Self-Renormalization of Quasi-Light-Front Correlators on the Lattice

(Lattice Parton Collaboration (LPC))

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Nuclear Physics B (2021): 115443

arXiv: 2103.02965

Speaker: Yushan Su
Supervisor: Xiangdong Ji

Background

X. Ji, Phys. Rev. Lett. 110, 262002 (2013)

Large Momentum Effective Theory (LaMET) proposes an effective way to get parton physics from lattice data:

Quasi-Light-Front correlator on lattice

e.g. $\mathcal{M} = \langle H|\bar{\psi}(0)\Gamma U(0,z)\psi(z)|H\rangle_{large\ P^z}$

Renormalization
Fourier Transform
Matching

Parton Physics

Many nice works:

PDF (Parton Distribution Function)

C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen,A. Scapellato, and F. Steffens, Phys. Rev. D98, 091503(2018)

Y.-S. Liu et al.(Lattice Parton), Phys. Rev.D101,034020 (2020)

Z. Fan, X. Gao, R. Li, H.-W. Lin, N. Karthik, S. Mukher-jee, P. Petreczky, S. Syritsyn, Y.-B. Yang, and R. Zhang,Phys. Rev. D102, 074504 (2020)

GPD (Generalized Parton Distribution)

J.-W. Chen, H.-W. Lin, and J.-H. Zhang, Nucl. Phys. B952, 114940 (2020)

C. Alexandrou, K. Cichy, M. Constantinou, K. Had-jiyiannakou, K. Jansen, A. Scapellato,and F. Stef-fens, PoSLATTICE2019, 036 (2019)

DA (Distribution Amplitude)

J.-H. Zhang, J.-W. Chen, X. Ji, L. Jin, and H.-W. Lin,Phys. Rev.D95, 094514 (2017)

Jun Hua et al. arXiv: 2011.09788

TMD (Transverse Momentum Dependent) Distributions

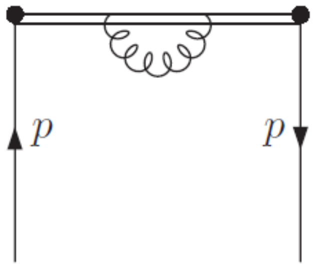
P. Shanahan, M. Wagman, and Y. Zhao, Phys. Rev. D102, 014511 (2020)

Q.-A. Zhang et al.(Lattice Parton), Phys. Rev. Lett.125, 192001 (2020)

To do precision calculation, renormalization is extremely important

Wilson Link

Self-Energy



$$\sim -\frac{\alpha_s C_F}{2}\frac{\pi}{a}$$

linear divergence

a is lattice spacing

J.-W. Chen, X. Ji, and J.-H. Zhang,
Nucl. Phys. B915, 1 (2017)

Ji, Zhang, Zhao,
Phys.Rev.Lett. 120 (2018) 11, 112001

$$M(\mathrm{z},\mathrm{a}) = \mathrm{Exp}\left[-\frac{m_{-1}}{a}\mathrm{z}\right]\mathrm{f}(\mathrm{z},\mathrm{a})$$

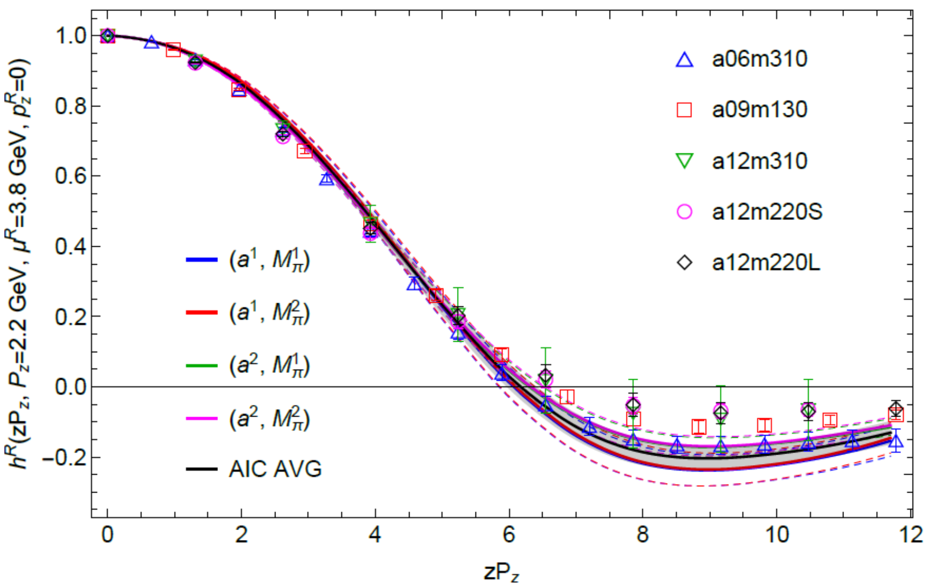
We must get rid of this linear divergence factor!

Purposes of renormalization:

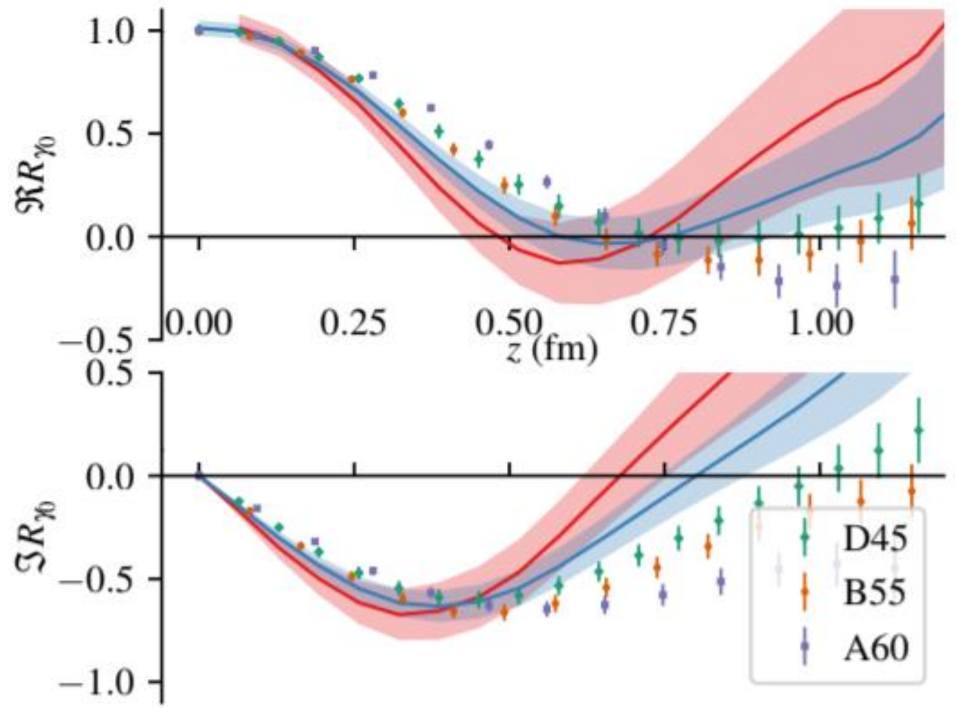
1. Achieve nice continuum limit

2. Avoid introducing extra non-perturbative effect

Continuum Limit Study in Literature



Lin, Huey-Wen, Jiunn-Wei Chen, and Rui Zhang. *arXiv:2011.14971* (2020).



Alexandrou, Constantia, et al. "Lattice continuum-limit study of nucleon parton quasidistribution functions." *Physical Review D* 103.9 (2021): 094512.

Our Methods

Purposes of renormalization:

1. Achieve nice continuum limit → Self-renormalization Method
2. Avoid introducing extra non-perturbative effect → Hybrid renormalization Method

X. Ji, Y. Liu, A. Schafer, W. Wang, Y.-B. Yang, J.-H. Zhang, and Y. Zhao, Nuclear Physics B 964 (2021): 115311, *arXiv:2008.03886*

Self-renormalization Fitting Function

Extract the linear divergence factor from a matrix element itself without using another matrix element

$\mathcal{M}$ is a lattice matrix element

$$
\begin{aligned}
&ln(\mathcal{M}) \\
&= \frac{kz}{a ln\left(a\Lambda_{QCD}\right)} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases} + \frac{3C_F}{b0} ln\left[\frac{\ln[1/\left(a\Lambda_{QCD}\right)]}{ln\left[\mu/\Lambda_{QCD}\right]}\right] + \ln[1 + \frac{d}{\ln\left(a\Lambda_{QCD}\right)}]
\end{aligned}
$$

- Linear divergence
- Residual
- Discretization error
- Resummation of Log divergence

Self-renormalization Steps

Step1: For each z (z can be chosen in a large range), extract $g(z)$ by fitting the a dependence:

$$ln(M) = \frac{kz}{aln(a\Lambda_{QCD})} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases} + \frac{3C_F}{b0} ln\left[\frac{\ln[1/(a\Lambda_{QCD})]}{ln[\mu/\Lambda_{QCD}]}\right] + \ln[1 + \frac{d}{\ln(a\Lambda_{QCD})}]$$

Step2: Fit the $m0$:

$$g(z) - \ln(Z_{\overline{MS}}) = m0\, z$$

$Z_{\overline{MS}}(z, \mu, \Lambda_{\overline{MS}})= 1 + \frac{\alpha_s(\mu,\Lambda_{\overline{MS}})C_F}{2\pi}\left[\frac{3}{2}\ln\frac{\mu^2 z^2 e^{2\gamma_E}}{4} + \frac{5}{2}\right]$,
for small z (a < z < 0.24 fm)

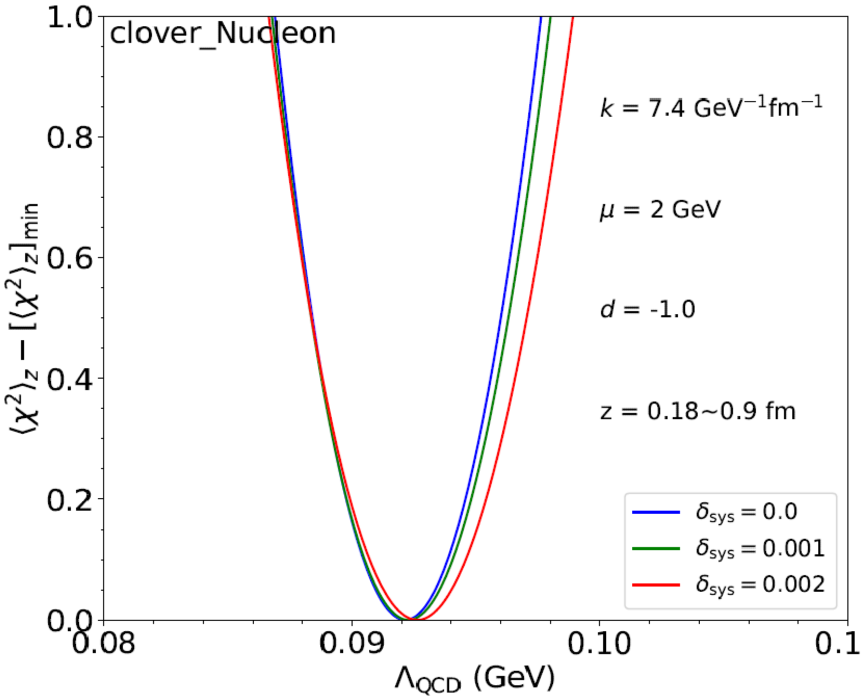
Fixed parameters:
k= 7.4 $GeV^{-1}fm^{-1}$(for hyp0),
$\Lambda_{\overline{MS}}$=0.3 GeV, μ = 2 GeV
Fitted parameters: k (for hyp1), Λ_{QCD}, g(z), f1(z), f2(z), m0;
Fine-tuning parameter: d

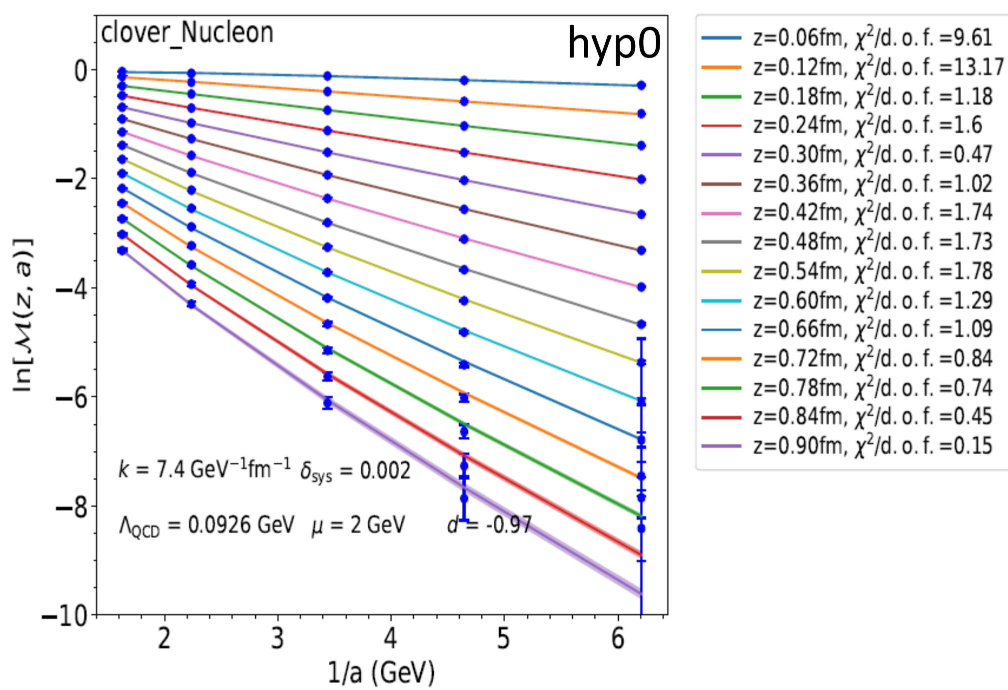
Step3: $m0\, z$ subtraction (The result works in a large range of z):

Exp$[g(z) - m0\, z]$ ← Residual intrinsic physics e.g. To self-renormalize a zero-momentum nucleon state quasi-light-front correlator: $M(\mathrm{z,a}) = \langle N|O_{\gamma_t}(z)|N\rangle_{\vec{P}=0}$

Step1: For each z (z can be chosen in a large range), extract $g(z)$ by fitting the a dependence:

$$ln(M) = \frac{kz}{a ln(a\Lambda_{QCD})} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases} + \frac{3C_F}{b0} ln\left[\frac{\ln[1/(a\Lambda_{QCD})]}{ln[\mu/\Lambda_{QCD}]}\right] + \ln[1 + \frac{d}{\ln(a\Lambda_{QCD})}]$$



