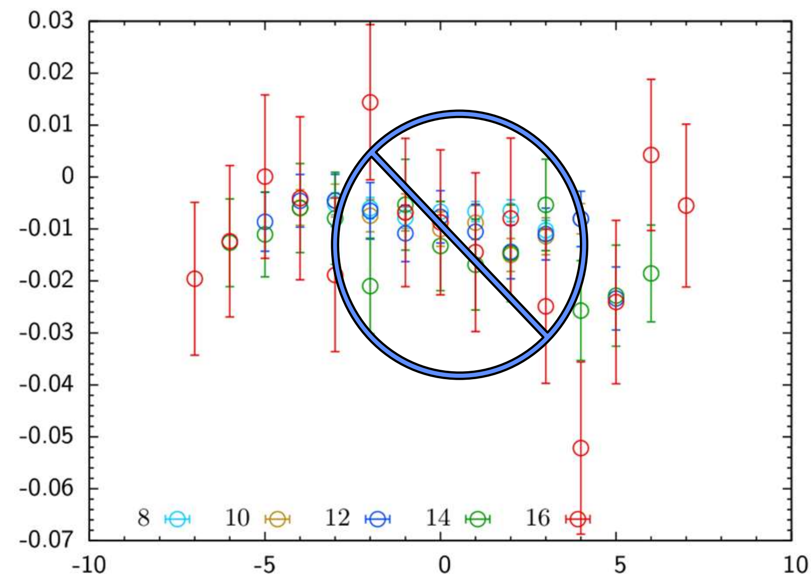
Physical M_π Ensemble: $a \approx 0.09 fm, \; M_\pi = 135\, MeV, \; M_\pi L = 3.9$

- Source-sink time separation τ =8,10,12,14,16 (0.7~1.4fm)
- 1290/1270 configs
- 128 measurements of connected per config
- 10,000 sources for disconnected loop per config

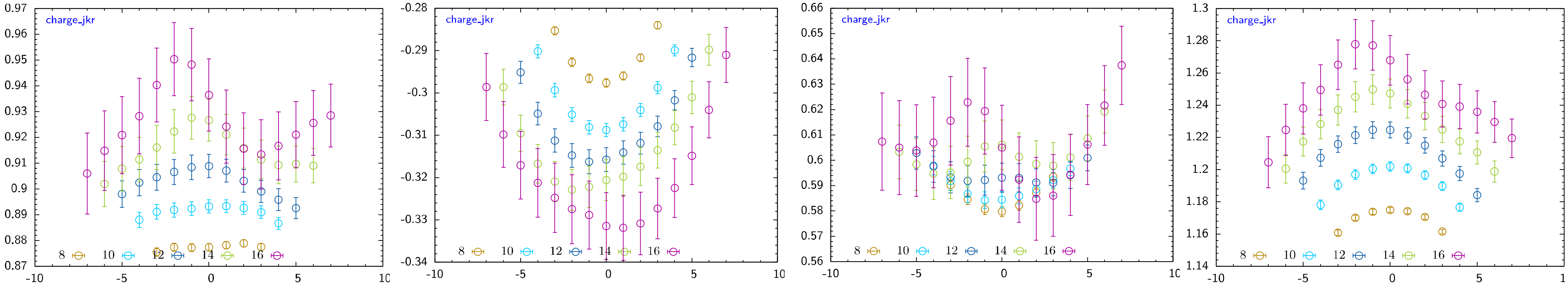
When are n-state ESC fits reliable?

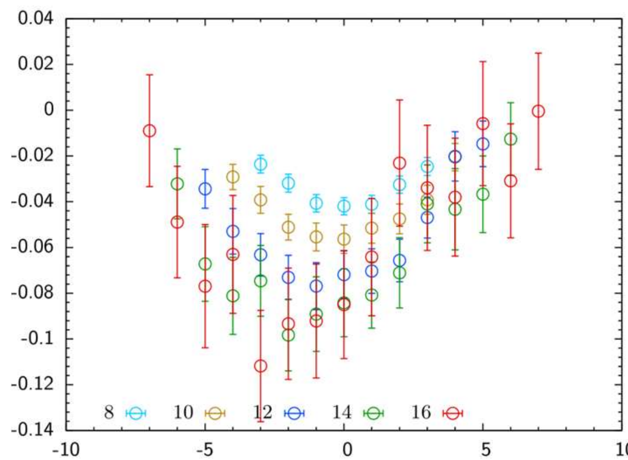
- Spectral decomposition of 3-point function tells us
  - Data for a given τ should be symmetric about $\tau/2$
  - Convergence should be monotonic for large enough τ especially when only "one" excited state contribution is left
  - Only positive parity intermediate states contribute in the fits



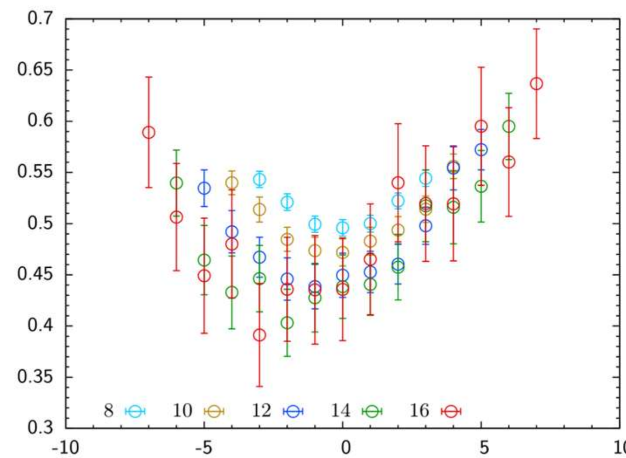


$g_A^{u,conn}$ $g_A^{d,conn}$ $g_A^{u+d,conn}$ g_A^{u-d}





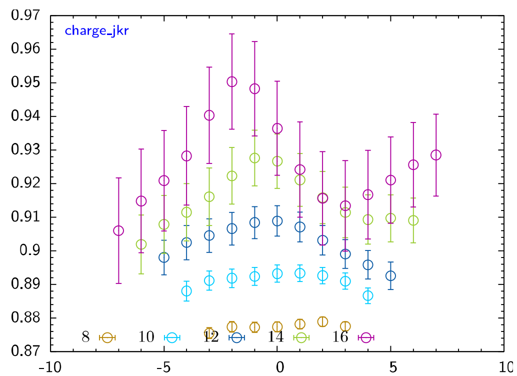
$g_A^{l,disc}$



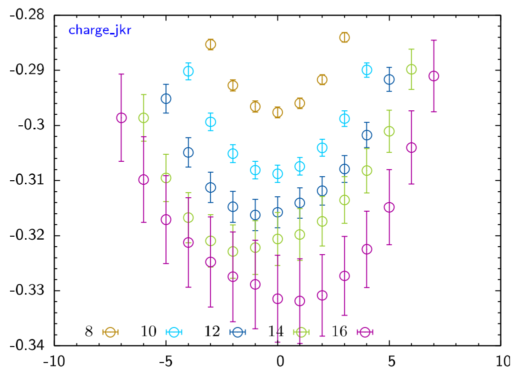
$g_A^{u+d} = g_A^{u+d,\mathrm{conn}} + 2g_A^{l,\mathrm{disc}}$

