Lattice QCD determination of Collins-Soper Kernel through TMD Wave Function in LaMET

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Outline

- Why Collins-Soper kernel?
- TMD wave function
- Collins-Soper kernel from lattice QCD
- Numerical results

Why Collins-Soper kernel ?

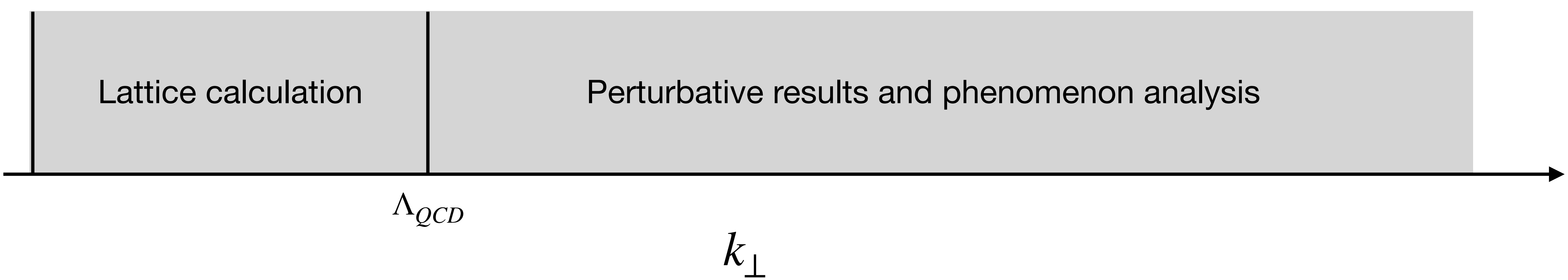
The Collins-Soper kernel relates the evolution of transverse momentum-dependent parton distributions and hadron wave function.

P. Shanahan, M. Wangman and Y. Zhao, arxiv 2003.06063(2020)

$$f^{TMD}(x, b_\perp, \mu, \zeta) = f^{TMD}(x, b_\perp, \mu_0, \zeta_0)\ \exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'}\gamma(\mu', \zeta_0)\right] \exp\left[\frac{1}{2}K(\mu, b_\perp)\ln\frac{\zeta}{\zeta_0}\right]$$

Transverse momentum of parton $k_\perp$ represents the scale of TMD physics.

Energy scale of parton in hadron

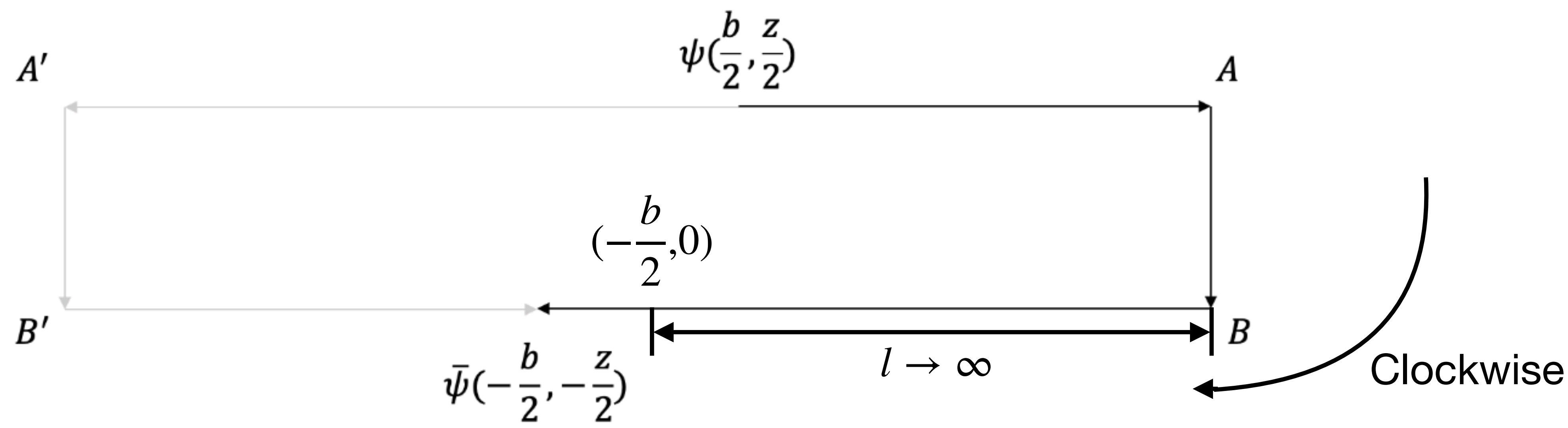


TMD wave function

leading twist for pion: $\Gamma_1 = \gamma^t\gamma_5,\ \ \gamma^z\gamma_5$

TMDWF: $\tilde{\Phi}(b, z, \overrightarrow{P}) = \lim_{l\to\infty} \langle 0 | \bar{\psi}_1(-\frac{b}{2}\hat{x} - \frac{z}{2}\hat{z})\Gamma_1 U_c \psi_2(\frac{b}{2}\hat{x} + \frac{z}{2}\hat{z}) | \pi(\overrightarrow{P})\rangle$