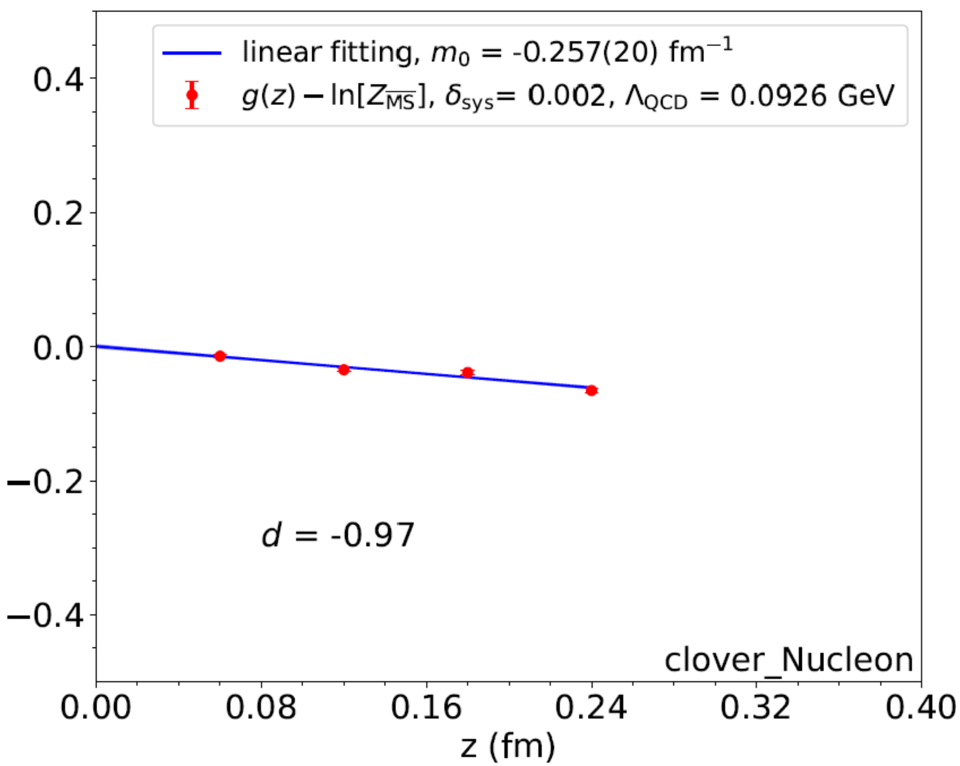


$$\sigma_{lnM\_new} = \sqrt{(\sigma_{lnM\_old})^2 + (\delta_{sys}\, a\, \mu)^2}$$

Step2: Fit the $m0$:

$$g(z) - \ln(Z_{\overline{MS}}) = m0\ z$$

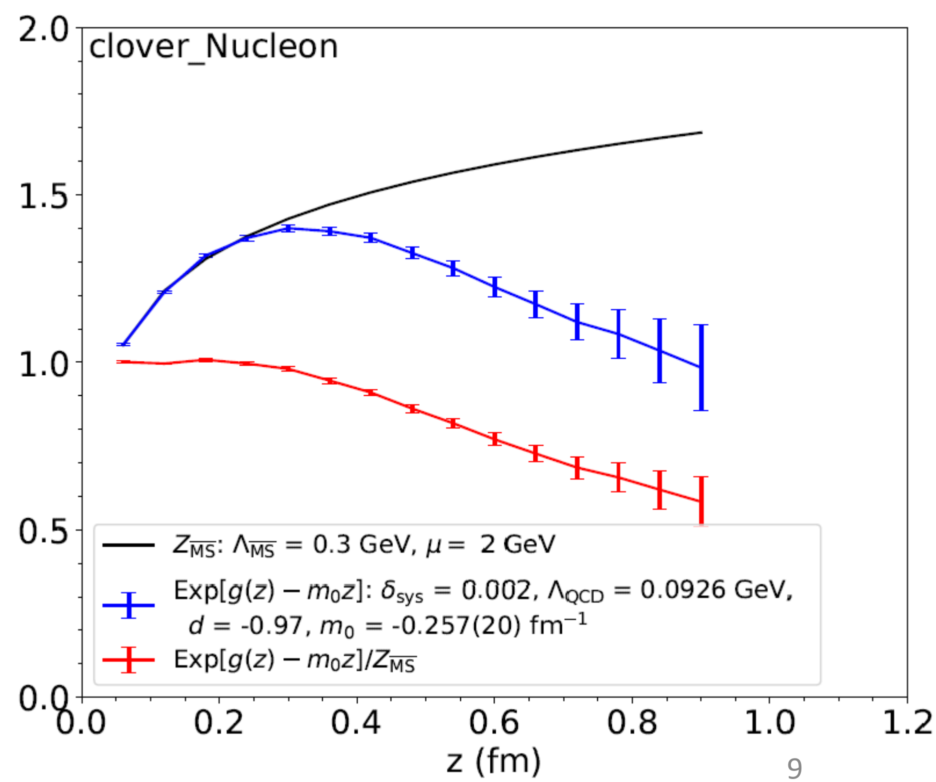
$Z_{\overline{MS}}(z, \mu, \Lambda_{\overline{MS}}) = 1 + \frac{\alpha_s(\mu, \Lambda_{\overline{MS}}) C_F}{2\pi}\left[\frac{3}{2}\ln\frac{\mu^2 z^2 e^{2\gamma_E}}{4} + \frac{5}{2}\right]$, for small z (0.06fm < z < 0.24 fm)



Step3: $m0\ z$ subtraction (The result works in a large range of z):

Exp$[g(z) - m0\ z]$

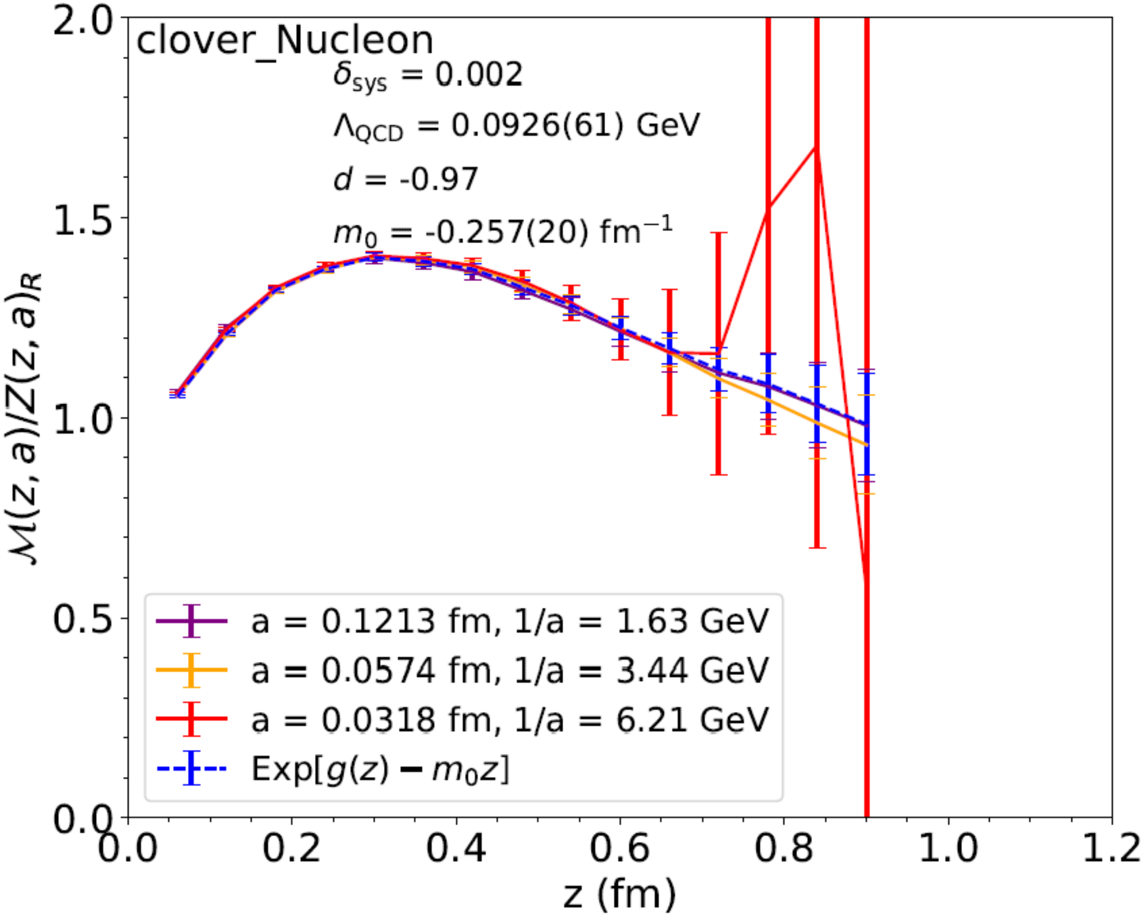
Residual intrinsic physics



Test of Self-renormalization method

Renormalization factor:

$$Z(z,a)_R = Exp\left\{\frac{kz}{a\ln(a\Lambda_{QCD})} + m_0 z + \left\{\begin{matrix} f1(z)a, MILC \\ f2(z)a, RBC \end{matrix}\right. + \frac{3C_F}{b0} ln\left[\frac{\ln[1/(a\Lambda_{QCD})]}{ln[\mu/\Lambda_{QCD}]}\right] + \ln[1 + \frac{d}{\ln(a\Lambda_{QCD})}]\right\}$$



Nice continuum limit

Different matrix elements

Linear divergence parameters k and Λ_{QCD}

$$M = \langle state | O_{\gamma_t}(z) | state \rangle_{P^z=0}$$

$$\ln M = \frac{kz}{a \ln(a\Lambda_{QCD})} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases} + \;\ldots$$

hyp0 (Unsmearing)

Cases	k	Λ_{QCD}(GeV)	d	m_0(GeV)
overlap quark	1.46	0.109(02)	-1.29	0.0462(16)
clover quark	1.46	0.135(02)	-1.35	0.1513(16)
overlap pion	1.46	0.093(10)	-1.17	-0.0357(46)
clover pion	1.46	0.086(14)	-0.92	-0.0715(50)
clover nucleon	1.46	0.093(06)	-0.97	-0.0508(40)

hyp1 (one-step of HYP smearing)

Cases	k	Λ_{QCD}(GeV)
overlap quark	0.5521(07)	0.39
clover quark	0.6328(05)	0.39
overlap pion	0.5191(14)	0.39
clover pion	0.5178(18)	0.39
clover nucleon	0.5139(54)	0.39

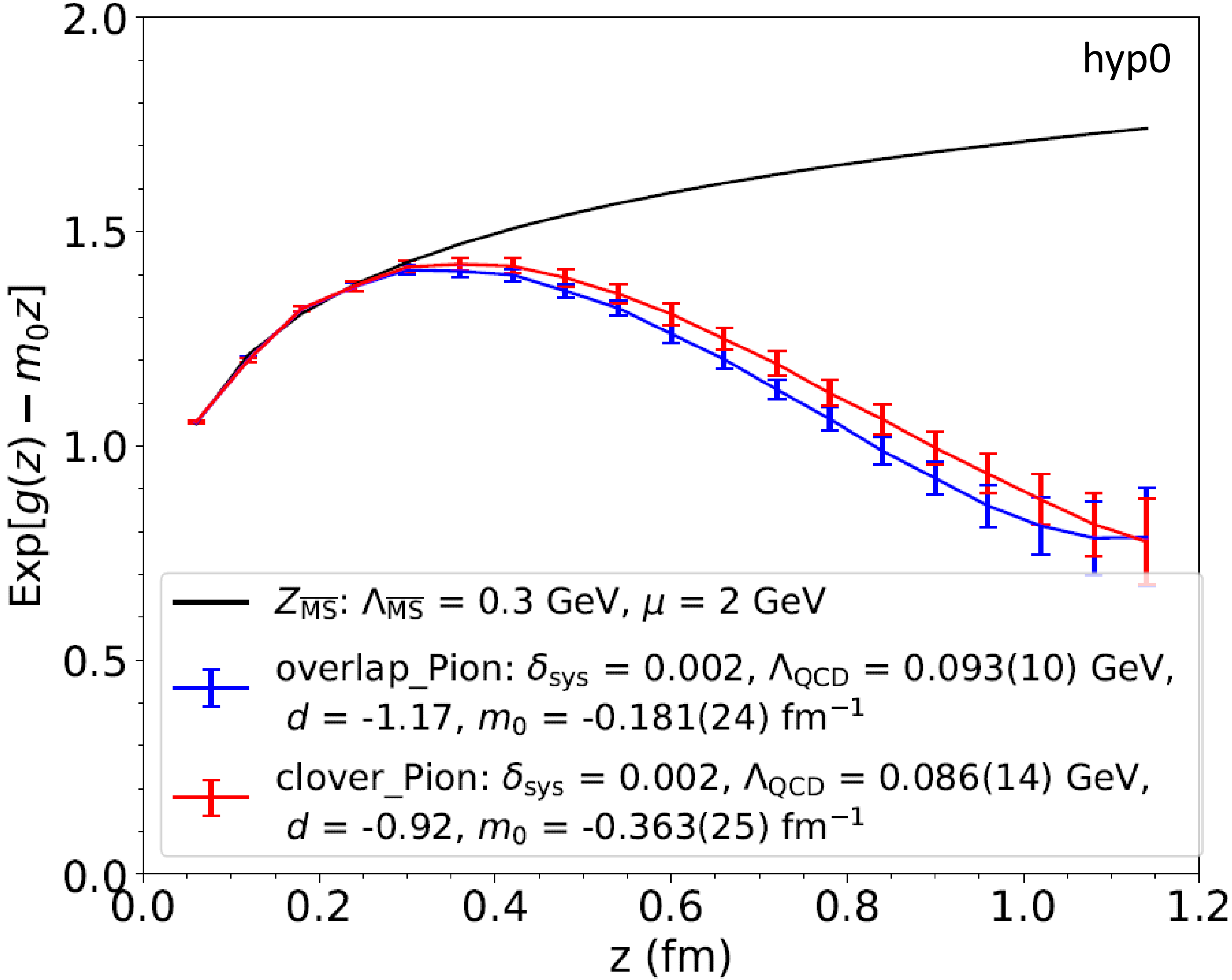
Valence fermion action

External state (quark means RI/MOM factor)

Linear divergences for hadron matrix elements are similar to each other.