Also see “anatomy of ESC” in appendix D in arXiv:2103.05599

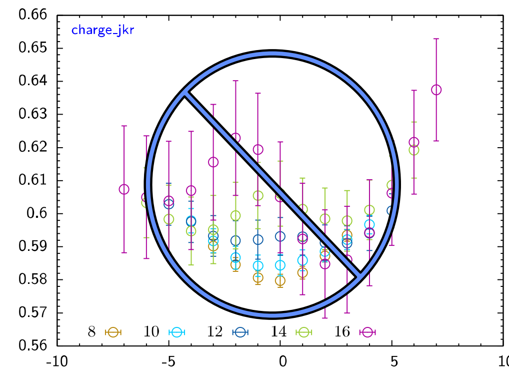
$g_A^{u,conn}$



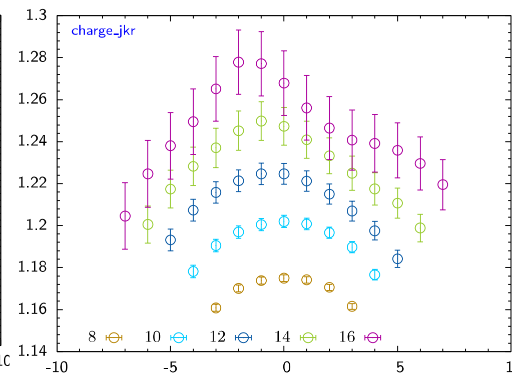
$g_A^{d,conn}$

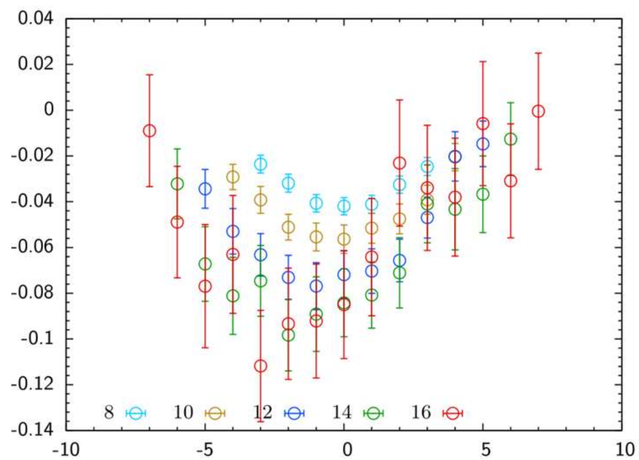


$g_A^{u+d,conn}$

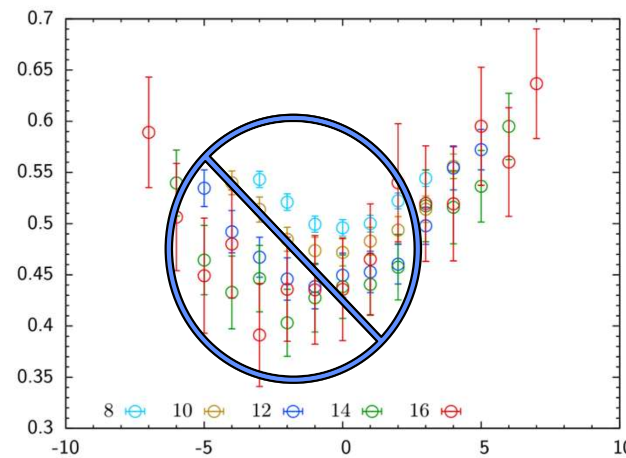


g_A^{u-d}





$g_A^{l,disc}$



$$g_A^{u+d} = g_A^{u+d,\mathrm{conn}} + 2g_A^{l,\mathrm{disc}}$$

g_A

- Signal in $g_A^{u,conn}$, $g_A^{d,conn}$, $g_A^{l,disc}$ versus τ improves together
- ESC in g_A^{u-d} adds

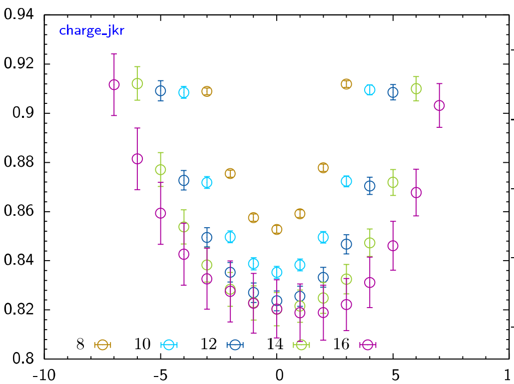
  ⇒ Excited-state fits are reasonable

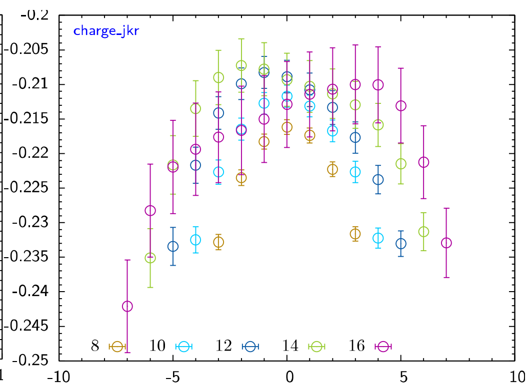
  ⇒ Open issue of contribution of $N\pi$ state. (arXiv:2103.05599)
- ESC in $g_A^{u+d,conn}$ subtracts

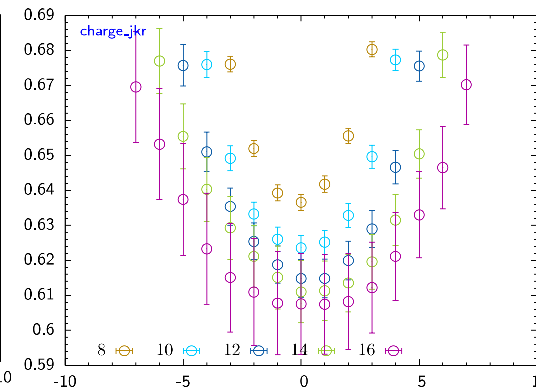
  ⇒ τ dependence hard to resolve
- $g_A^{l,disc}$ converges to a more negative value ≈ -0.1