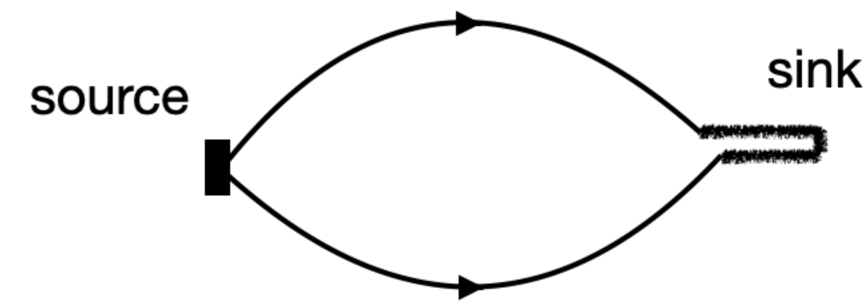
U_c :



Two point correlation function:

$$C_2(b, z, t, \overrightarrow{P}) = \int d^3x e^{-i\overrightarrow{P}\overrightarrow{x}} \langle 0 | \chi(\overrightarrow{x}, t, b, z, \Gamma_1)\bar{\chi}(0,0,0,0,\gamma_5) | 0\rangle$$

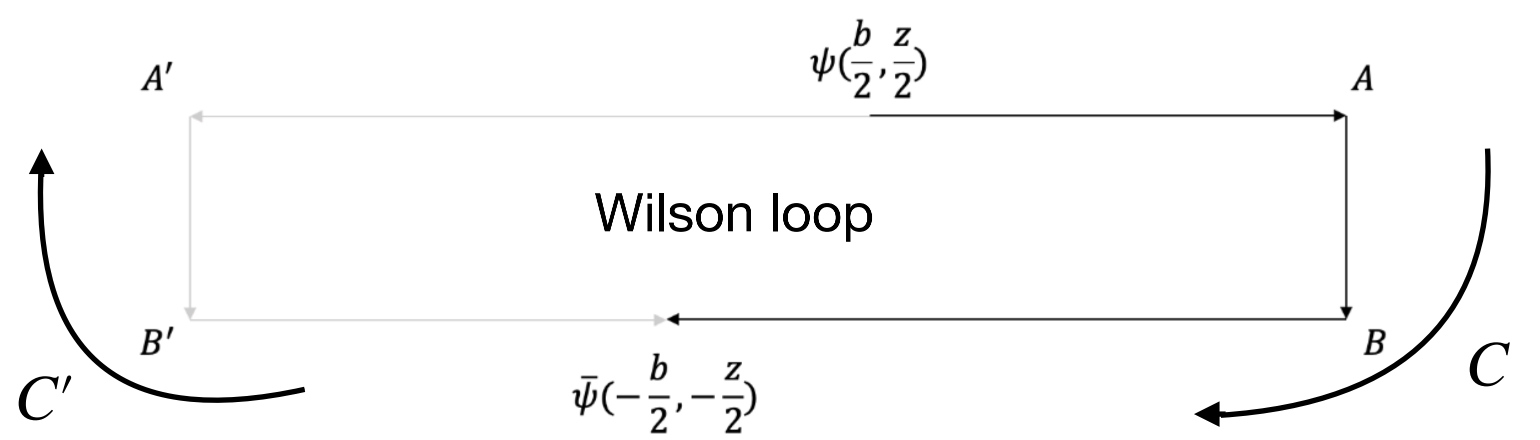
$$\chi(\overrightarrow{x}, t, b, z) = \bar{\psi}_1(\overrightarrow{x} - \frac{b}{2}\hat{x} - \frac{z}{2}\hat{z})\Gamma_1 U_c \psi_2(\overrightarrow{x} + \frac{b}{2}\hat{x} + \frac{z}{2}\hat{z})$$



Computation

Subtracted quasi-TMDWF:

$$\tilde{\phi}(z,b,P_z)_{sub} = \lim_{l\to\infty} \frac{\tilde{\phi}(l,z,b,P_z)}{\sqrt{U_{loop}}} = \lim_{l\to\infty} \frac{\langle 0|\bar{\psi}_1(-\frac{b}{2}\hat{x}-\frac{z}{2}\hat{z})\Gamma_1 U_c \psi_2(\frac{b}{2}\hat{x}+\frac{z}{2}\hat{z})|\pi(\overrightarrow{P})\rangle}{\langle 0|\bar{\psi}_1(0)\Gamma_1\psi_2(0)|\pi(\overrightarrow{P})\rangle\sqrt{\langle 0|U_{c+c'}|\pi\rangle}}$$