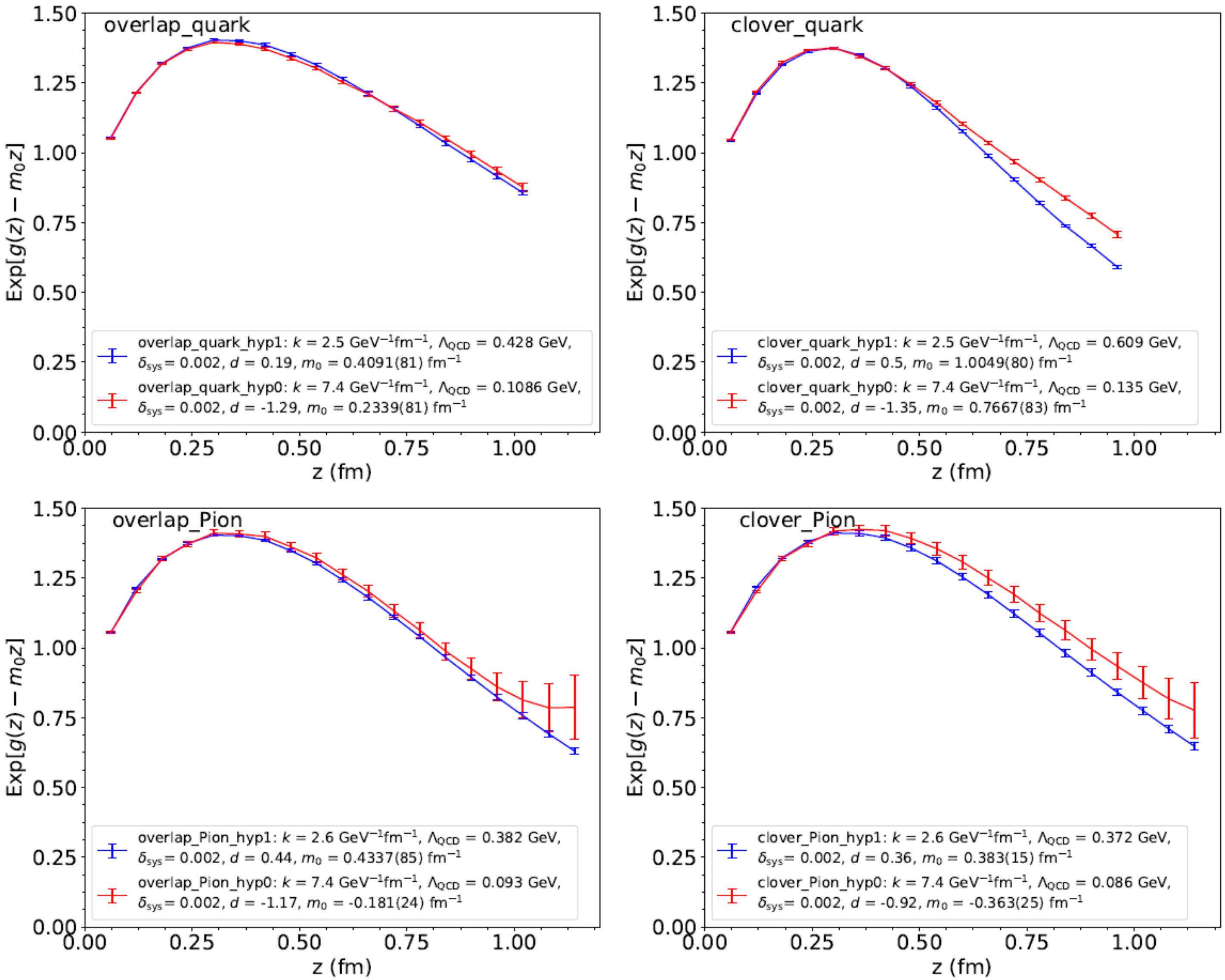
Linear divergences for RI/MOM factors are different from those of hadron matrix elements.

Residual intrinsic physics



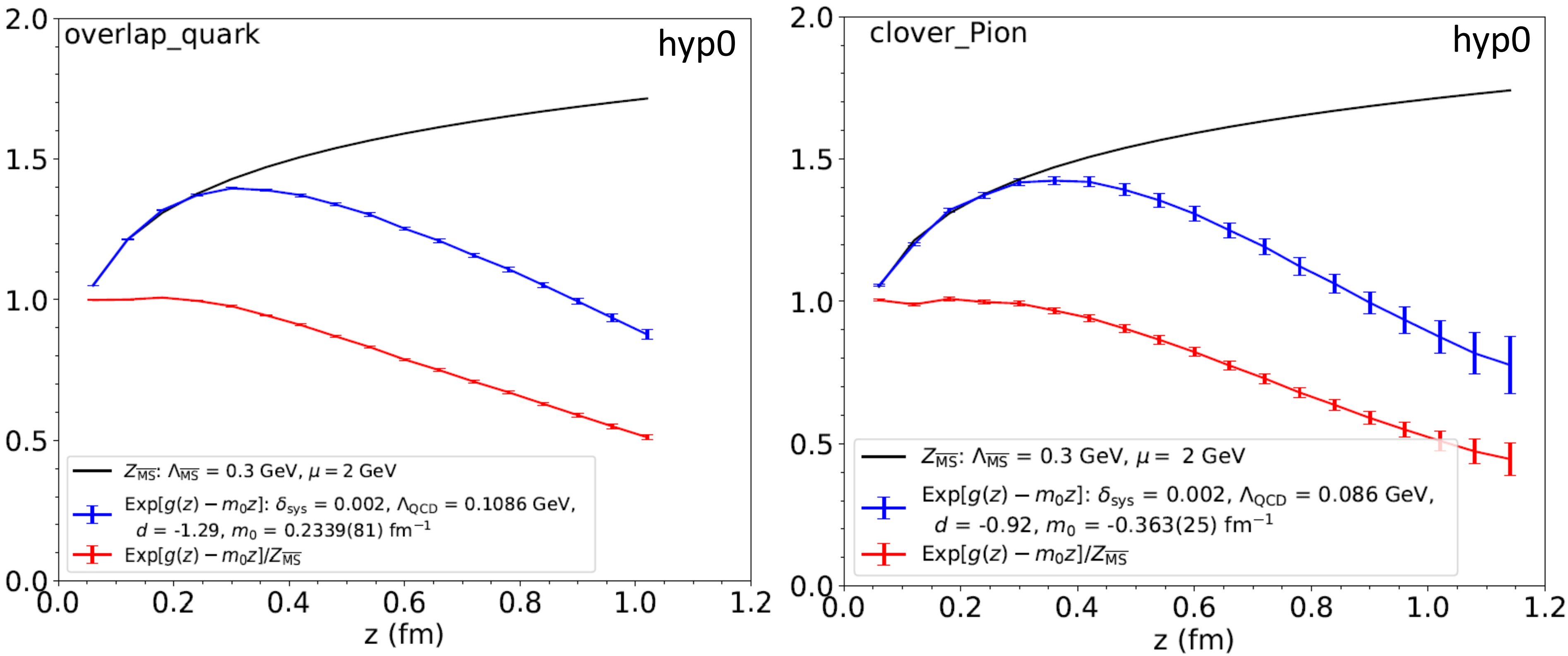
Clover and overlap valence fermion actions
lead to similar residual intrinsic physics

Residual intrinsic physics



HYP smearing does not change residual intrinsic physics

Residual intrinsic physics



Large extra non-perturbative effect at large distance in RI/MOM and Ratio scheme, which supports Hybrid renormalization method

X. Ji, Y. Liu, A. Schafer, W. Wang, Y.-B. Yang, J.-H. Zhang, and Y. Zhao, (2020), arXiv:2008.03886

Hybrid Renormalization Method

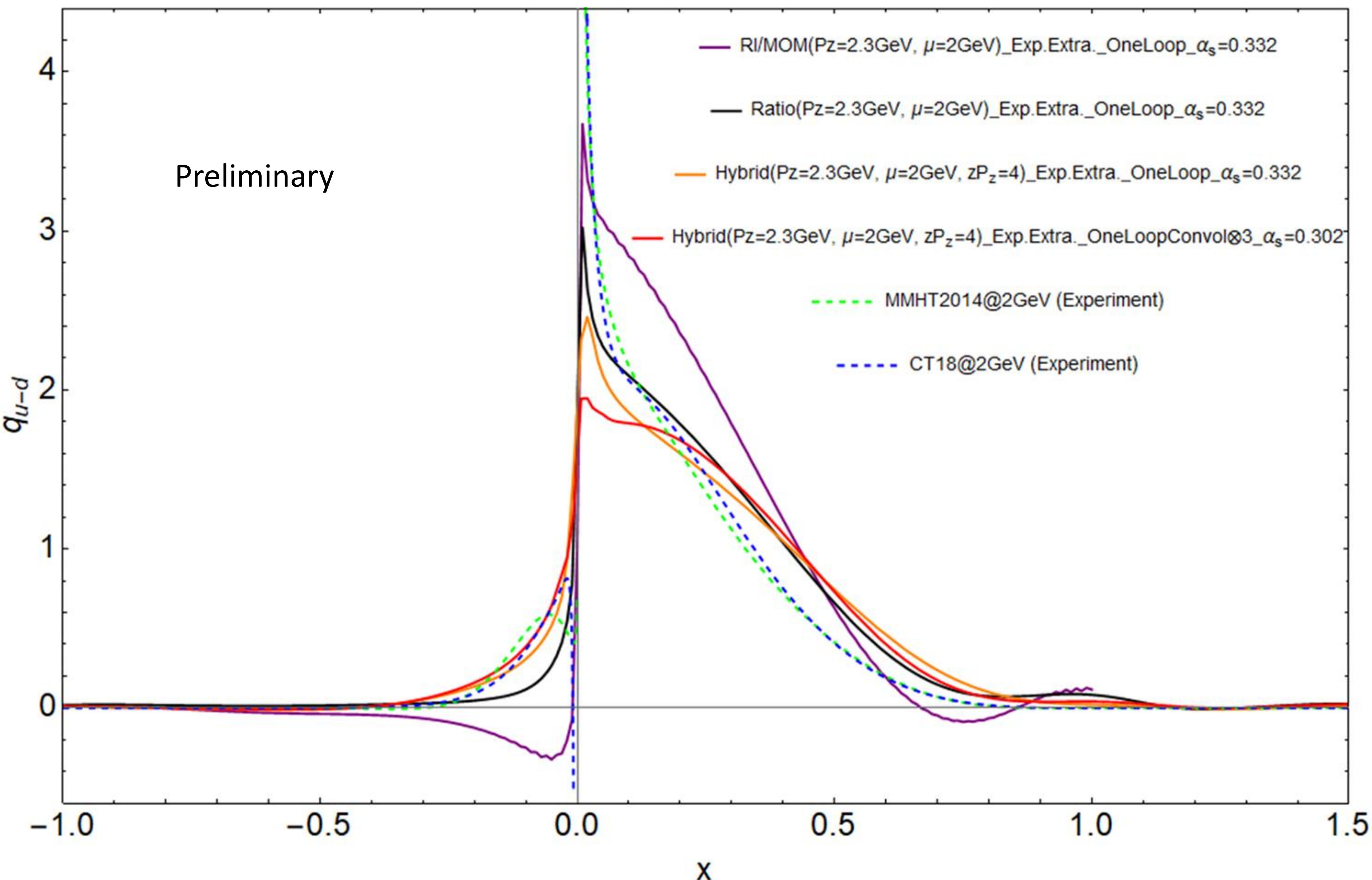
$$\tilde{h}^R(z,P_z) = \frac{\tilde{h}(z,P_z)}{\tilde{h}(z,0)}\theta(z_S - |z|) + \tilde{h}(z,P_z)e^{-\delta m|z|}Z_{\text{hybrid}}(z_S)\theta(|z| - z_S)$$

Short Distance: Ratio Scheme, divided by zero momentum matrix element

Long Distance: divided by the renormalization factor
It will not introduce extra non-perturbative effect.

X. Ji, Y. Liu, A. Schafer, W. Wang, Y.-B. Yang, J.-H. Zhang, and Y. Zhao, Nuclear Physics B 964 (2021): 115311, *arXiv:2008.03886*

Isovector unpolarized proton PDF



Hybrid Renormalization Method gives better result for negative x

arXiv:1510.02332

arXiv:1908.11394

Conclusion

1. We propose the self-renormalization method. It works well in disentangling the renormalization factor (containing linear divergence, log divergence, discretization effect, renormalon effect) from residual intrinsic physics;

2. Linear divergences for RI/MOM factors are different from those of hadron matrix elements;

3. Clover and overlap, hyp0 and hyp1 lead to similar residual intrinsic physics;

4. Large non-perturbative effect at large distance in RI/MOM and Ratio scheme. Hybrid Renormalization Method can solve this problem and give closer result to global fit PDF.

Appendix

A. Explanation of each term in the fitting function

For each z (z can be chosen in a large range), extract $g(z)$ by fitting the a dependence:

$$ln(Z) = \frac{kz}{aln(a\Lambda_{QCD})} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases} + \frac{3C_F}{b0} ln\left[\frac{\ln[1/(a\Lambda_{QCD})]}{ln[\mu/\Lambda_{QCD}]}\right] + \ln[1 + \frac{d}{\ln(a\Lambda_{QCD})}]$$

g(z) is a fitted parameter for each z

Fit the $m0$:

$$g(z) - \ln(Z_{\overline{MS}}) = m0\ z$$

$$Z_{\overline{MS}}(z, \mu, \Lambda_{\overline{MS}}) = 1 + \frac{\alpha_s(\mu,\Lambda_{\overline{MS}})C_F}{2\pi}\left[\frac{3}{2}\ln\frac{\mu^2 z^2 e^{2\gamma_E}}{4} + \frac{5}{2}\right],$$

for small z (a < z < 0.2 fm)

$m0\ z$ subtraction:

$$g(z) - m0\ z$$

Our final result, the physical result

Λ_{QCD} as a fitted parameter

We only consider leading order term, but there should be higher order terms, and the higher order terms may be different for different actions and ensembles.

$$ln(Z) = \frac{kz}{aln(a\Lambda_{QCD})} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases} + \frac{3C_F}{b0} ln\left[\frac{\ln[1/(a\Lambda_{QCD})]}{ln[\mu/\Lambda_{QCD}]}\right] + \ln[1 + \frac{d}{\ln(a\Lambda_{QCD})}]$$

Choose a proper Λ_{QCD}, we can use the leading order term to represent the results up to higher order corrections:

$$\alpha_s(Q, \Lambda_{QCD}) \sim \alpha_s(Q, \Lambda) + c1 * \alpha_s(Q, \Lambda)^2 + c2 * \alpha_s(Q, \Lambda)^3 \ldots$$

G.Peter Lepage (Cornell U., LNS), Paul B. Mackenzie (Fermilab), On the viability of lattice perturbation theory, DOI: 10.1103/PhysRevD.48.2250)