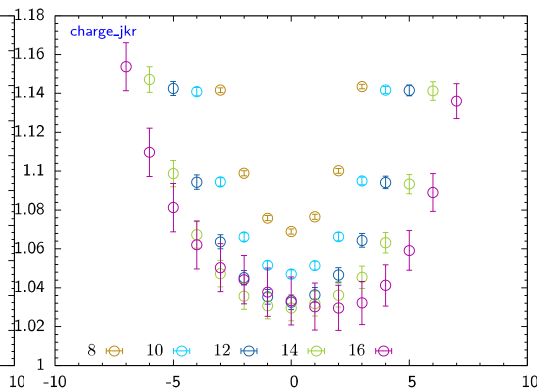
  ⇒ Makes the quark contribution to the proton spin smaller

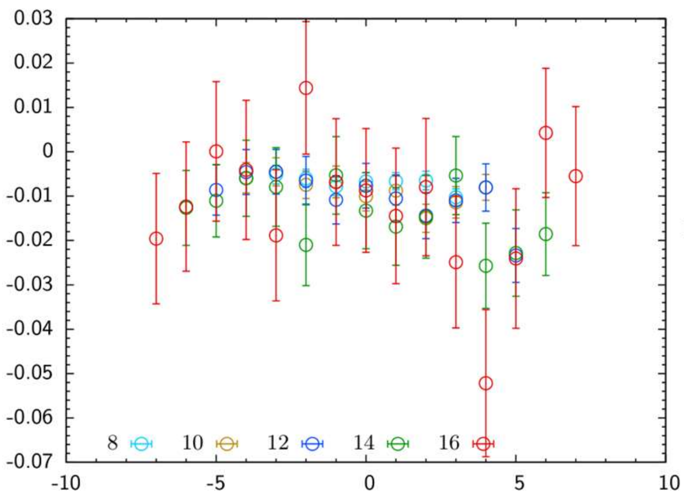
$g_T^{u,conn}$ $g_T^{d,conn}$ $g_T^{u+d,conn}$ g_T^{u-d}



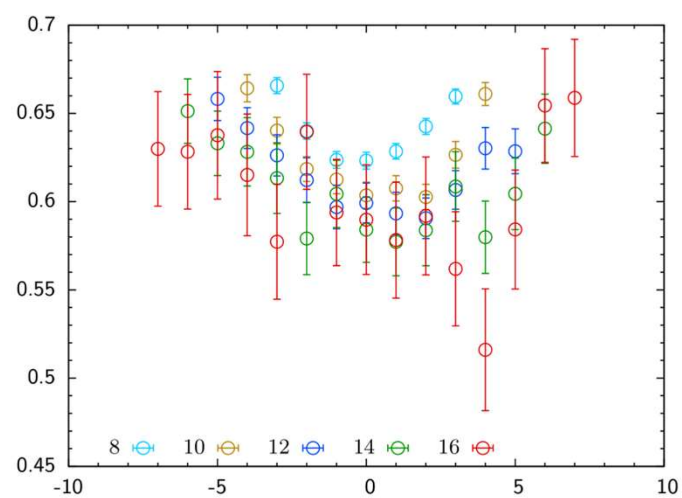






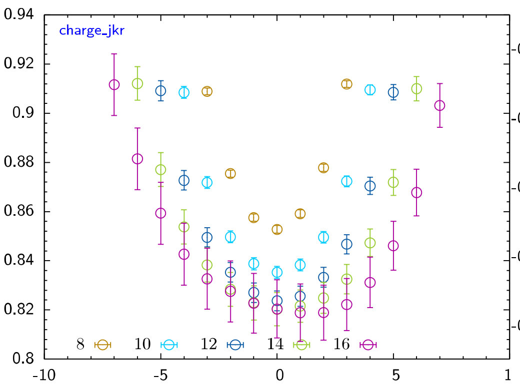


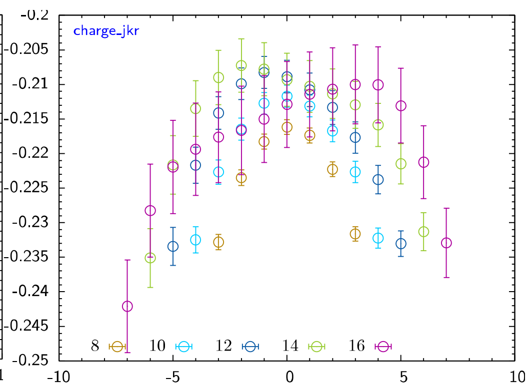
$g_T^{l,disc}$

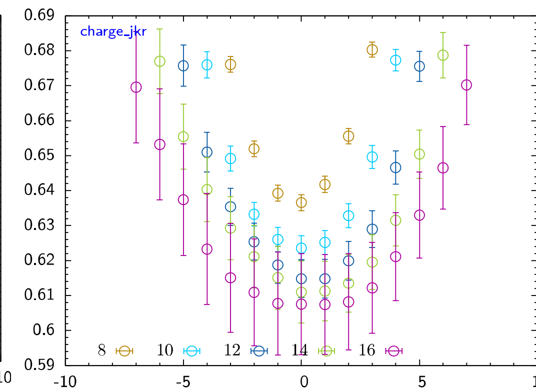


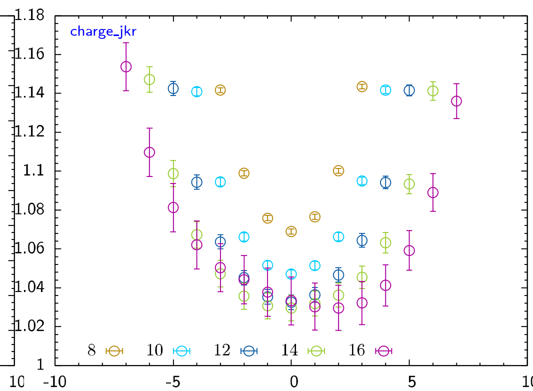
$$g_T^{u+d} = g_T^{u+d,\mathrm{conn}} + 2g_T^{l,\mathrm{disc}}$$

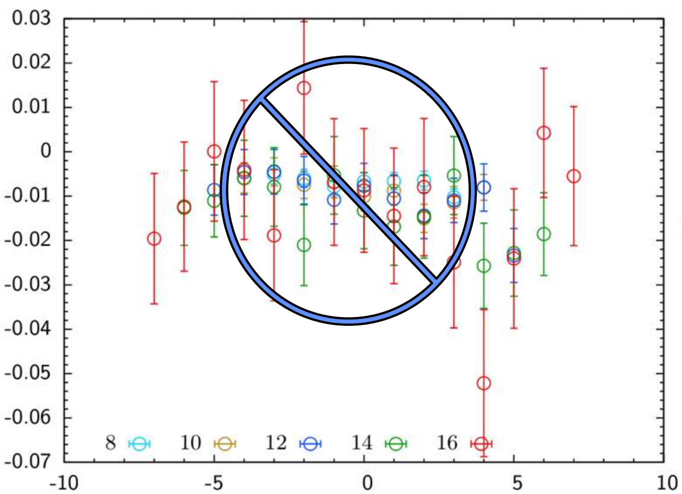
$g_T^{u,conn}$ $g_T^{d,conn}$ $g_T^{u+d,conn}$ g_T^{u-d}