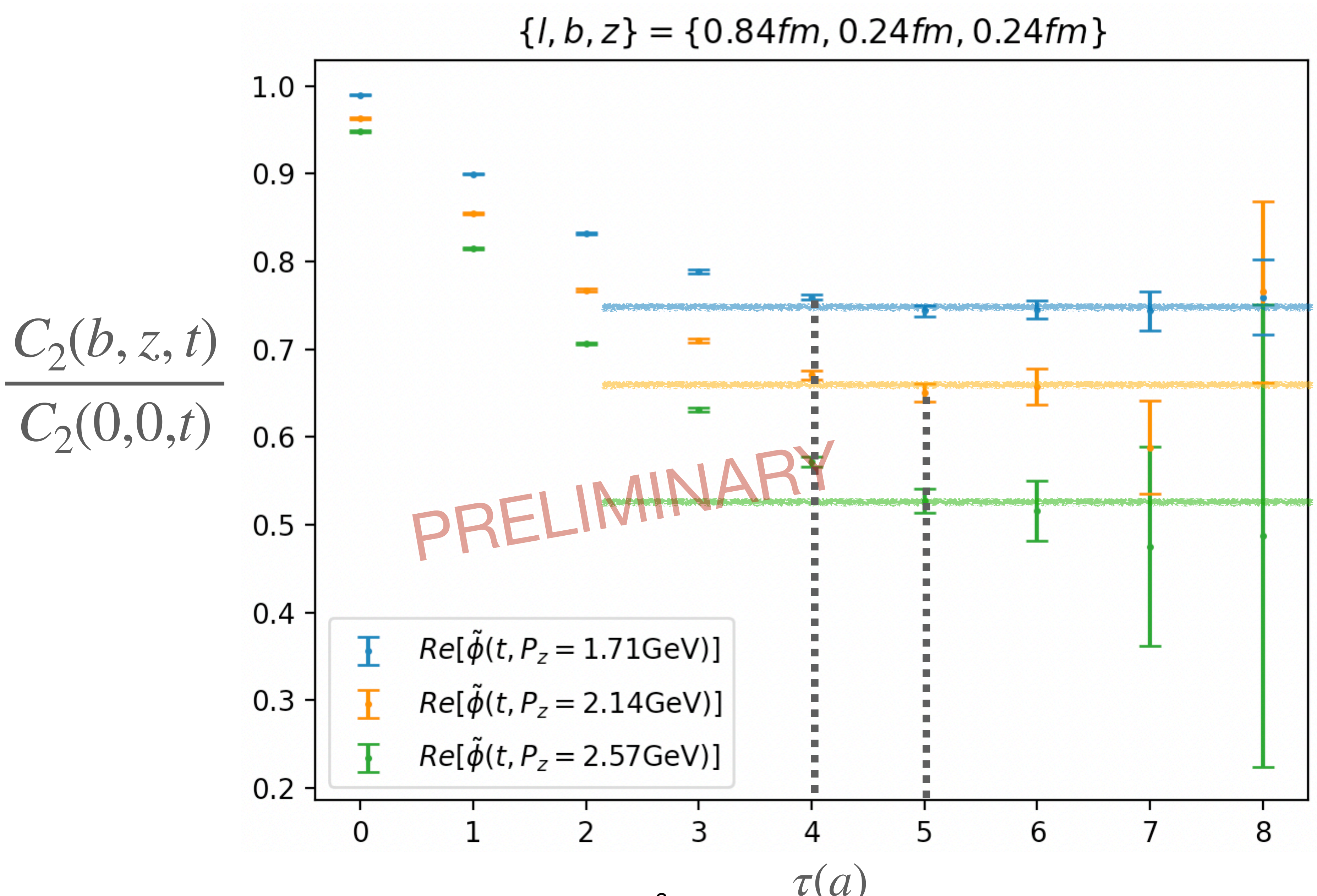
Lattice setup: MILC configurations with $a = 0.12$fm and choose three hadron momenta $P_z = \{1.72, 2.15, 2.58\}$ GeV. We used 190 configurations and in each we measured 4 times, so the total measurement is $4 \times 190 = 760$

Ensemble	a(fm)	$L^3 \times T$	$m_{\pi,sea}$(MeV)	$m_{\pi,val}$(MeV)	momentum(γ)
a12m130	0.12	48×64	140	670	$1.72GeV(2.57)$, $2.15GeV(3.21)$, $2.58GeV(3.85)$
a12m130	0.12	48×64	140	310	$1.72GeV(5.55)$, $2.15GeV(6.93)$, $2.58GeV(8.32)$