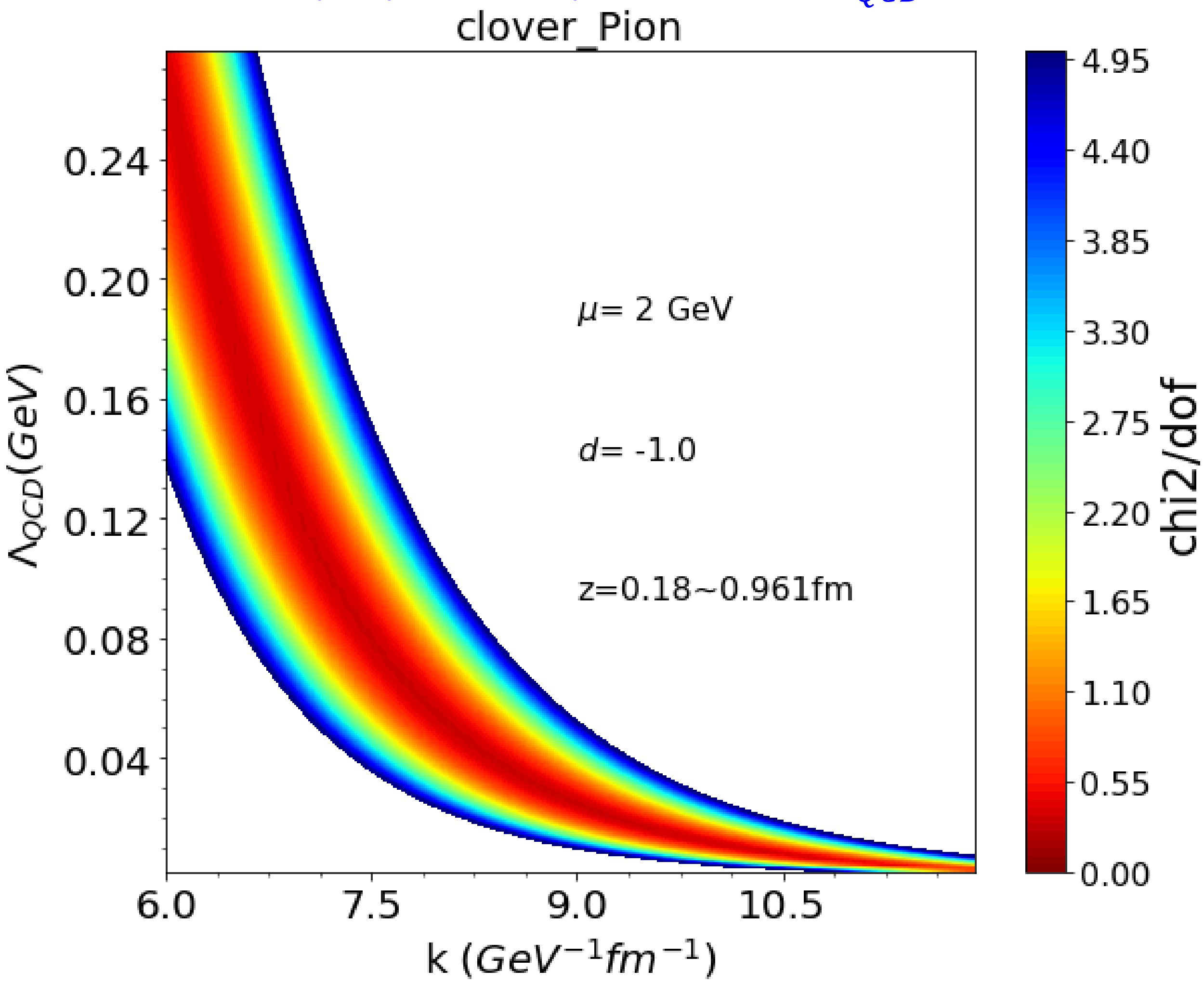
Let the data help us choose the Λ_{QCD}!

Renormalon Uncertainty

Fit the matrix element with the formula:

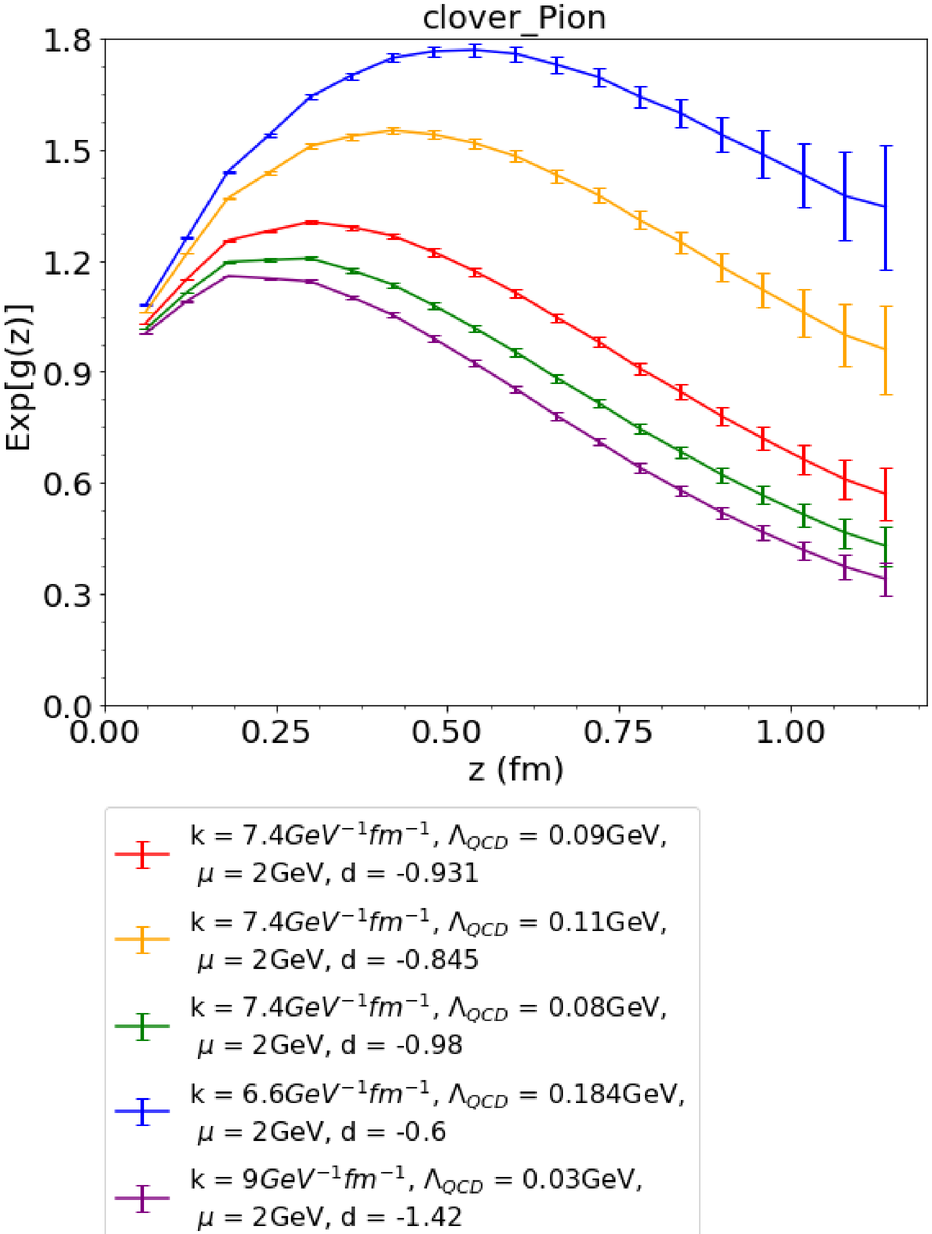
$$ln(Z) = \frac{kz}{aln(a\Lambda_{QCD})} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases} + \frac{3C_F}{b0} ln\left[\frac{\ln[1/(a\Lambda_{QCD})]}{ln[\mu/\Lambda_{QCD}]}\right] + \ln[1 + \frac{d}{\ln(a\Lambda_{QCD})}]$$

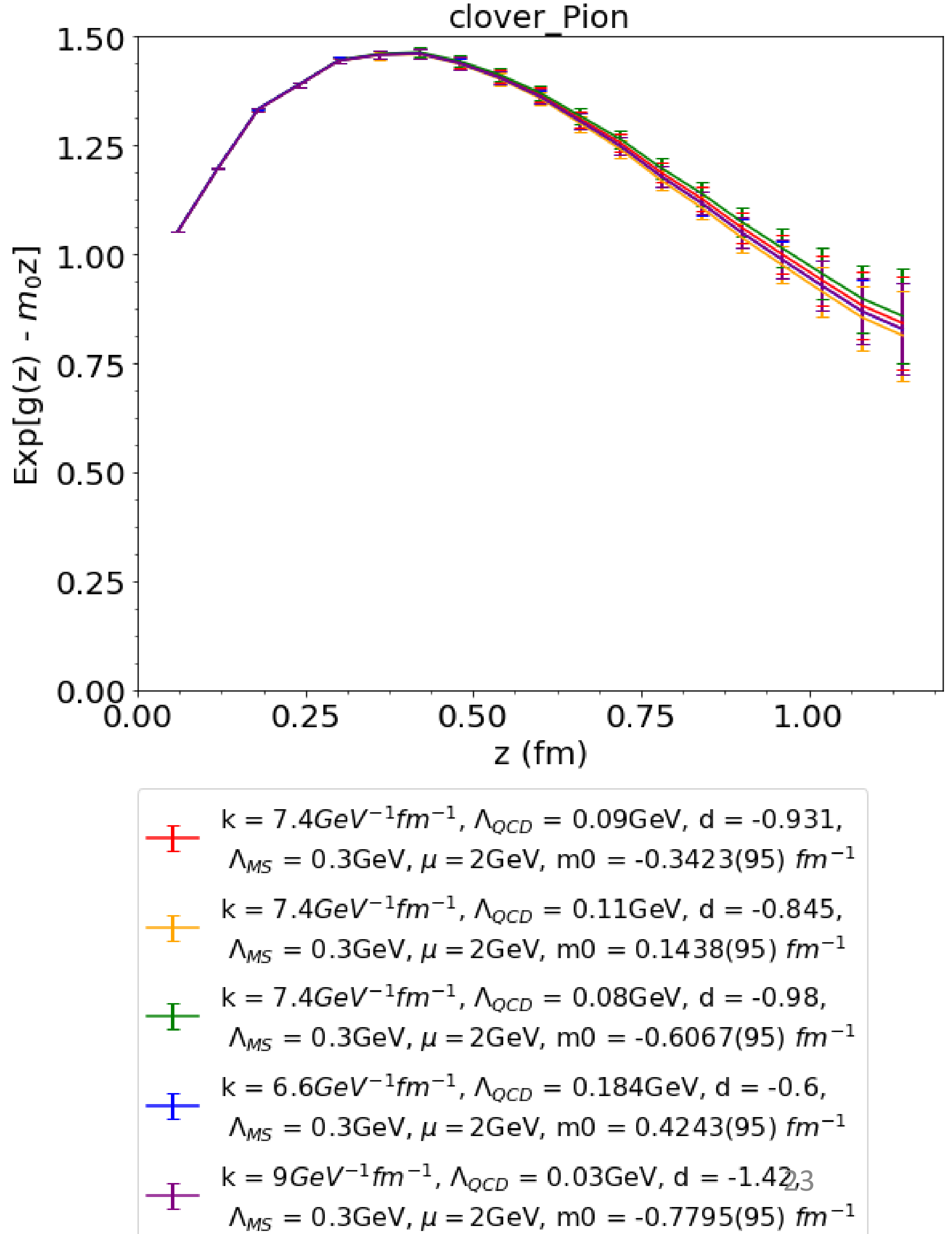
Chisqmap with respect to k and Λ_{QCD}:



g(z), f1(z), f2(z) fitted for each set of {k, Λ_{QCD}}

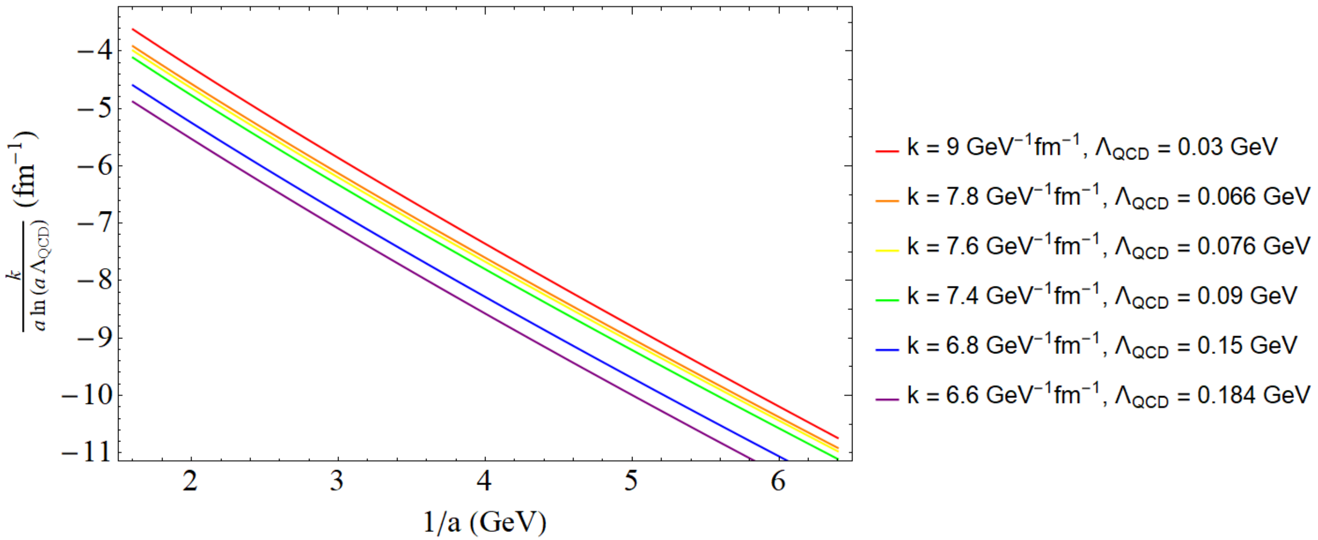
Choose some sets of parameters on the small chisq band

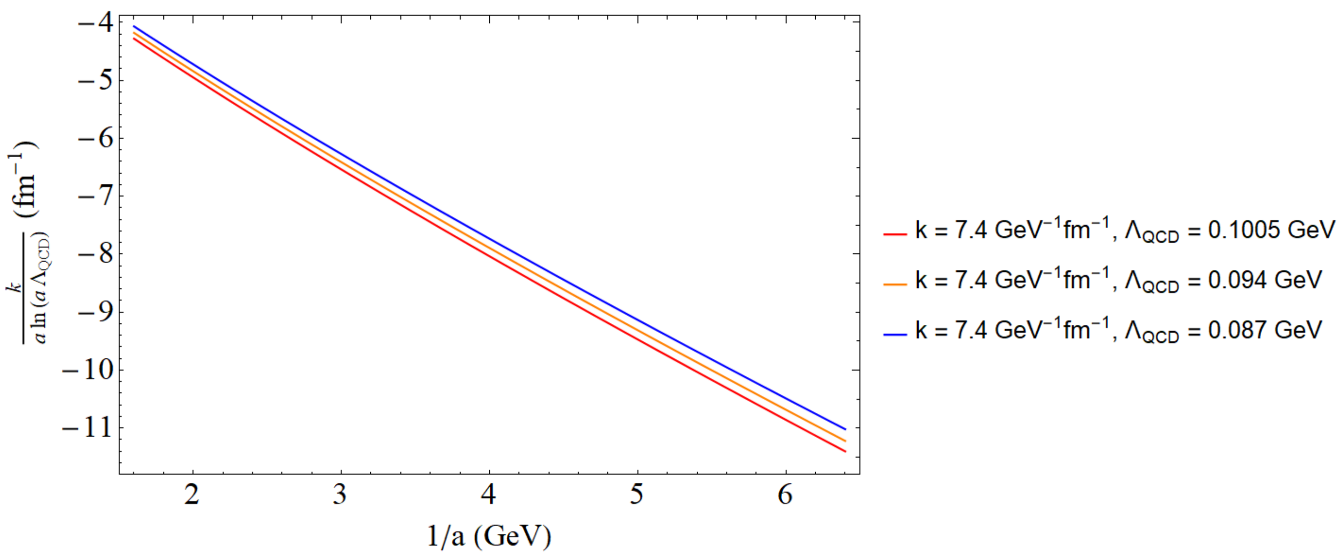




$$ln(Z) = \frac{kz}{aln(a\Lambda_{QCD})} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases} + \frac{3C_F}{b0} ln\left[\frac{\ln[1/(a\Lambda_{QCD})]}{ln[\mu/\Lambda_{QCD}]}\right] + \ln[1 + \frac{d}{\ln(a\Lambda_{QCD})}]$$

In our data range (lattice spacing range), varying k and Λ_{QCD} along the small chisq band just translates $\frac{k}{aln(a\Lambda_{QCD})}$ by a constant:





$$ln(Z) = \frac{kz}{aln(a\Lambda_{QCD})} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases} + \frac{3C_F}{b0} ln\left[\frac{\ln[1/(a\Lambda_{QCD})]}{ln[\mu/\Lambda_{QCD}]}\right] + \ln[1 + \frac{d}{\ln(a\Lambda_{QCD})}]$$

varying k and Λ_{QCD} along the small chisq band

↓

In our data range

$\frac{kz}{aln(a\Lambda_{QCD})}$ -> $\frac{kz}{aln(a\Lambda_{QCD})}$ + C*z

↓

$g(z)$ -> $g(z)$ - C*z

↓

The physical result
Exp(g(z) – m0 z)
will not change

We cannot determine k and Λ_{QCD} from our data. But the uncertainty of k and Λ_{QCD} doesn't influence the physical result.