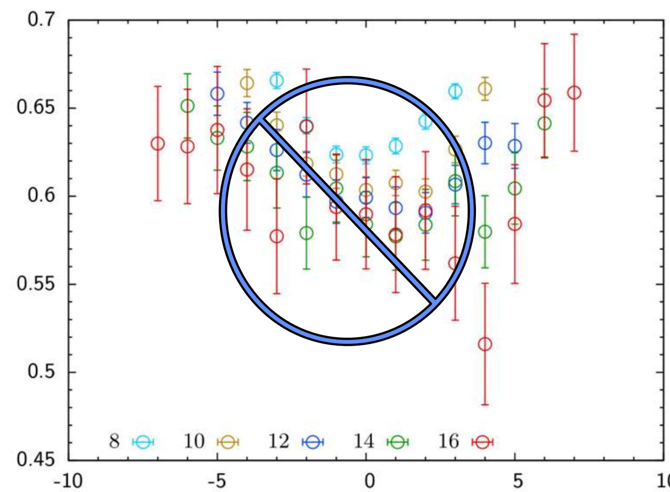








$g_T^{l,disc}$



$$g_T^{u+d} = g_T^{u+d,\text{conn}} + 2g_T^{l,\text{disc}}$$

g_T

- ESC in $g_T^{u,conn}$ larger than in $g_T^{d,conn}$

  ⇒ ESC $g_T^{u,conn}$ dominates all combinations

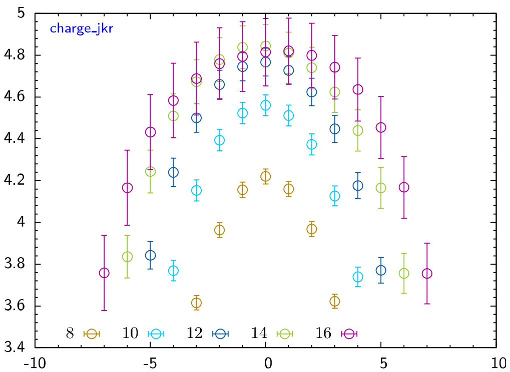
- ESC in g_T^{u-d} and $g_T^{u+d,conn}$ mainly driven by $g_T^{u,conn}$