Computation

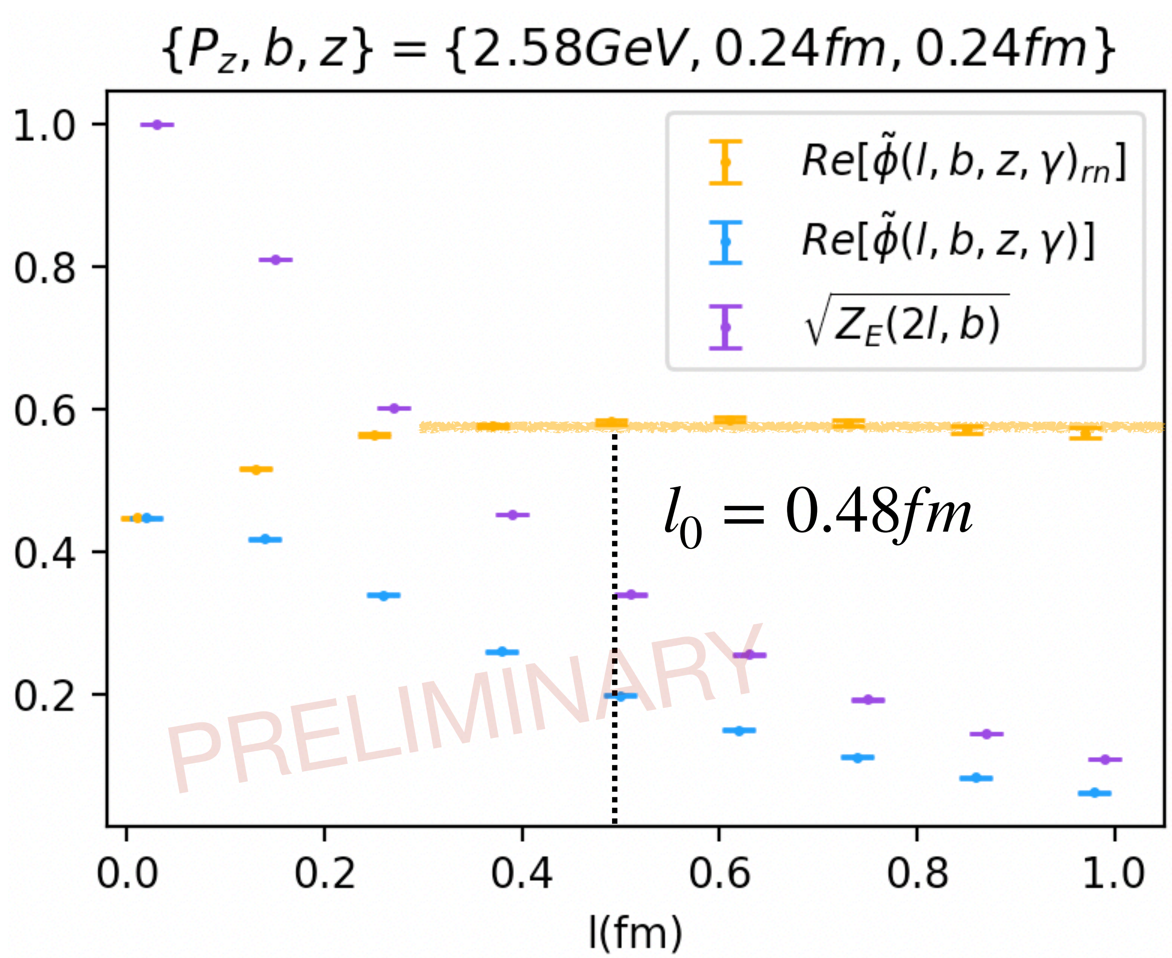
Two point correlation function $\overrightarrow{P} = (P_0,0,0,P_z)$, $\Gamma_1 = \gamma^t\gamma_5$

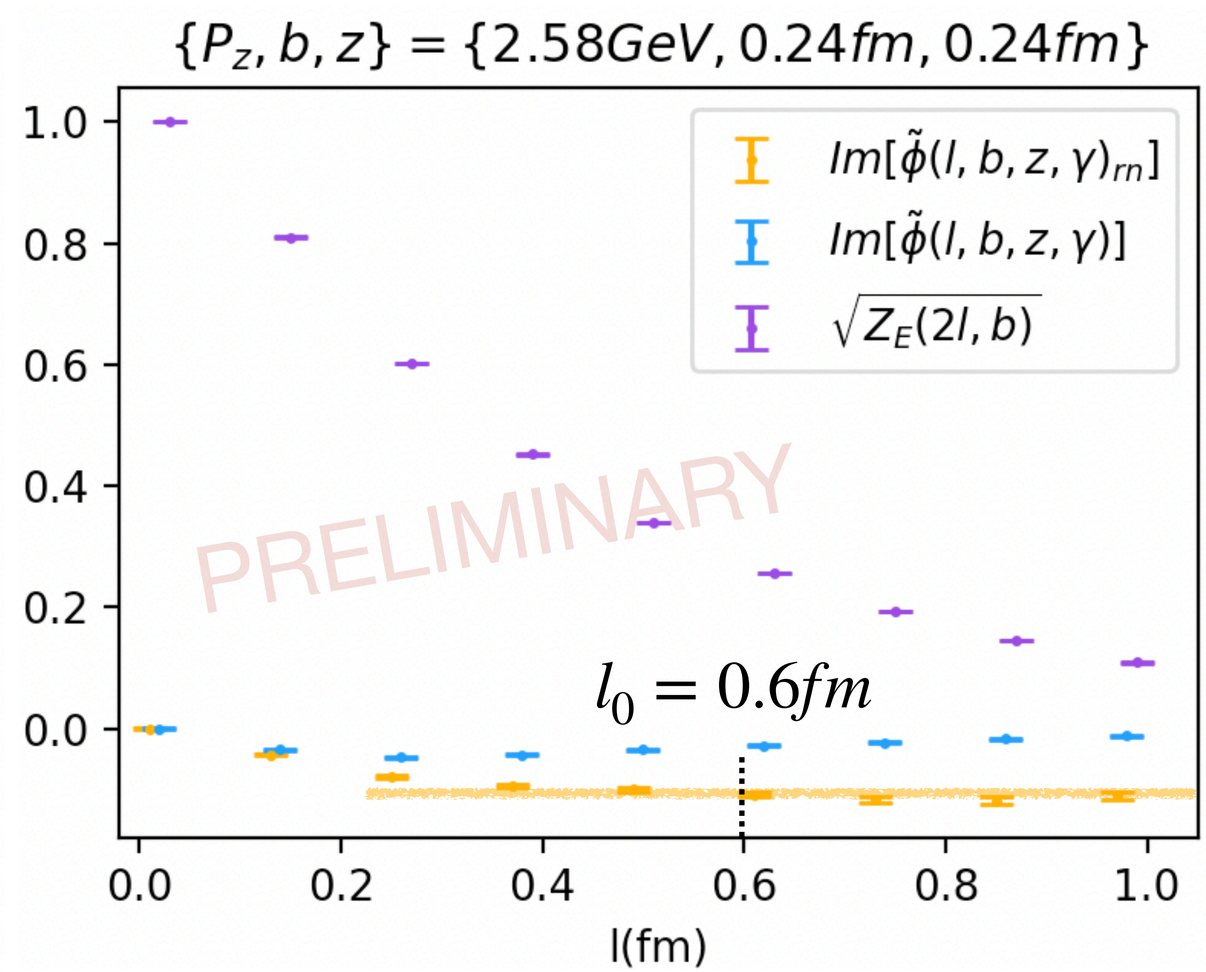
$$\frac{C_2(l,b,z,t)}{C_2(0,0,0,t)} \sim \tilde{\phi}(l,b,z)_{sub}(1 + A(l,b,z)e^{-\Delta E\tau} + \dots)$$