We can fix k at the one-loop perturbative value with nf=3:

$$k = \frac{(2\text{Pi})^2}{3\left(11 - \frac{2}{3} * \text{nf}\right)} \frac{1}{0.1973269631\ GeV\ fm}$$
$$= 7.4\ GeV^{-1} fm^{-1}$$

Ji, Zhang, Zhao,
Phys.Rev.Lett. 120 (2018) 11, 112001

Then choose Λ_{QCD} as the best fitted value

Resummation of the logarithmic divergence term

$$ln(Z) = \frac{kz}{a ln(a\Lambda_{QCD})} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases} + \frac{3C_F}{b0} ln\left[\frac{\ln[1/(a\Lambda_{QCD})]}{\ln[\mu/\Lambda_{QCD}]}\right] + \ln[1 + \frac{d}{\ln(a\Lambda_{QCD})}]$$

logarithmic divergence term from wavefunction and vertex renormalizations at one-loop order:

$$Z_{\psi,z} = 1 + \frac{3\ C_F\ \alpha_s}{2\pi}\frac{1}{4-d}$$

Ji, X., Liu, Y.-S., Liu, Y., Zhang, J.-H., and Zhao, Y., "Large-Momentum Effective Theory", arXiv:2004.03543v1, 2020.

Solve the renormalization group equation to get the QCD rescaling factor:

$$\frac{dZ_Q}{dln(Q)} = \gamma Z_Q, where\ \gamma = -\frac{3\ C_F\ \alpha_s(Q, \Lambda_{QCD})}{2\pi}$$

$$\Downarrow$$

$$Z_{1/a} = [\frac{\ln[1/(a\Lambda_{QCD})]}{\ln[\mu/\Lambda_{QCD}]}]^{\frac{3C_F}{b0}} Z_\mu$$

we can fix μ = 2 GeV

Resummation of the logarithmic divergence term to higher order

$$ln(Z) = \frac{kz}{a ln(a\Lambda_{QCD})} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases} + \frac{3C_F}{b0} ln\left[\frac{\ln[1/(a\Lambda_{QCD})]}{\ln[\mu/\Lambda_{QCD}]}\right] + \ln[1 + \frac{d}{\ln(a\Lambda_{QCD})}]$$

Solve the renormalization group equation to get the QCD rescaling factor:

$$\frac{dZ_Q}{dln(Q)} = \gamma Z_Q, where\ \gamma = \frac{3\ C_F\ \alpha_s(Q,\Lambda_{QCD})}{2\pi} + \text{c2}{*}\alpha_s(Q,\Lambda_{QCD})^2$$

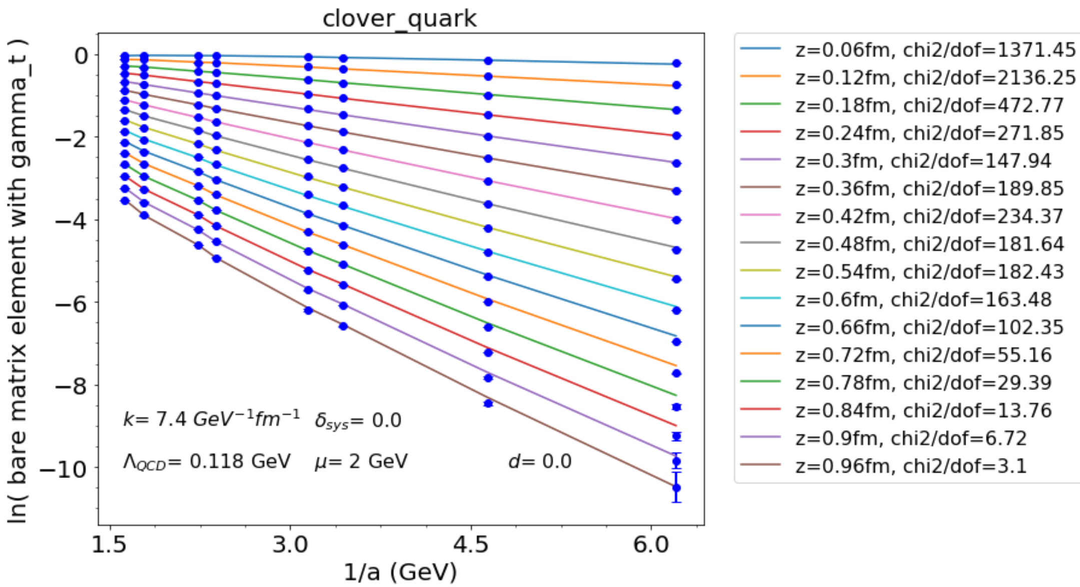
$$\Downarrow$$

$$Z_{1/a} = [\frac{\ln[1/(a\Lambda_{QCD})]}{\ln[\mu/\Lambda_{QCD}]}]^{\frac{3C_F}{b0}}[1 + \frac{d}{\ln(a\Lambda_{QCD})}]Z_\mu$$

d is related to the actions and ensembles.

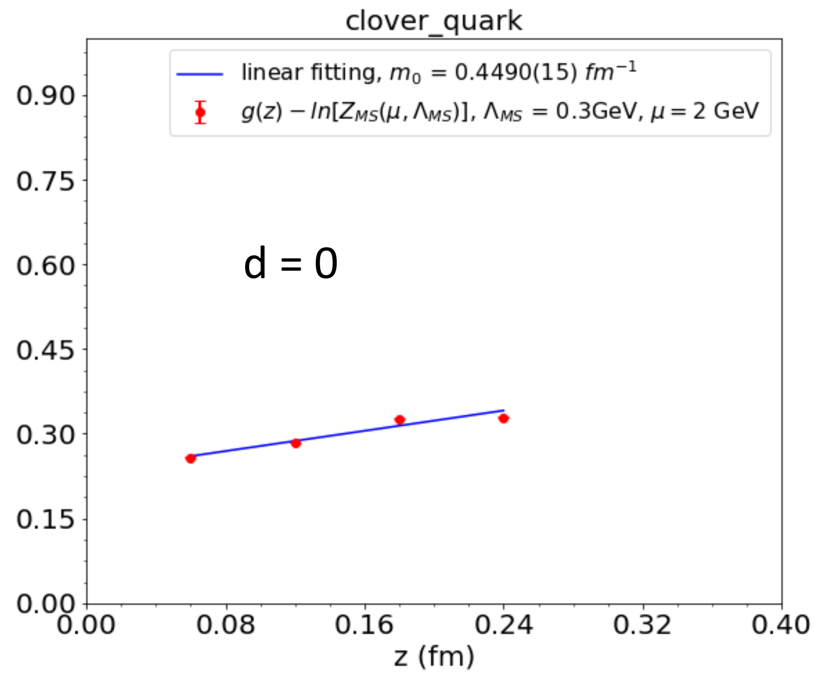
However, we will treat d as a fine-tuning parameter.

Xiangdong Ji, M.J. Musolf, Sub-leading logarithmic mass-dependence in heavy-meson form-factors, Physics Letters B, Volume 257, Issues 3–4, 1991

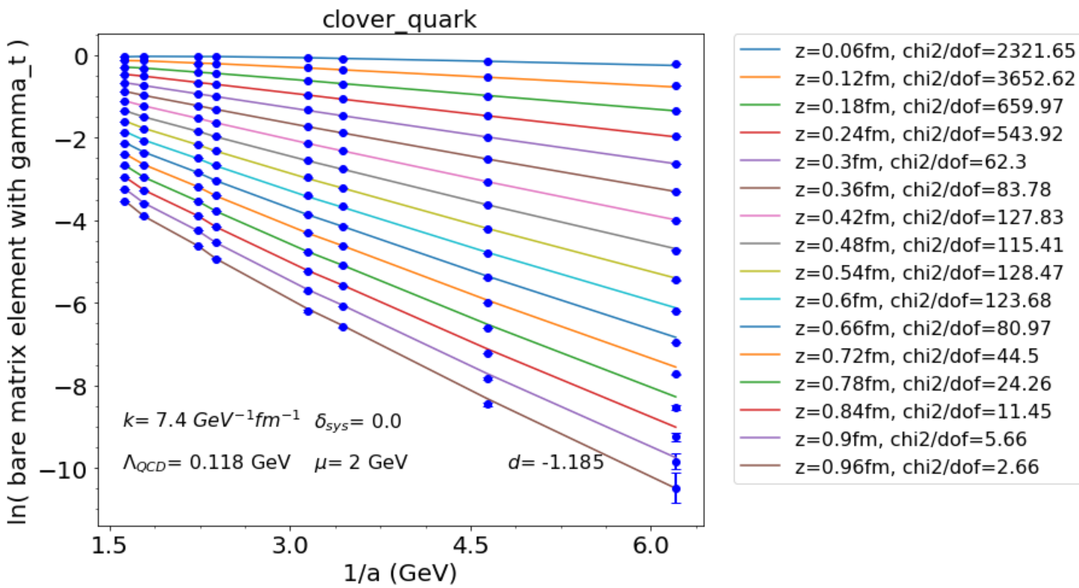


$kz\frac{1}{a\ln(a\Lambda_{QCD})} + g(z) + \frac{3C_f}{b0}\ln[\frac{\ln(1/(a\Lambda_{QCD}))}{\ln(\mu/\Lambda_{QCD})}] + \ln(1+\frac{d}{\ln(a\Lambda_{QCD})}) + f1(z)a$ for MILC

$kz\frac{1}{a\ln(a\Lambda_{QCD})} + g(z) + \frac{3C_f}{b0}\ln[\frac{\ln(1/(a\Lambda_{QCD}))}{\ln(\mu/\Lambda_{QCD})}] + \ln(1+\frac{d}{\ln(a\Lambda_{QCD})}) + f2(z)a$ for RBC