  ⇒ Excited-state fits are reasonable

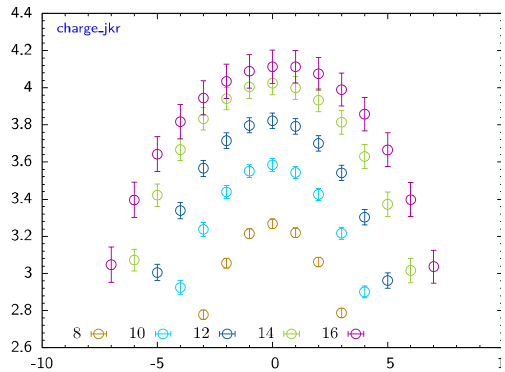
- $g_T^{l,disc}$ ~ -0.01 and noisy

  ⇒ A small contribution but makes g_T^{u+d} noisy

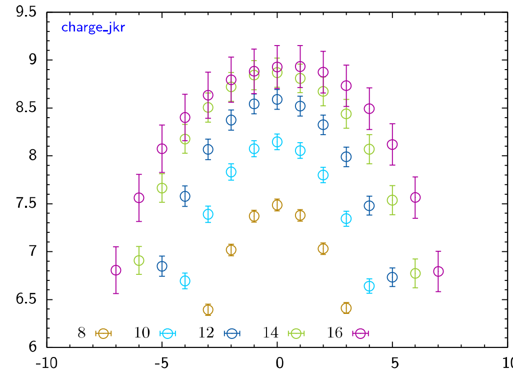
$g_S^{u,conn}$



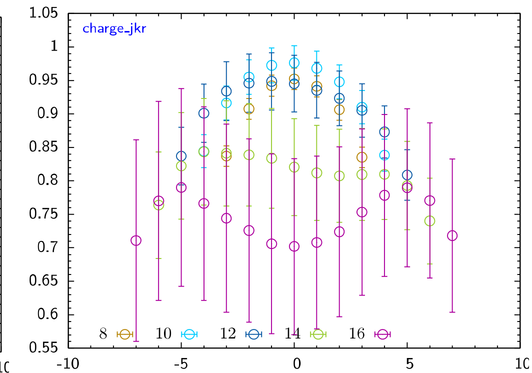
$g_S^{d,conn}$

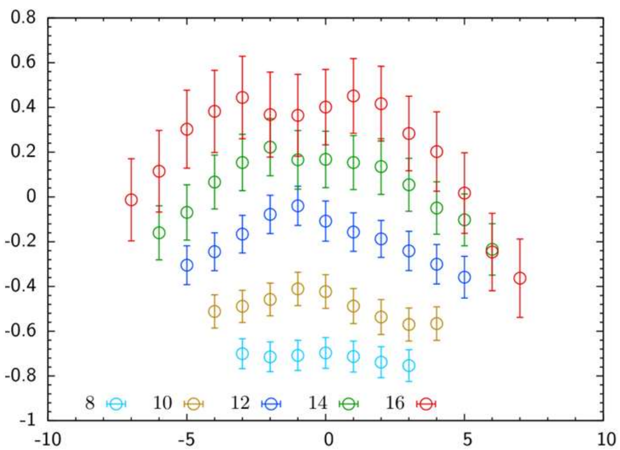


$g_S^{u+d,conn}$

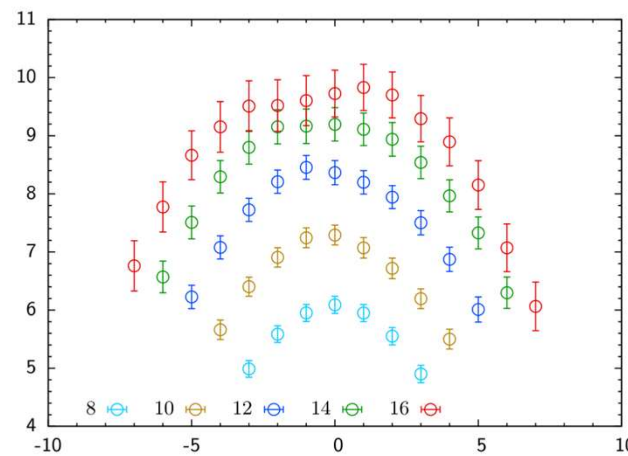


g_S^{u-d}



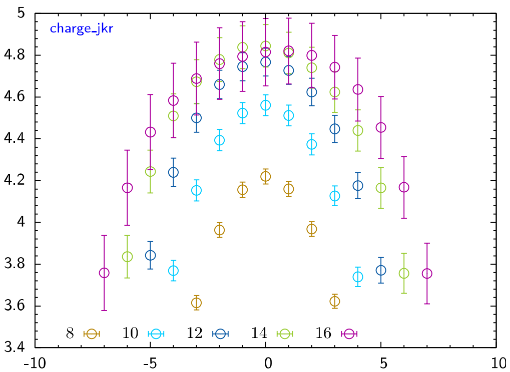


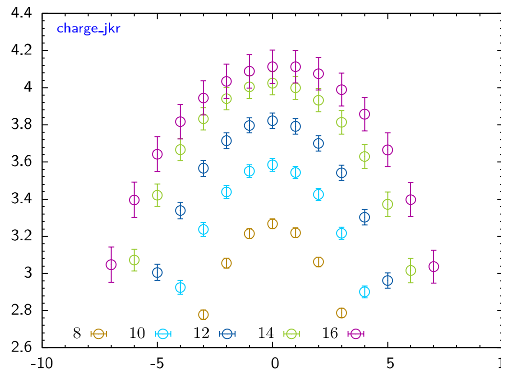
$g_S^{l,disc}$

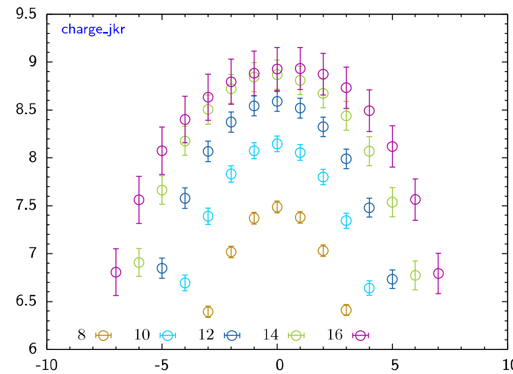


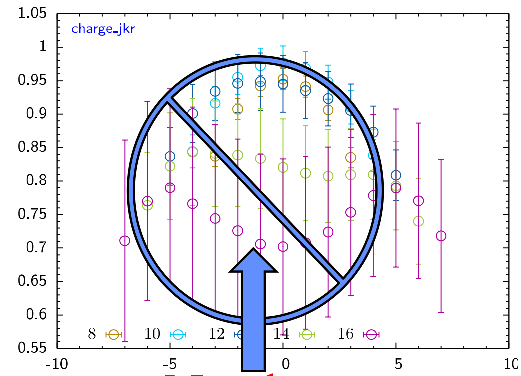
$$g_S^{u+d} = g_S^{u+d,\mathrm{conn}} + 2 g_S^{l,\mathrm{disc}}$$

$g_S^{u,conn}$ $g_S^{d,conn}$ $g_S^{u+d,conn}$ g_S^{u-d}

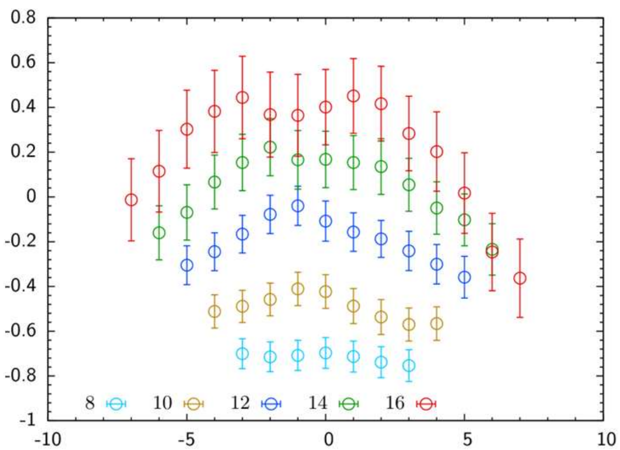




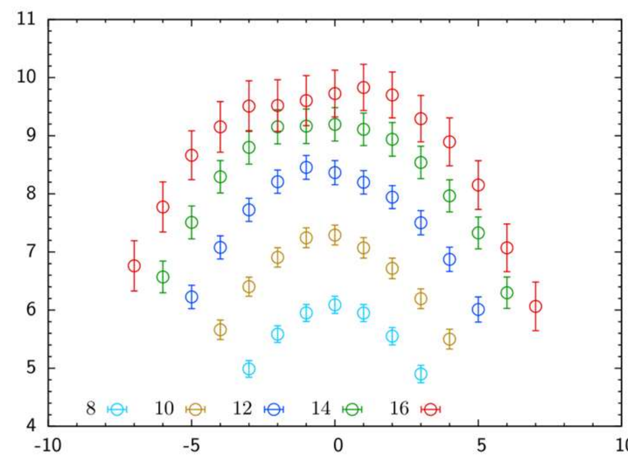




Needs very high statistics



$g_S^{l,disc}$



$$g_S^{u+d} = g_S^{u+d,\text{conn}} + 2g_S^{l,\text{disc}}$$

g_S

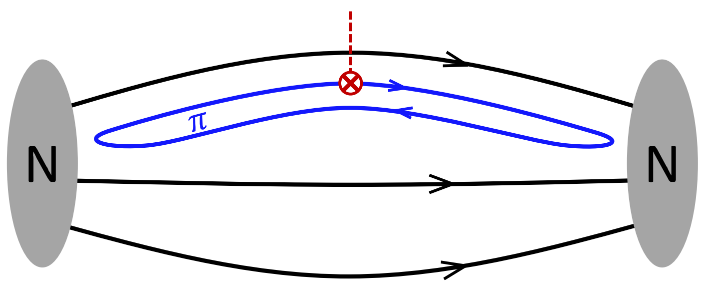
- ESC in $g_S^{u,conn}$ and $g_S^{d,conn}$ are similar
- ESC in g_S^{u-d} cancels

  ⇒ Excited-state fits need high statistics
- ESC in $g_A^{u+d,conn}$ add

  ⇒ τ dependence well-resolved
- $g_S^{l,disc}$ has a good signal

  ⇒ Makes a big contribution

χPT analysis indicates $N\pi/N\pi\pi$ states have significant coupling



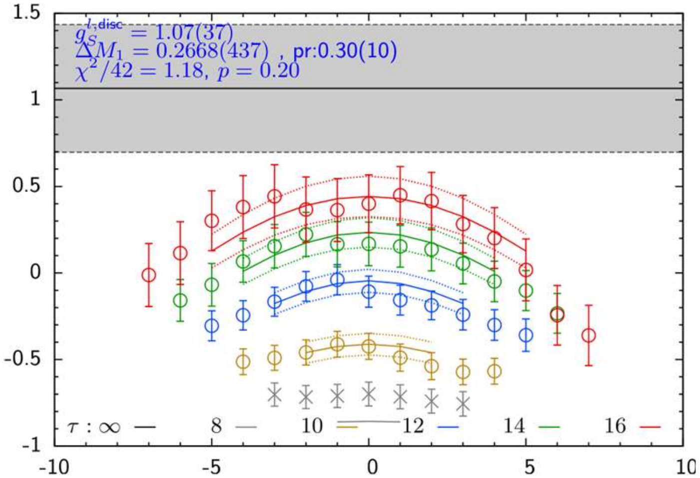
$a \approx 0.09 fm$
$M_\pi \approx 135 MeV$