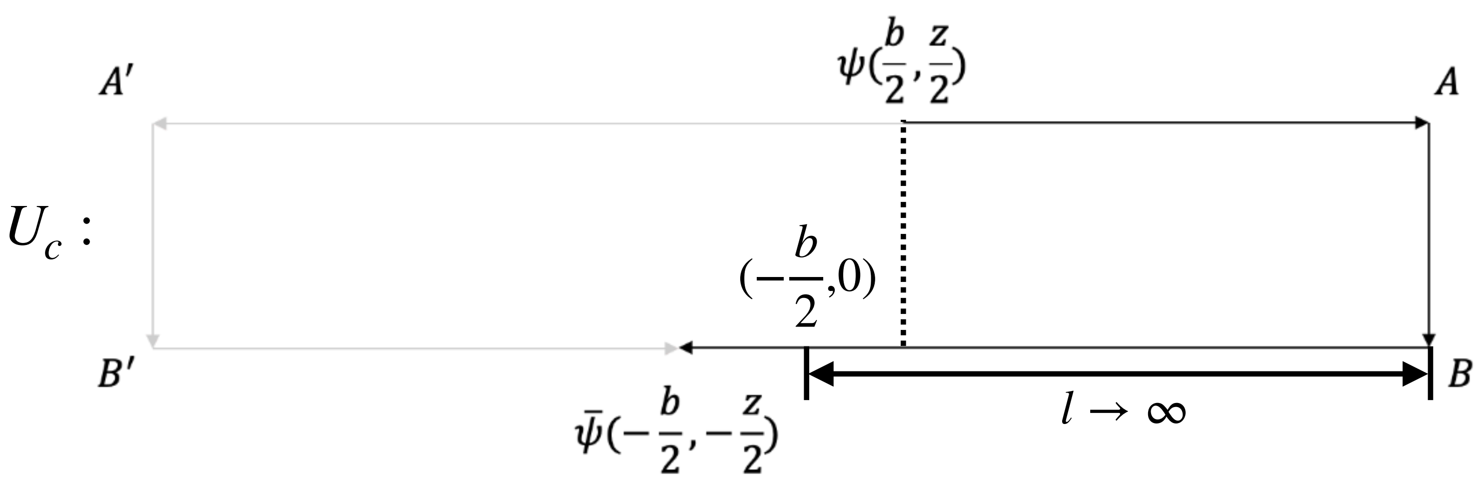


Computation

TMDWF: $\tilde{\phi}(b,z) = \lim_{l\to\infty} \tilde{\phi}(l,b,z)$





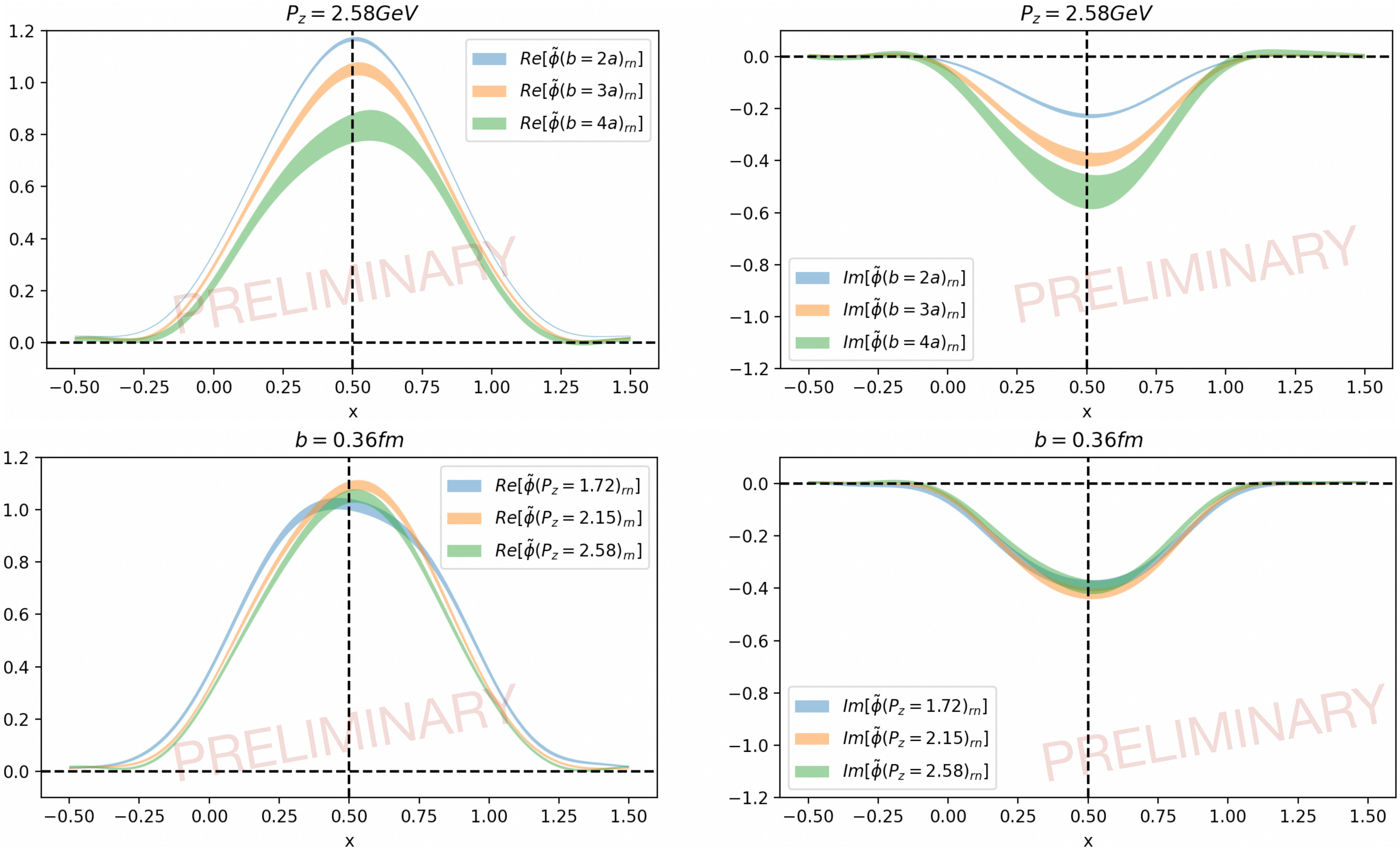


Due to $l_0 \geq \frac{z}{2}$, we choose $l_0 = \frac{z_{max}}{2}$

In our data, it is $l_0 = 6a = 0.72fm$

Fourier Transformation

TMDWF: $\tilde{\phi}(l = l_0, b, z, t = t_0) \rightarrow \tilde{\phi}(b, \boxed{z}) \xrightarrow{\text{Fourier transformation}} \tilde{\phi}(b, \boxed{x})$



Collins-Soper kernel from lattice QCD

The pz dependence of quasi-TMDWF is related to CS kernel:

X.D. Ji and Y.Z Liu, arxiv 2106.05310 (2021)

$$\phi(b,x,\zeta) = H_1^{-1}\left(4x^2P_z^2, 4(1-x)^2P_z^2\right) e^{-\frac{1}{2}\ln\left(\frac{4x^2P_z^2}{\zeta}\right)K(b)} S_r^{\frac{1}{2}}(b)\tilde{\phi}(b,x,P_z)$$

$$\downarrow$$

$$\boxed{K(b) = \frac{1}{\ln(P_2/P_1)}\ln\left[\frac{H_1(4x^2P_1^2, 4(1-x)^2P_1^2)\tilde{\phi}(b,x,P_2)}{H_1(4x^2P_2^2, 4(1-x)^2P_2^2)\tilde{\phi}(b,x,P_1)}\right]}$$

$$\downarrow$$

$$\boxed{K(b)_{tree} = \frac{1}{\ln(P_2/P_1)}\ln\left[\frac{\tilde{\phi}(b,z=0,P_2)}{\tilde{\phi}(b,z=0,P_1)}\right]}$$

Tree level: $H_1 = 1$

Fourier transformation: $\int_0^1 \tilde{\phi}(b,x)dx = \tilde{\phi}(b,z=0)$

Matching kernel up to 1-loop level:

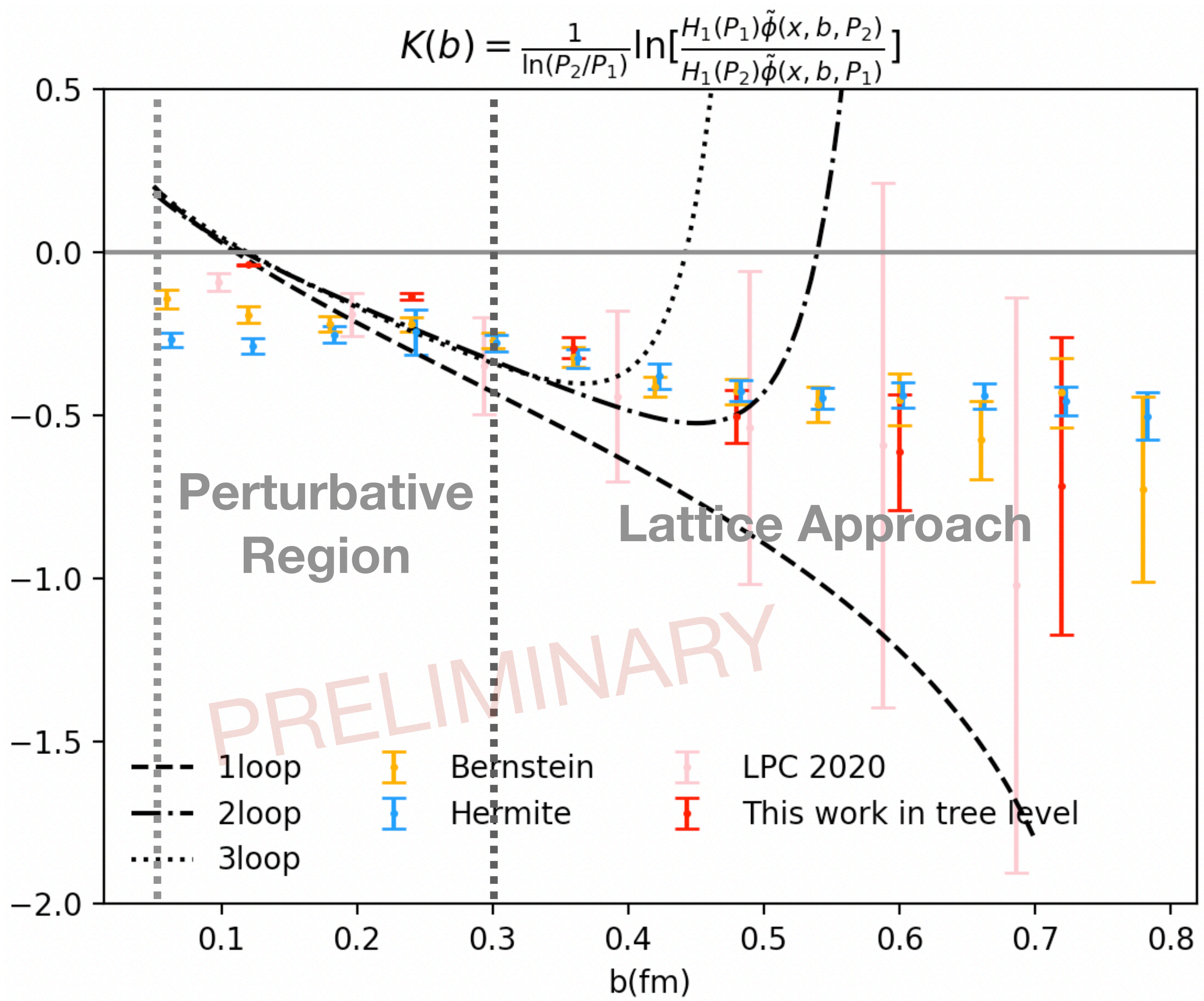
X.D. Ji and Y.Z Liu, arxiv 2106.05310 (2021)

$$H_1(l,\bar{l}) = e^h, \quad h = \frac{\alpha_s C_F}{4\pi}\left[-\frac{5\pi^2}{6} - 4 + \ln\frac{-l-i0}{\mu^2} + \ln\frac{-\bar{l}-i0}{\mu^2} - \frac{1}{2}\left(\ln^2\frac{-l-i0}{\mu^2} + \ln^2\frac{-\bar{l}-i0}{\mu^2}\right)\right] + O(\alpha_s^2)$$