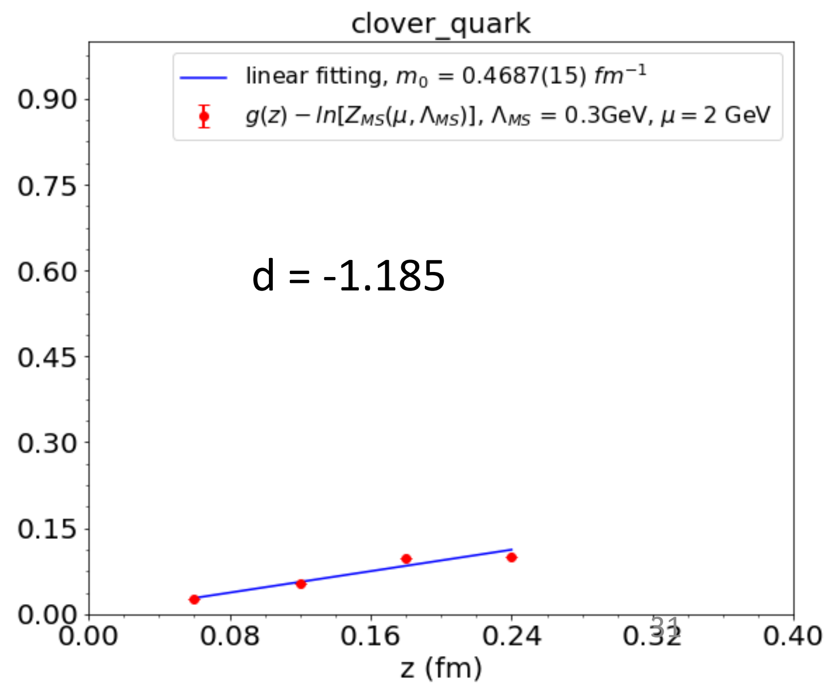


Tune d to make $g(z) - \ln(Z_{\overline{MS}}) \propto z$

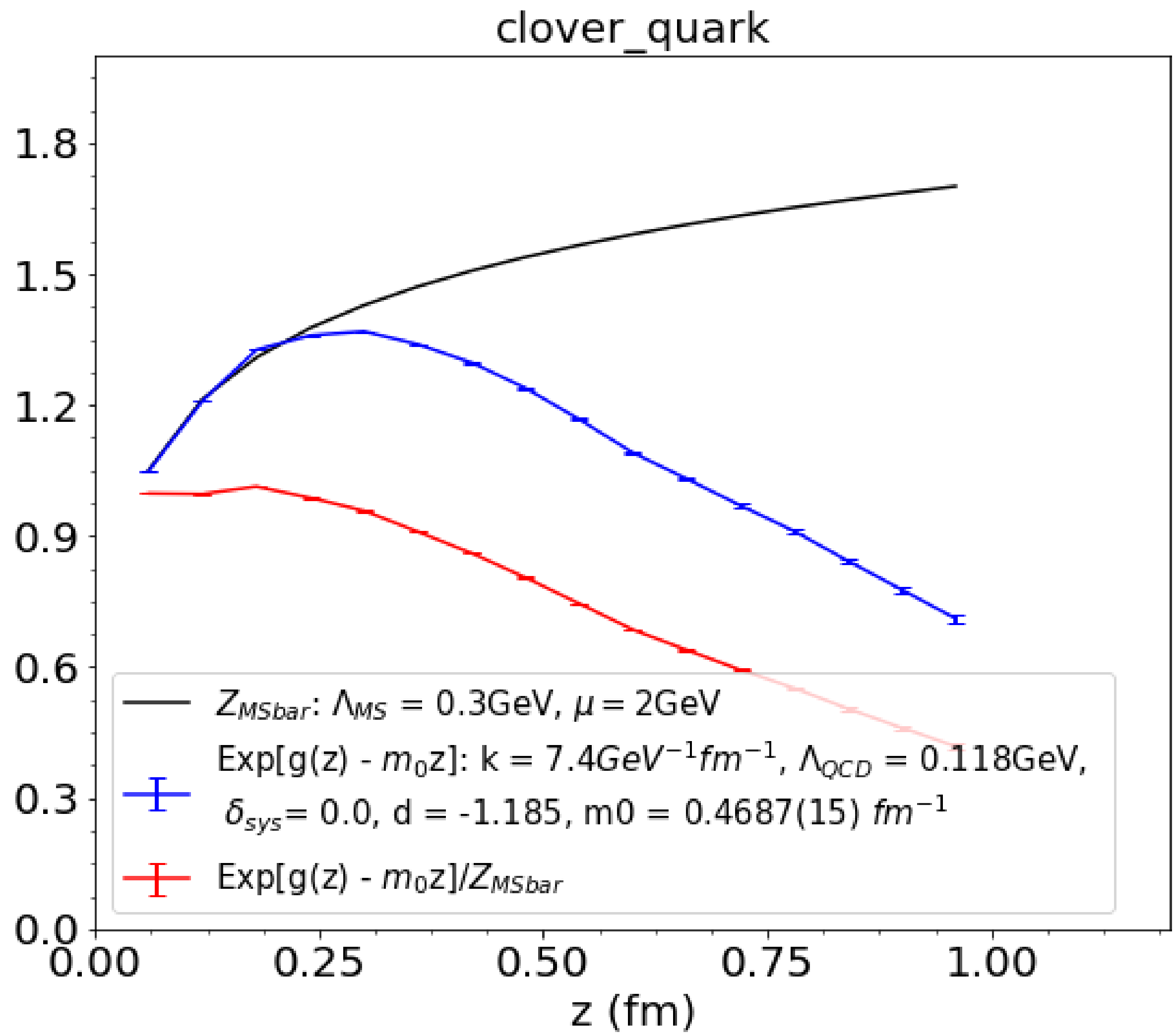


$kz\frac{1}{a\ln(a\Lambda_{QCD})} + g(z) + \frac{3C_f}{b0}\ln[\frac{\ln(1/(a\Lambda_{QCD}))}{\ln(\mu/\Lambda_{QCD})}] + \ln(1+\frac{d}{\ln(a\Lambda_{QCD})}) + f1(z)a$ for MILC

$kz\frac{1}{a\ln(a\Lambda_{QCD})} + g(z) + \frac{3C_f}{b0}\ln[\frac{\ln(1/(a\Lambda_{QCD}))}{\ln(\mu/\Lambda_{QCD})}] + \ln(1+\frac{d}{\ln(a\Lambda_{QCD})}) + f2(z)a$ for RBC



After m0 z subtraction, the results from lattice and the results from continuum perturbation theory can match in the region where a < z < 0.2 fm.



Large differences between perturbative and non-perturbative RI/MOM factors at large z

B. Different Fitting functions

$$\ln(Z) = \frac{kz}{aln(a\Lambda)} + g(z)$$

Not accurate enough

$$\ln(Z) = \frac{kz}{aln(a\Lambda)} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases}$$

Our current function

$$ln(Z) = \frac{kz}{aln(a\Lambda_{QCD})} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases} + \frac{3C_F}{b0} ln\left[\frac{\ln[1/(a\Lambda_{QCD})]}{ln[\mu/\Lambda_{QCD}]}\right] + \ln[1 + \frac{d}{\ln(a\Lambda_{QCD})}]$$

$$\ln(Z) = \frac{kz}{aln(a\Lambda)} b0 \ln\left(\frac{1}{a^2\Lambda^2}\right) \alpha s(1 + \lambda\, \alpha s) + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases}$$

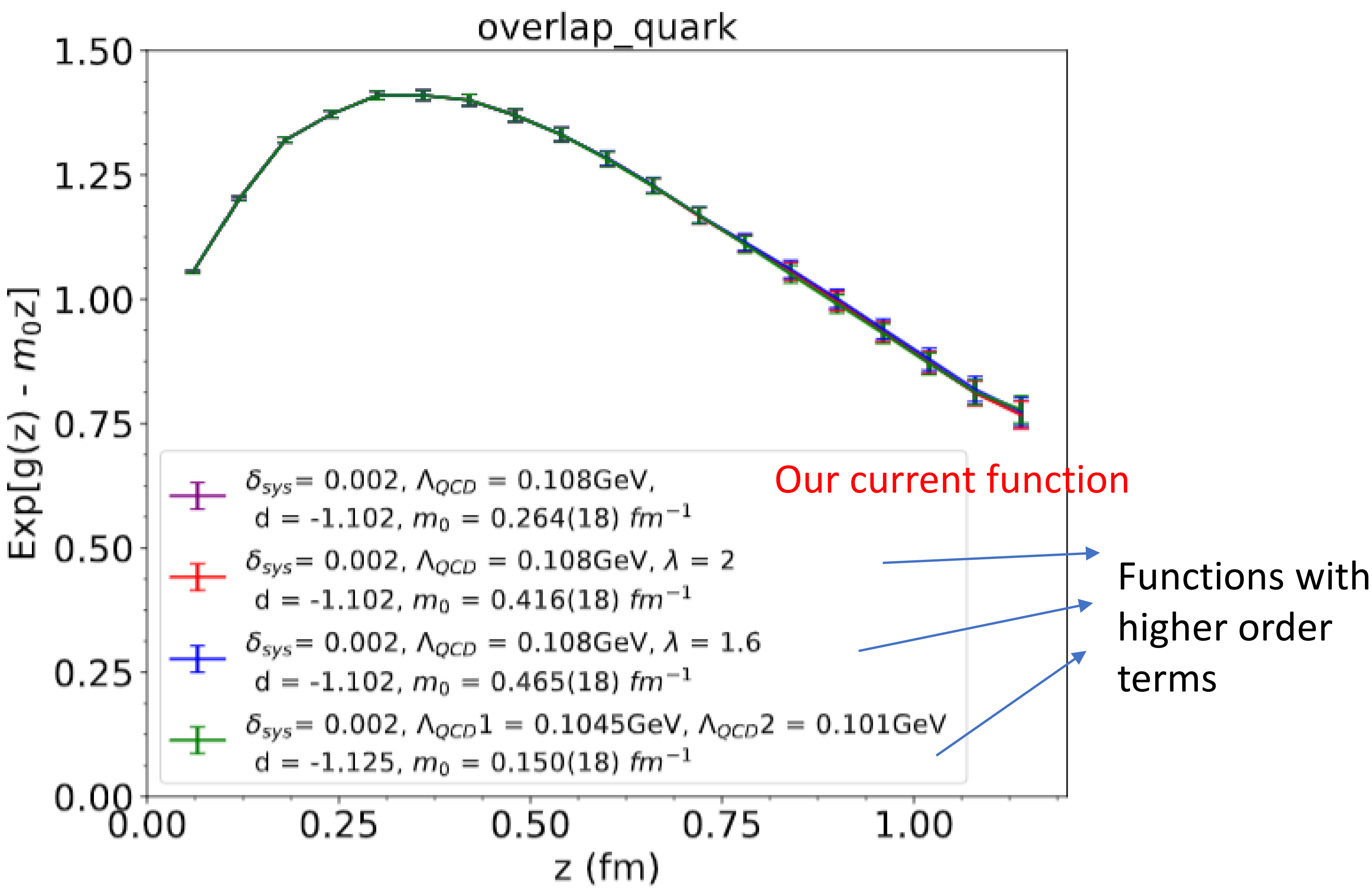
$$\ln(Z) = \frac{kz}{aln(a\Lambda)} b0 \ln\left(\frac{1}{a^2\Lambda^2}\right) \frac{\alpha s}{1 - \lambda\, \alpha s} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases}$$

Same physical results as our current one

$$ln(Z) = g(z) + \begin{cases} \frac{kz}{aln(a\Lambda_{QCD1})} + \frac{3C_F}{b0} ln\left[\frac{\ln[1/(a\Lambda_{QCD1})]}{ln[\mu/\Lambda_{QCD1}]}\right] + \ln[1 + \frac{d}{\ln(a\Lambda_{QCD1})}] + f1(z)a, MILC \\ \frac{kz}{aln(a\Lambda_{QCD2})} + \frac{3C_F}{b0} ln\left[\frac{\ln[1/(a\Lambda_{QCD2})]}{ln[\mu/\Lambda_{QCD2}]}\right] + \ln[1 + \frac{d}{\ln(a\Lambda_{QCD2})}] + f2(z)a, RBC \end{cases}$$

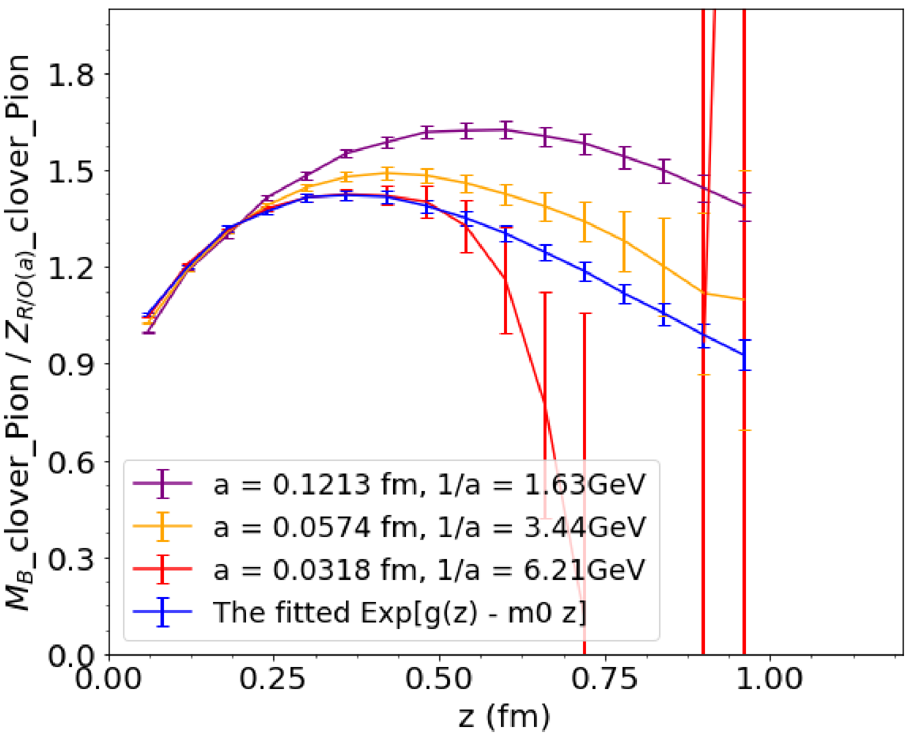
$$\ln(Z) = \frac{kz}{aln(a\Lambda)} b0 \ln\left(\frac{1}{a^2\Lambda^2}\right) \alpha s(1 + \lambda\, \alpha s) + g(z) + \begin{cases} f11(z)a + f12(z)a^2, MILC \\ f21(z)a + f22(z)a^2, RBC \end{cases}$$

Unstable

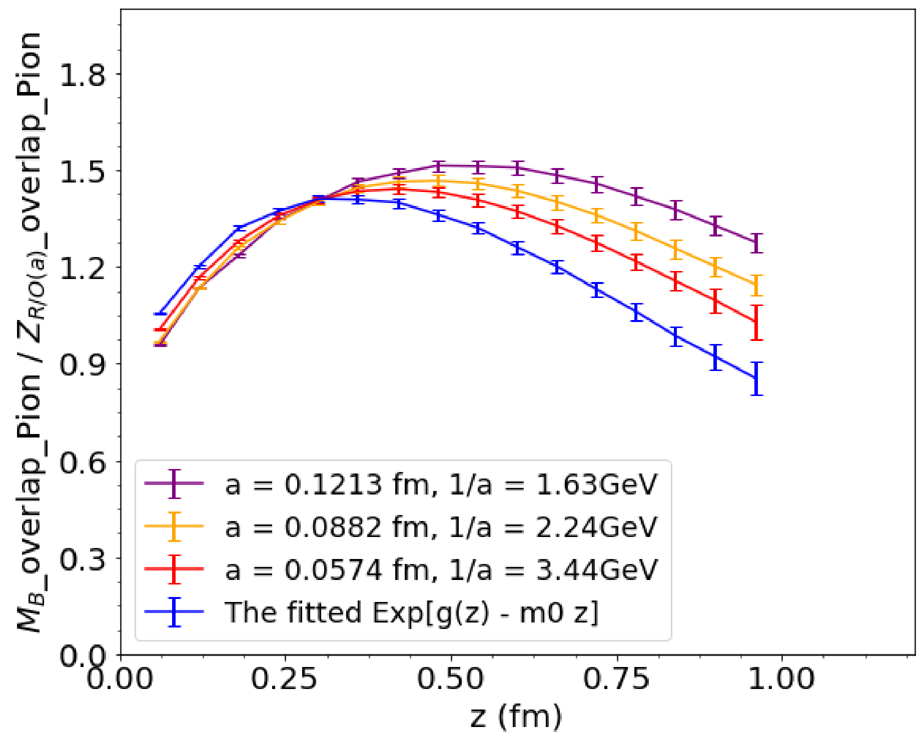


overlap_quark
Exp[g(z) - m_0z]
z (fm)
1.50
1.25
1.00
0.75
0.50
0.25
0.00
0.00
0.25
0.50
0.75
1.00
δ_{sys}= 0.002, Λ_{QCD} = 0.108GeV, d = -1.102, m_0 = 0.264(18) fm^{-1}
δ_{sys}= 0.002, Λ_{QCD} = 0.108GeV, λ = 2 d = -1.102, m_0 = 0.416(18) fm^{-1}
δ_{sys}= 0.002, Λ_{QCD} = 0.108GeV, λ = 1.6 d = -1.102, m_0 = 0.465(18) fm^{-1}
δ_{sys}= 0.002, $\Lambda_{QCD}1$ = 0.1045GeV, $\Lambda_{QCD}2$ = 0.101GeV d = -1.125, m_0 = 0.150(18) fm^{-1}
Our current function
Functions with higher order terms