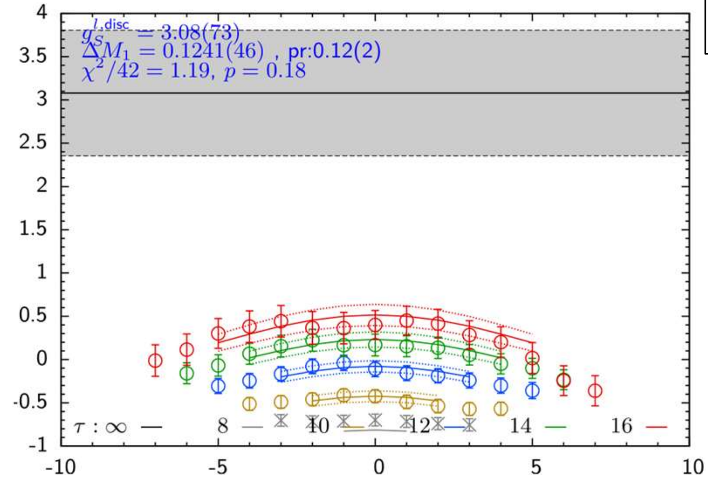
without $N\pi$

with $N\pi$

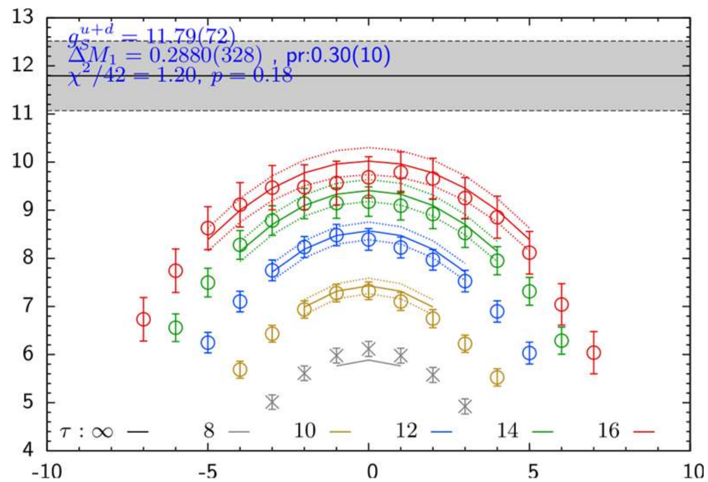
$N(1)\pi(-1)$
or, $N(0)\pi(0)\pi(0)$

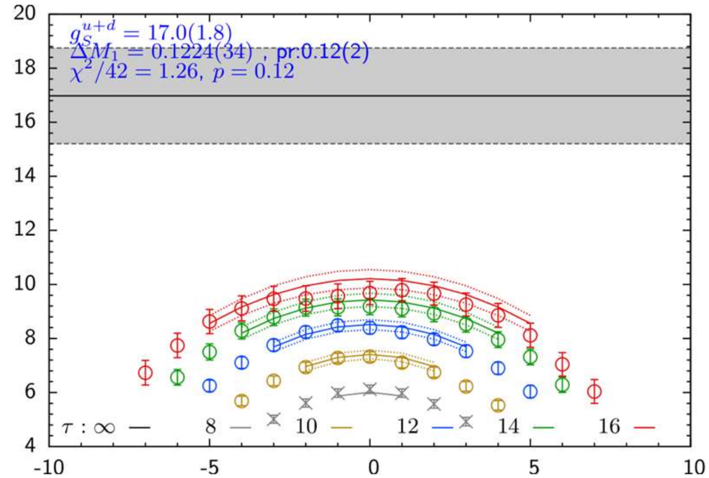
$g_S^{l,\text{disc}}$





$g_S^{u+d} =$
$g_S^{u+d,\text{conn}}$
$+2 g_S^{l,\text{disc}}$





$\sigma_{\pi N} = m_l g_S^{u+d} \sim 40$ MeV

$\sigma_{\pi N} = m_l g_S^{u+d} \sim 60$ MeV

g_S and the nucleon sigma-term

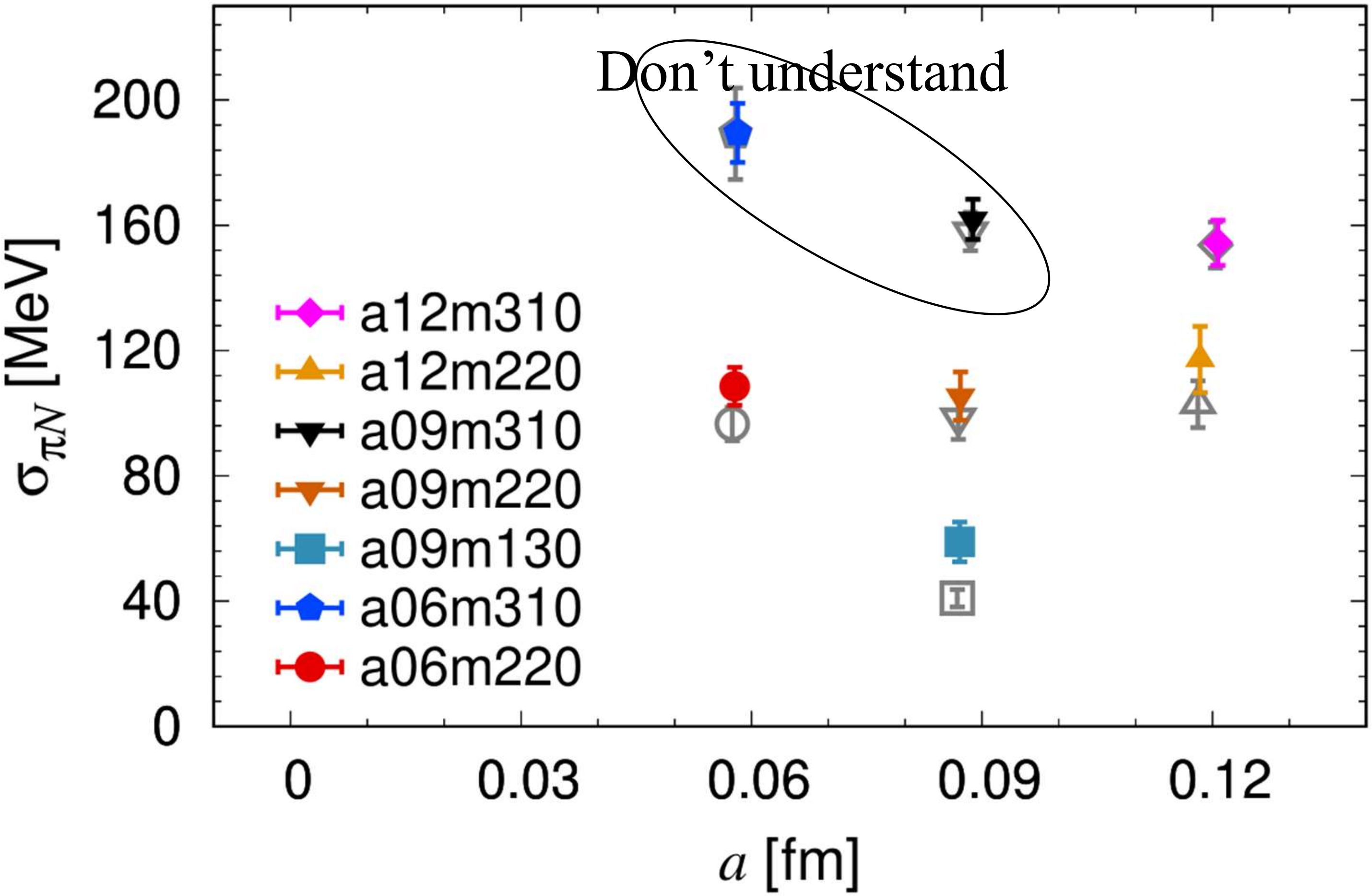
- Signal in $g_S^{u+d,conn}$ and $g_S^{l,disc}$ is good