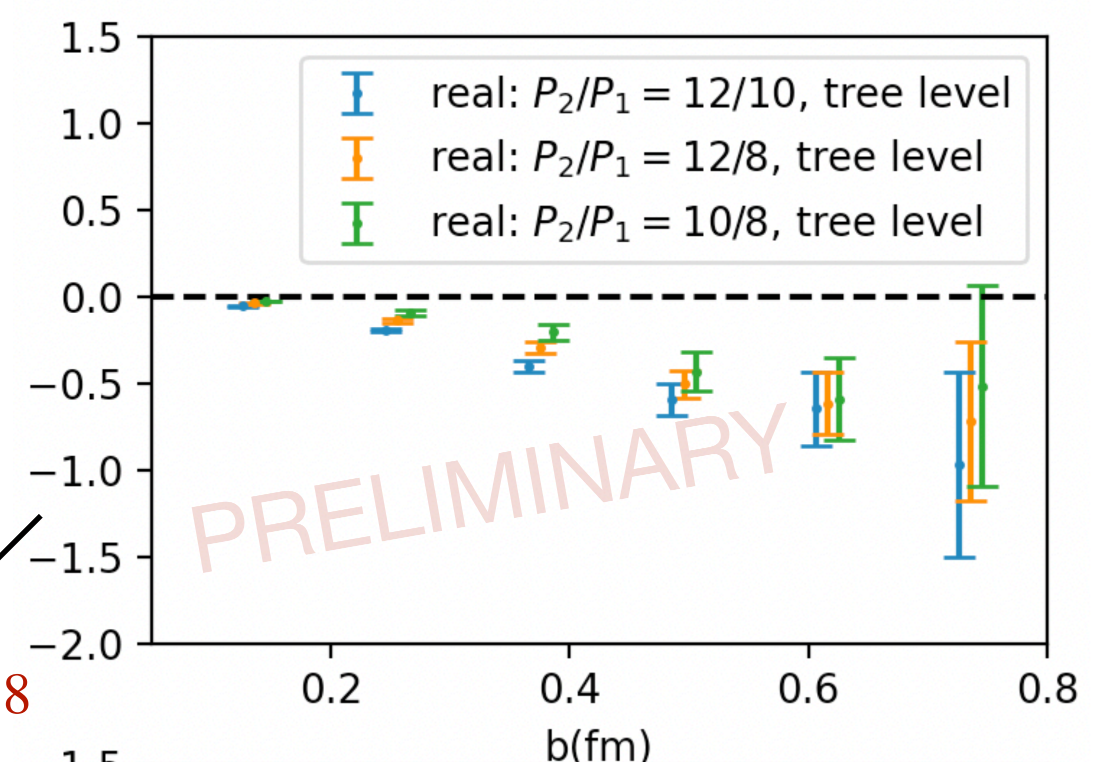
Numerical Results

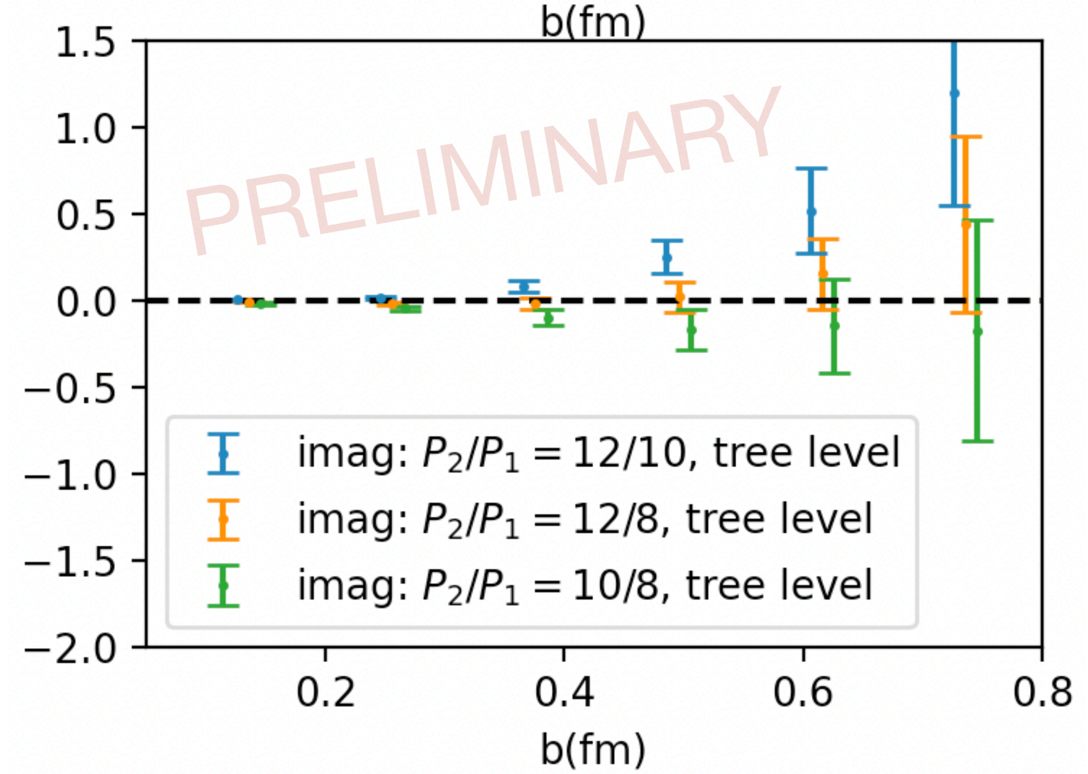
Collins-Soper kernel (tree level): $K(b)_{tree} = \frac{1}{\ln(P_2/P_1)} \ln\left[\frac{\tilde{\phi}(b, z=0, P_2)}{\tilde{\phi}(b, z=0, P_1)}\right]$

- Collins-Soper kernel $K(b_\perp)$ is almost real.
- Stable with different momentum combinations.



real part
$P_2/P_1 = 12/8$





Promote to 1-loop level (in progress)

- Collins-Soper kenel:

$$K(b_\perp) = \frac{1}{\ln(P_2/P_1)} \ln \left[\frac{H_1(4x^2P_1^2, 4(1-x)^2P_1^2)\tilde{\phi}(b,x,P_2)}{H_1(4x^2P_2^2, 4(1-x)^2P_2^2)\tilde{\phi}(b,x,P_1)} \right]$$

- Matching kernel up to 1-loop level:

X.D. Ji and Y.Z Liu, arxiv 2106.05310 (2021)

$$H_1(l,\bar{l}) = e^h, \quad h = \frac{\alpha_s C_F}{4\pi}\left[-\frac{5\pi^2}{6} - 4 + \ln\frac{-l-i0}{\mu^2} + \ln\frac{-\bar{l}-i0}{\mu^2} - \frac{1}{2}\left(\ln^2\frac{-l-i0}{\mu^2} + \ln^2\frac{-\bar{l}-i0}{\mu^2} \right) \right] + O(\alpha_s^2)$$

Kernel H_1 is given with an imaginary part, how to deal with it numerically.

- Momentum fraction dependence:

No momentum fraction dependence in tree level, what about 1-loop level?

Summary and outlook

- **Collins-Soper kernel can be extracted from TMDWF non-perturbatively.**
- **We are trying to compute Collins-Soper kernel through TMDWF with 1-loop matching kernel.**
- **Promote our results to continuum limit and physical pion mass.**
- **Collins-Soper kernel from the first principle can be used in global fit of the TMDPDFs. It will reveal the internal structure of hadrons.**

Thanks for your attention!