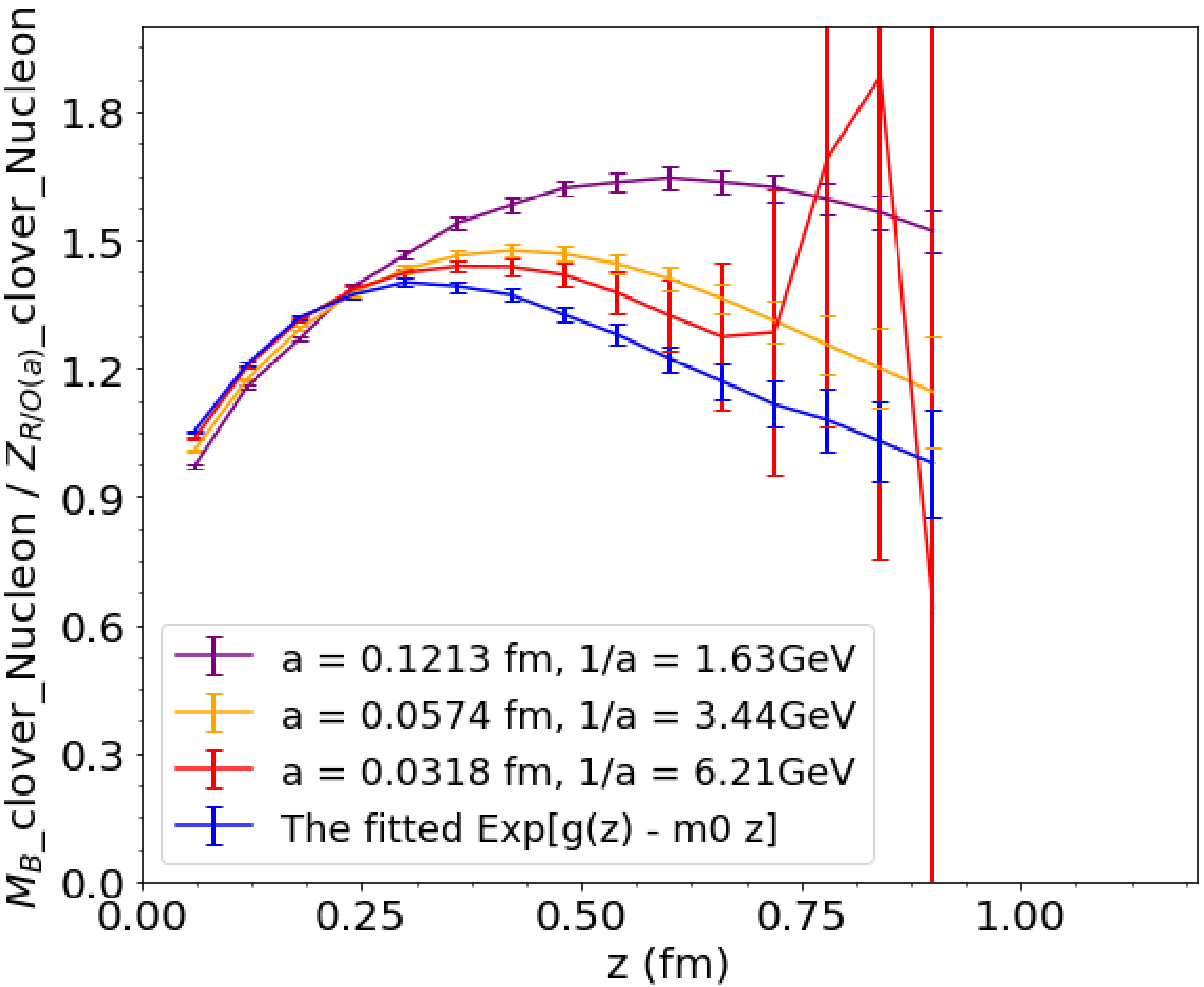
C. Discretization Effect



$$Z_{R/O(a)} = \text{Exp}[k\, z \frac{1}{a\, ln(a\,\Lambda_{QCD})} + \frac{3C_f}{b0} ln[\frac{ln(1/(a\Lambda_{QCD}))}{ln(\mu/\Lambda_{QCD})}] + ln(1+\frac{d}{ln(a\,\Lambda_{QCD})}) + m0\, z]$$

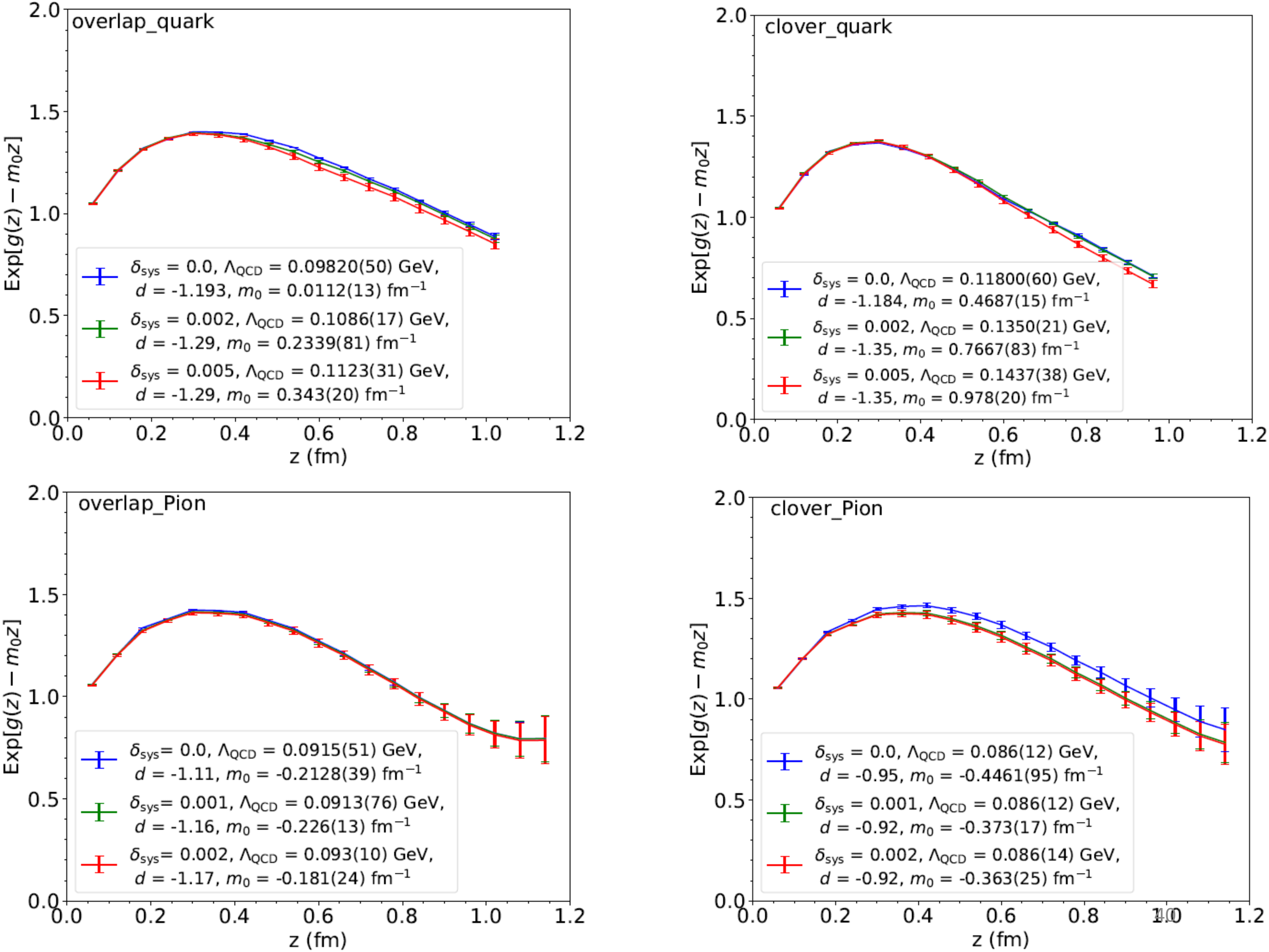


$$Z_{R/O(a)} = \text{Exp}[k\, z \frac{1}{a\, ln(a\,\Lambda_{QCD})} + \frac{3C_f}{b0} ln[\frac{ln(1/(a\Lambda_{QCD}))}{ln(\mu/\Lambda_{QCD})}] + ln(1+\frac{d}{ln(a\,\Lambda_{QCD})}) + m0\, z]$$



$$Z_{R/O(a)} = \mathrm{Exp}[k\, z \frac{1}{a\, ln(a\Lambda_{QCD})} + \frac{3C_f}{b0} ln[\frac{ln(1/(a\Lambda_{QCD}))}{ln(\mu/\Lambda_{QCD})}] + ln(1 + \frac{d}{ln(a\Lambda_{QCD})}) + m0\, z]$$

D. Dummy Systematic Errors


overlap_quark
δsys = 0.0, ΛQCD = 0.09820(50) GeV, d = -1.193, m0 = 0.0112(13) fm−1
δsys = 0.002, ΛQCD = 0.1086(17) GeV, d = -1.29, m0 = 0.2339(81) fm−1
δsys = 0.005, ΛQCD = 0.1123(31) GeV, d = -1.29, m0 = 0.343(20) fm−1
clover_quark
δsys = 0.0, ΛQCD = 0.11800(60) GeV, d = -1.184, m0 = 0.4687(15) fm−1
δsys = 0.002, ΛQCD = 0.1350(21) GeV, d = -1.35, m0 = 0.7667(83) fm−1
δsys = 0.005, ΛQCD = 0.1437(38) GeV, d = -1.35, m0 = 0.978(20) fm−1
overlap_Pion
δsys= 0.0, ΛQCD = 0.0915(51) GeV, d = -1.11, m0 = -0.2128(39) fm−1
δsys= 0.001, ΛQCD = 0.0913(76) GeV, d = -1.16, m0 = -0.226(13) fm−1
δsys= 0.002, ΛQCD = 0.093(10) GeV, d = -1.17, m0 = -0.181(24) fm−1
clover_Pion
δsys = 0.0, ΛQCD = 0.086(12) GeV, d = -0.95, m0 = -0.4461(95) fm−1
δsys = 0.001, ΛQCD = 0.086(12) GeV, d = -0.92, m0 = -0.373(17) fm−1
δsys = 0.002, ΛQCD = 0.086(14) GeV, d = -0.92, m0 = -0.363(25) fm−1
Exp[g(z) − m0z]
z (fm)

E. Some vacuum matrix elements

Analysis of Wilson Loop, Wilson Link, VEV

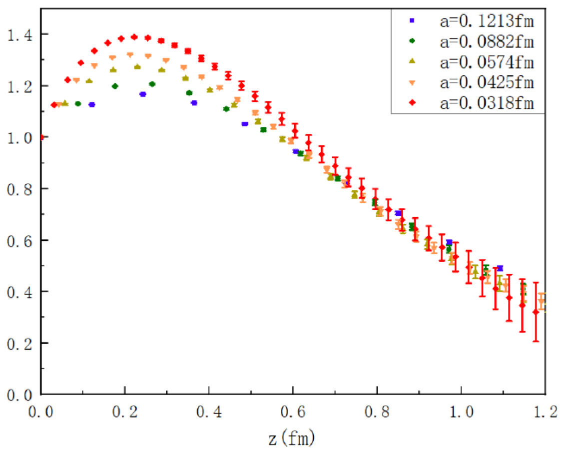


FIG. 6. The pion quasi-PDF matrix element renormalized by the linear divergence part of Wilson Loop.

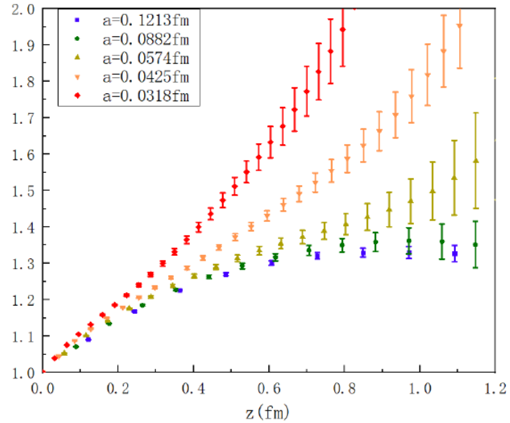


FIG. 7. The effective decay rate of Landau gauge fixed Wilson link (upper panel), and pion matrix element renormalized by the Wilson link.

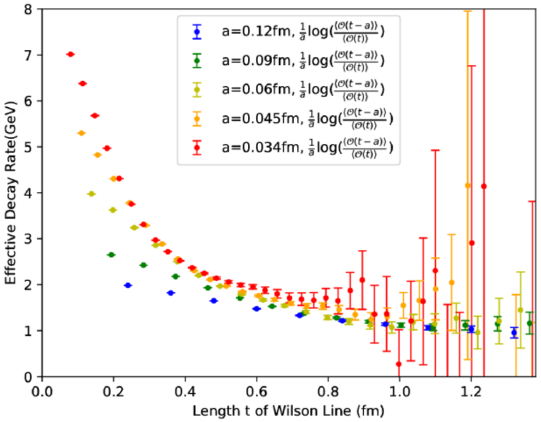
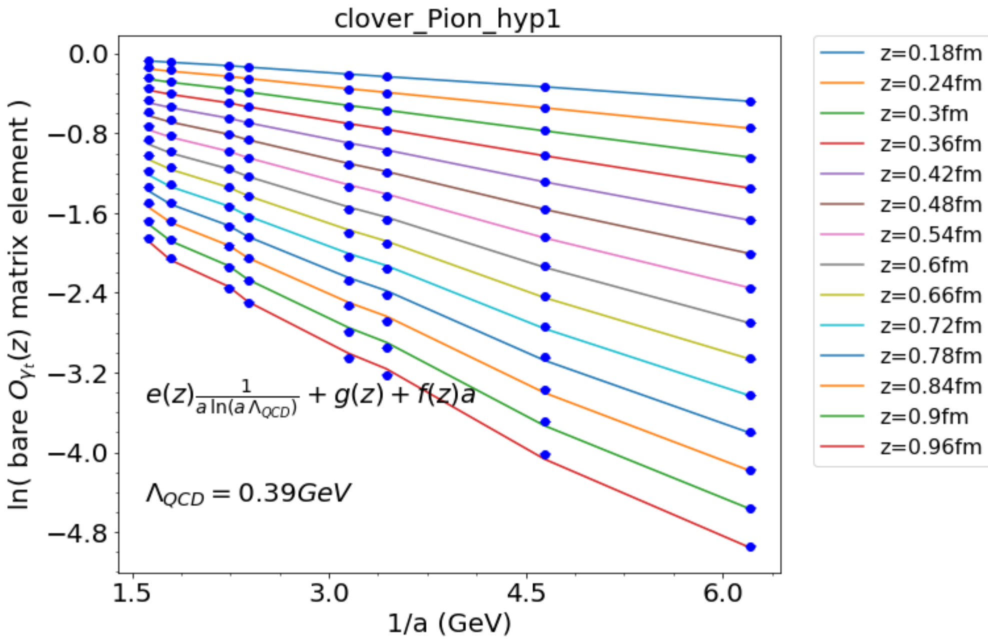
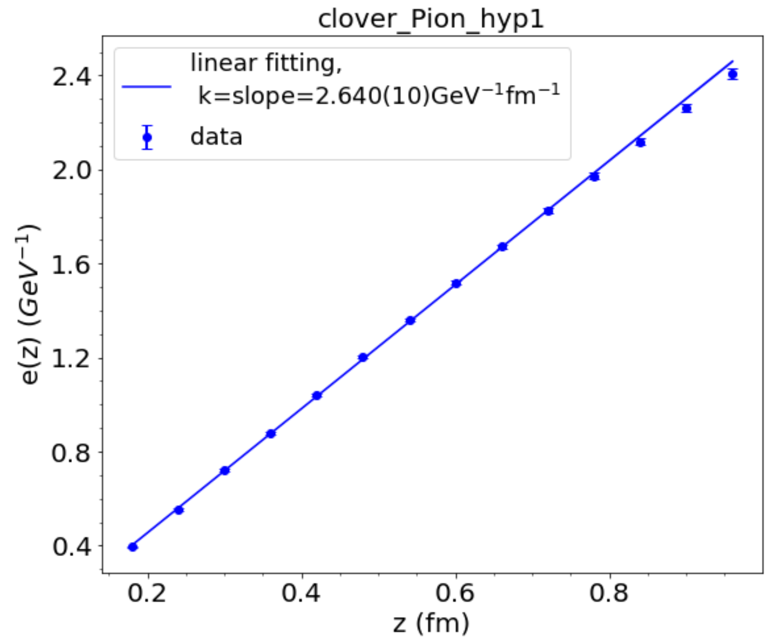