  $\Rightarrow \tau$ dependence well-resolved

- ESC in $g_S^{u+d,total}$ from u, d, $disc$ terms adds
- n-state fits to get the ground state ME are good
- ME (Nucleon σ-term) depends strongly on whether $N\pi$ is included in excited-state fits
  - χ^2 are equally good
- χPT analysis implies significant contribution of $N\pi$ and $N\pi\pi$ state

arXiv:2105.12095

a-dependence is not clear

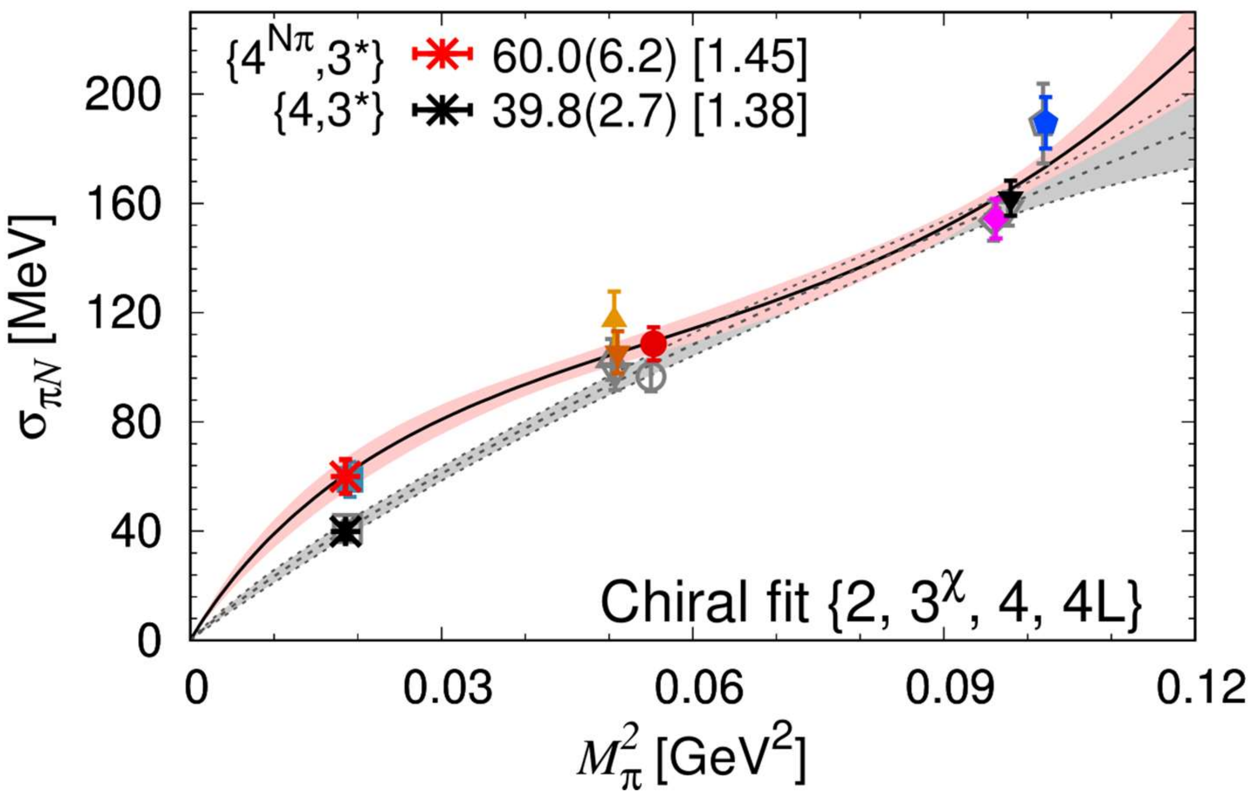


Light gray
{4,3*}: without $N\pi$

Other colors
$\{4^{N\pi},3^*\}$: with $N\pi$

Chiral fits

arXiv:2105.12095



Light gray
{4,3*}: without $N\pi$
Other colors
$\{4^{N\pi},3^*\}$: with $N\pi$

- a12m310
- a12m220
- a09m310
- a09m220
- a09m130
- a06m310
- a06m220

$$\sigma_{\pi N} = d_2 M_\pi^2 + d_3^\chi M_\pi^3 + d_4 M_\pi^4 + d_{4L} M_\pi^4 \log M_\pi^2$$