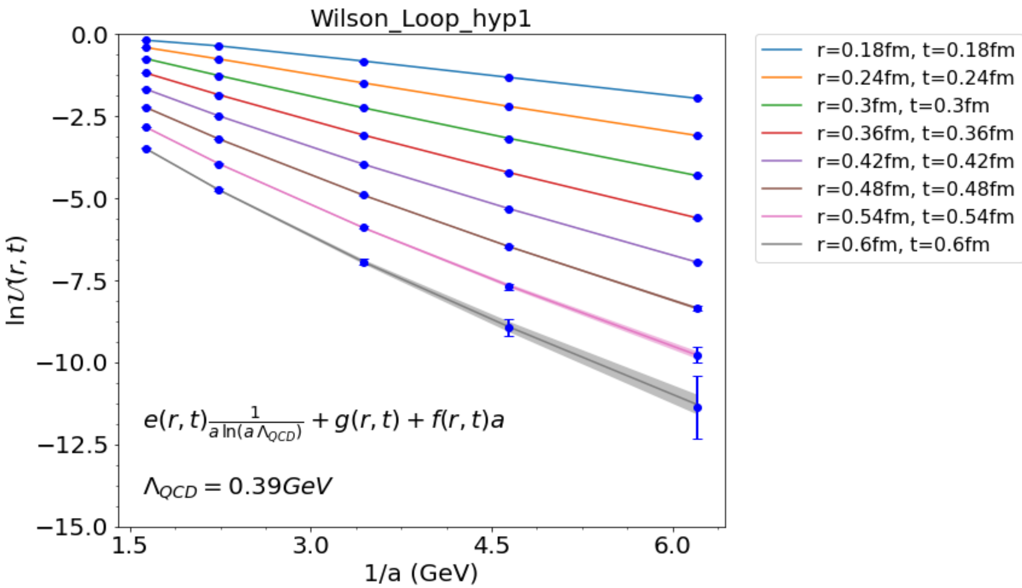
FIG. 4. Effective decay rate $\frac{1}{a}\log(\frac{O_{\gamma_t}(t-a)}{O_{\gamma_t}})$ of the O_{γ_t} VEV. The gaps between the results at different lattice spacings decreases on t.

$$ln(M) = \frac{e(z)}{aln\left(a\Lambda_{QCD}\right)} + g(z) + \begin{cases} f1(z)a, MILC \\ f2(z)a, RBC \end{cases}$$

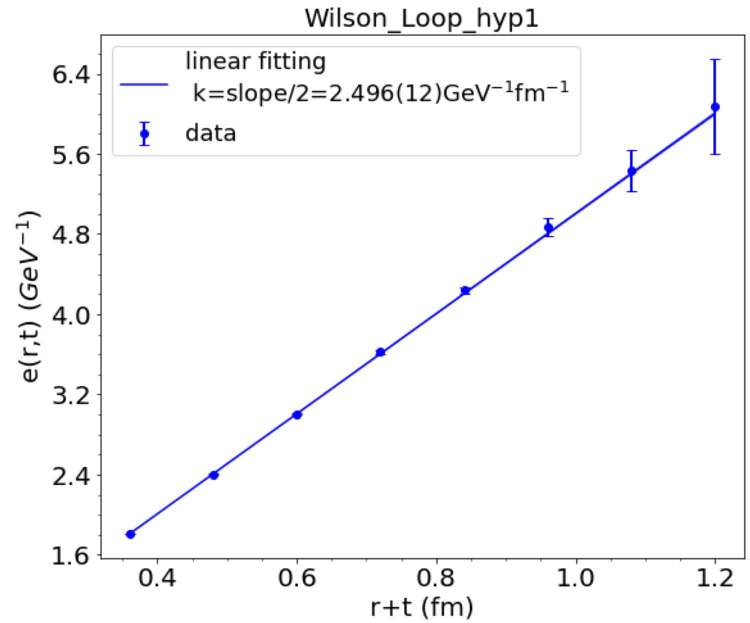
Step1: for any given z, fit dependence on a to find e(z)
step2: Plot e(z) as a function of z.
step3: Compare the slopes of e(z) between different matrix elements.


clover_Pion_hyp1
ln(bare $O_{\gamma_t}(z)$ matrix element)
1/a (GeV)
z=0.18fm
z=0.24fm
z=0.3fm
z=0.36fm
z=0.42fm
z=0.48fm
z=0.54fm
z=0.6fm
z=0.66fm
z=0.72fm
z=0.78fm
z=0.84fm
z=0.9fm
z=0.96fm
$e(z)\frac{1}{a\ln(a\Lambda_{QCD})} + g(z) + f(z)a$
$\Lambda_{QCD} = 0.39GeV$


clover_Pion_hyp1
linear fitting,
k=slope=2.640(10)GeV^{-1}fm^{-1}
data
e(z) (GeV^{-1})
z (fm)



Slope of e(z) of clover Pion hyp1:
2.640(10) GeV^{-1} fm^{-1}



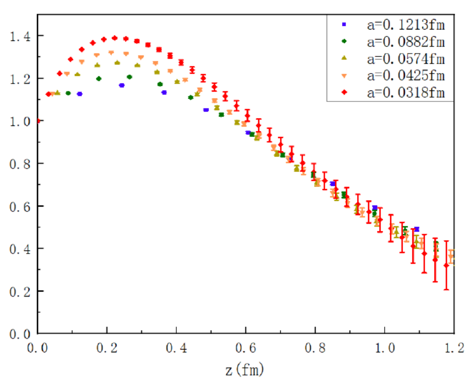
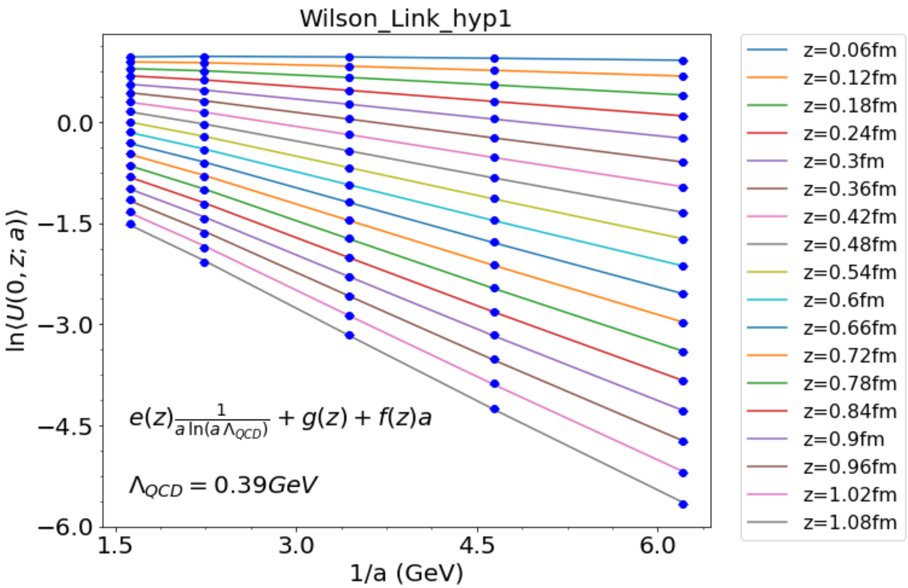


FIG. 6. The pion quasi-PDF matrix element renormalized by the linear divergence part of Wilson Loop.



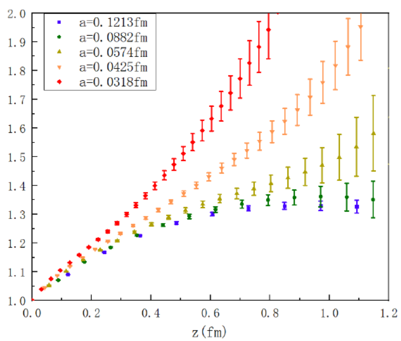
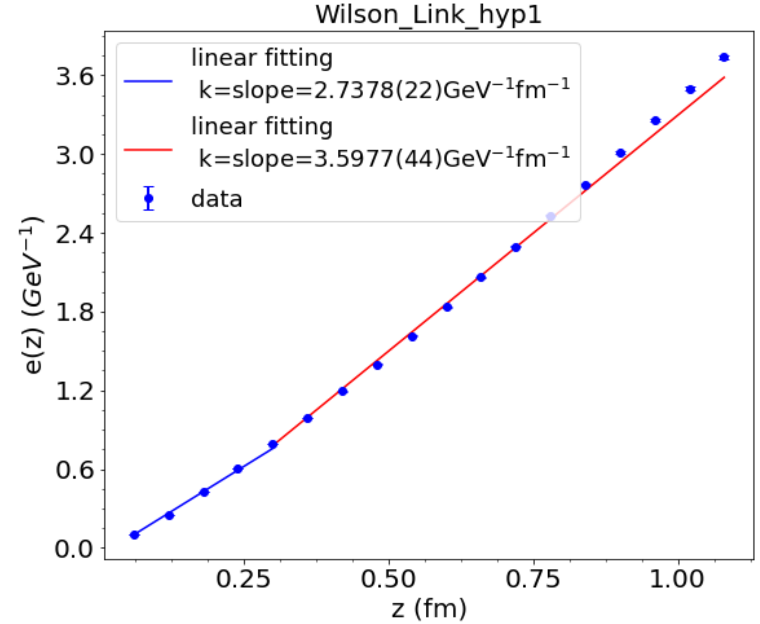
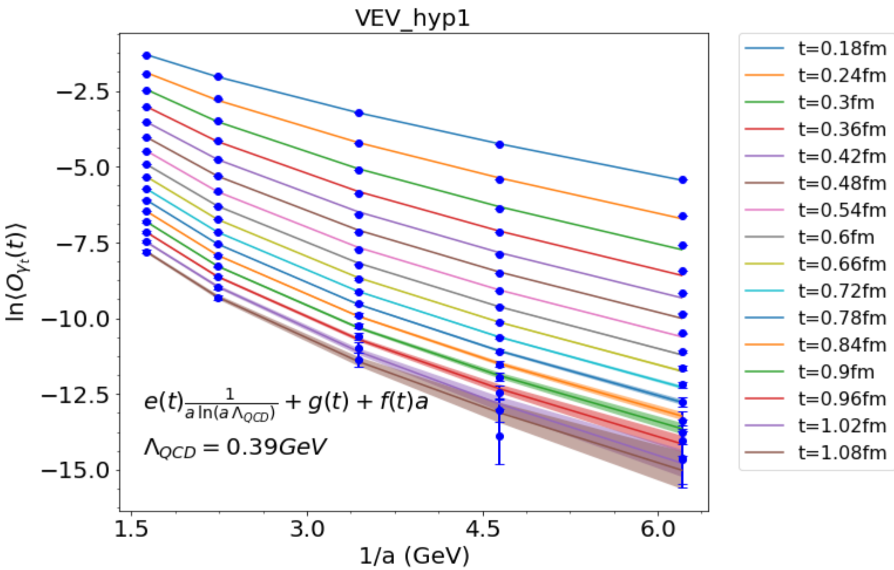


FIG. 7. The effective decay rate of Landau gauge fixed Wilson link (upper panel), and pion matrix element renormalized by the Wilson link.

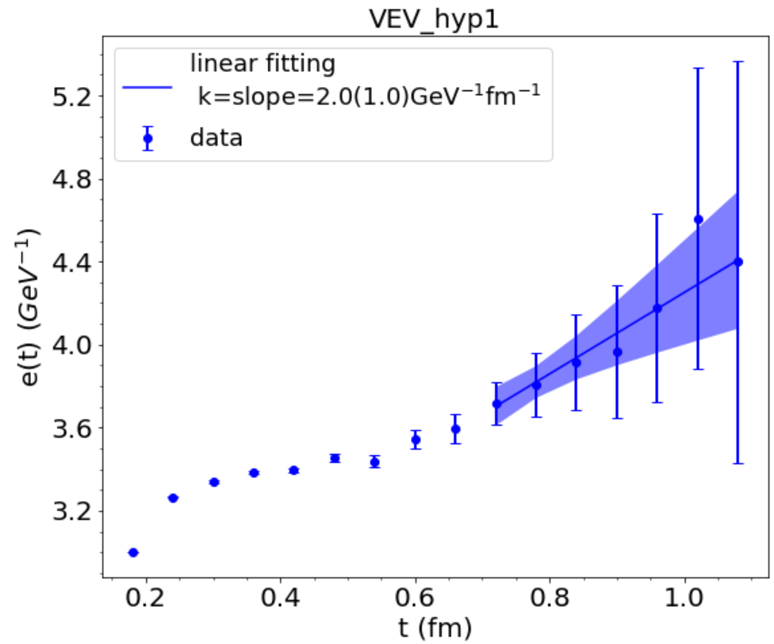
Slope of e(z) of clover Pion hyp1:
2.640(10) GeV^{-1} fm^{-1}



At small-z, it can be calculated in perturbation theory where the linear divergence works well. For large-z, the matrix element does not have a transfer-matrix interpretation in this gauge.



Slope of e(z) of clover Pion hyp1:
2.640(10) GeV^{-1} fm^{-1}



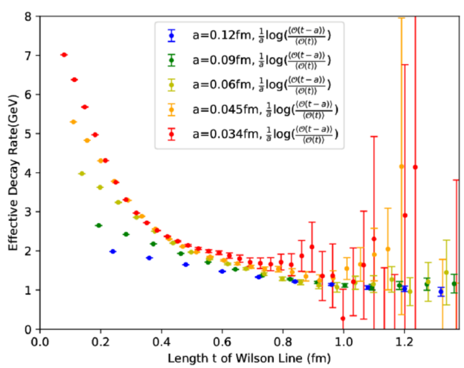


FIG. 4. Effective decay rate $\frac{1}{a}\log(\frac{O_{\gamma_t}(t-a)}{O_{\gamma_t}})$ of the O_{γ_t} VEV. The gaps between the results at different lattice spacings decreases on t.

$$O_{\gamma t}(z) = \mathrm{c0} * \bar{\psi}\gamma^t\psi + \ldots$$

$$\langle\bar{\psi}\gamma^t\psi\rangle = 0$$

At small z, $\langle O_{\gamma t}(z)\rangle$ only has higher twist terms. Small residual Exp[g(z)], large discretization error O(a).

Scalar matrix elements may avoid these problems