d_3^χ: fixed to χPT prediction

arXiv:2105.12095

Chiral fits

without $N\pi$ | with $N\pi$

chiral fit	$\{4,3^*\}_{\rm sim}$ d_2 (GeV^{-1})	d_3 (GeV^{-2})	d_4 (GeV^{-3})	d_{4L} (GeV^{-3})	$\frac{\chi^2}{dof}$	$\sigma_{\pi N}$ (MeV)	$\{4^{N\pi},3^*\}_{\rm sim}$ d_2 (GeV^{-1})	d_3 (GeV^{-2})	d_4 (GeV^{-3})	d_{4L} (GeV^{-3})	$\frac{\chi^2}{dof}$	$\sigma_{\pi N}$ (MeV)
χPT	4.44	−8.55	–	11.35	–	–	4.44	−8.55	–	11.35	–	–
$\{2,3\}$	2.53(20)	−2.88(71)	–	–	1.12	39.0(2.0)	3.29(29)	−5.15(99)	–	–	2.23	47.3(2.9)
$\{2,3,4\}$	2.80(76)	−5.3(6.5)	5(13)	–	1.37	39.7(2.8)	6.6(1.5)	−32(12)	50(23)	–	1.56	59.6(6.3)
$\{2^\chi,3^\chi,4\}$	4.44	−8.55	−2.69(44)	–	18.8	58.96(15)	4.44	−8.55	−1.30(43)	–	4.91	59.42(14)
$\{2,3^\chi,4,4L\}$	2.99(43)	−8.55	6(12)	−3.4(7.4)	1.38	39.8(2.7)	5.50(91)	−8.55	49(21)	27(13)	1.45	60.0(6.2)
$\{2,3^\chi,4,4L^\chi\}$	3.83(12)	−8.55	30.1(1.4)	11.35	1.91	44.0(1.8)	4.44(17)	−8.55	24.1(1.9)	11.35	1.45	53.3(2.5)
$\{2^\chi,3^\chi,4,4L\}$	4.44	−8.55	43.3(4.7)	20.2(2.1)	3.33	48.2(1.1)	4.44	−8.55	25.4(5.7)	11.9(2.5)	1.44	52.9(1.4)
$\{2^\chi,3^\chi,4,4L^\chi\}$	4.44	−8.55	23.10(44)	11.35	5.87	52.91(15)	4.44	−8.55	24.13(43)	11.35	1.20	53.25(14)

$$\sigma_{\pi N} = d_2 M_\pi^2 + d_3 M_\pi^3 + d_4 M_\pi^4 + d_{4L} M_\pi^4 \log M_\pi^2$$

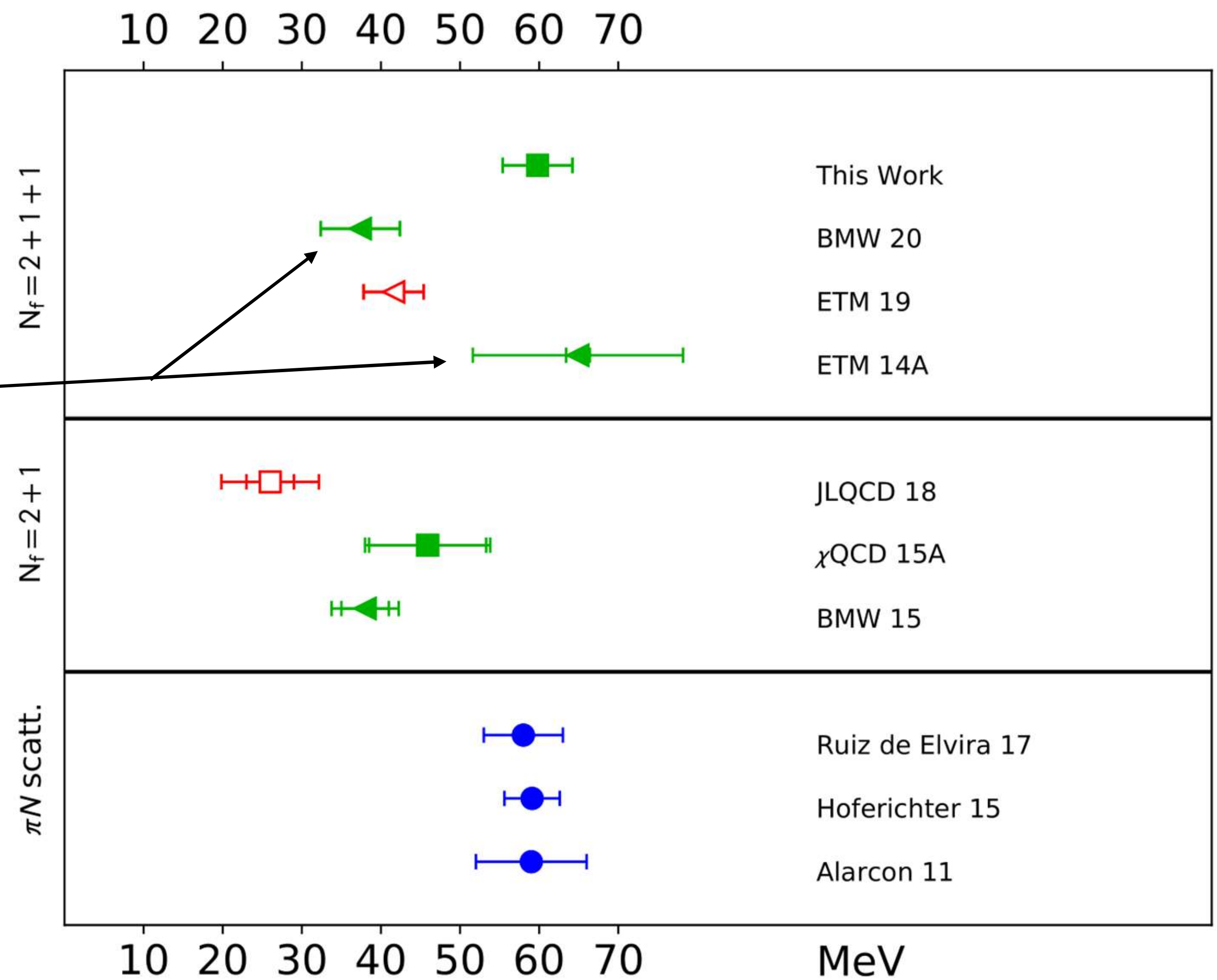
Coefficients with $N\pi$ "consistent" with χPT predicted d_i

arXiv:2105.12095

$\sigma_{\pi N}$

Tension in F-H method.

Both with $M_\pi \gtrsim 200$ MeV data



Summary

- Higher statistics needed to resolve τ dependence in
  - $g_A^{l,disc}$, $g_A^{u+d,conn}$
  - $g_T^{l,disc}$, $g_T^{u+d,conn}$
  - $g_S^{u-d,conn}$
- Evidence of large contribution of $N\pi$ & $N\pi\pi$ in ES fits to g_S^{u+d} Contribution becomes more significant as $M_\pi \rightarrow 135$ MeV.
- $\sigma_{\pi N}$ changes from ~40 MeV to ~60 MeV on including the $N\pi$ and $N\pi\pi$ excited states [arXiv:2105.12095]
- Need more data to improve the continuum & finite volume extrapolation of $\sigma_{\pi N}$