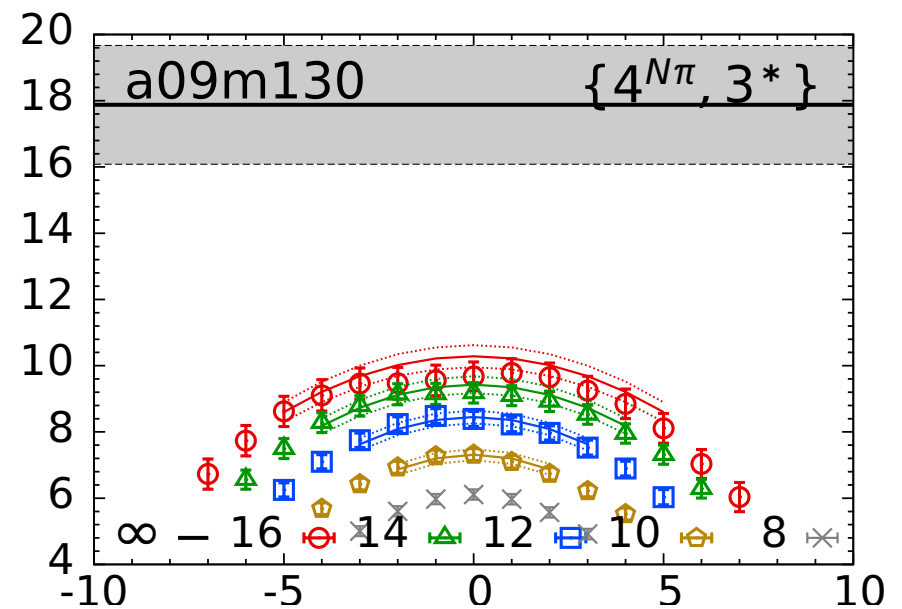
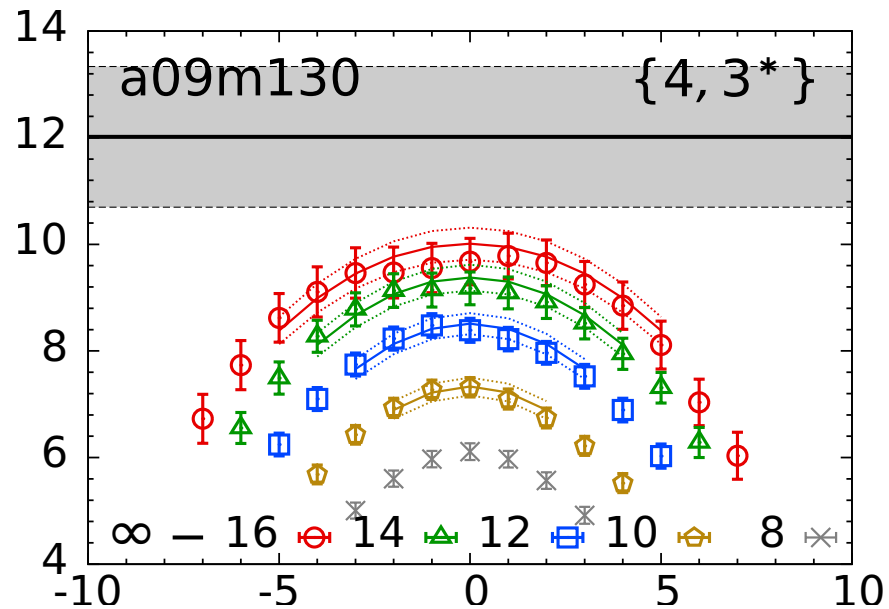


# Excited states and precision results for nucleon charges and form factors

Rajan Gupta

Work done with T. Bhattacharya, V. Cirigliano, M. Hoferichter, Y-C. Jang, B. Joo, H-W. Lin, E. Mereghetti, S. Mondal, S. Park, F. Winter, B. Yoon



# Discussion based on

- AFF: Phys Rev Lett 124 (2019) 072002
- $\Theta$ -term: Phys Rev D103 (2021) 114507
- $\sigma$ -term: arXiv:2105.12095
- $g_i$  and FF: arXiv:2103.05599

# Outline

- Excited States (ES) in nucleon correlation functions
- Intuitive understanding of enhanced contributions from  $N\pi$ ,  $N\pi\pi$ , ... states
- Three strategies
- Examples
  - PCAC relation and axial form factors
  - Axial charge
  - Electric and magnetic form factors
  - Nucleon  $\sigma$ -term
  - $\Theta$ -term contribution to neutron EDM
- Summary: Which ES contribute to which NME?

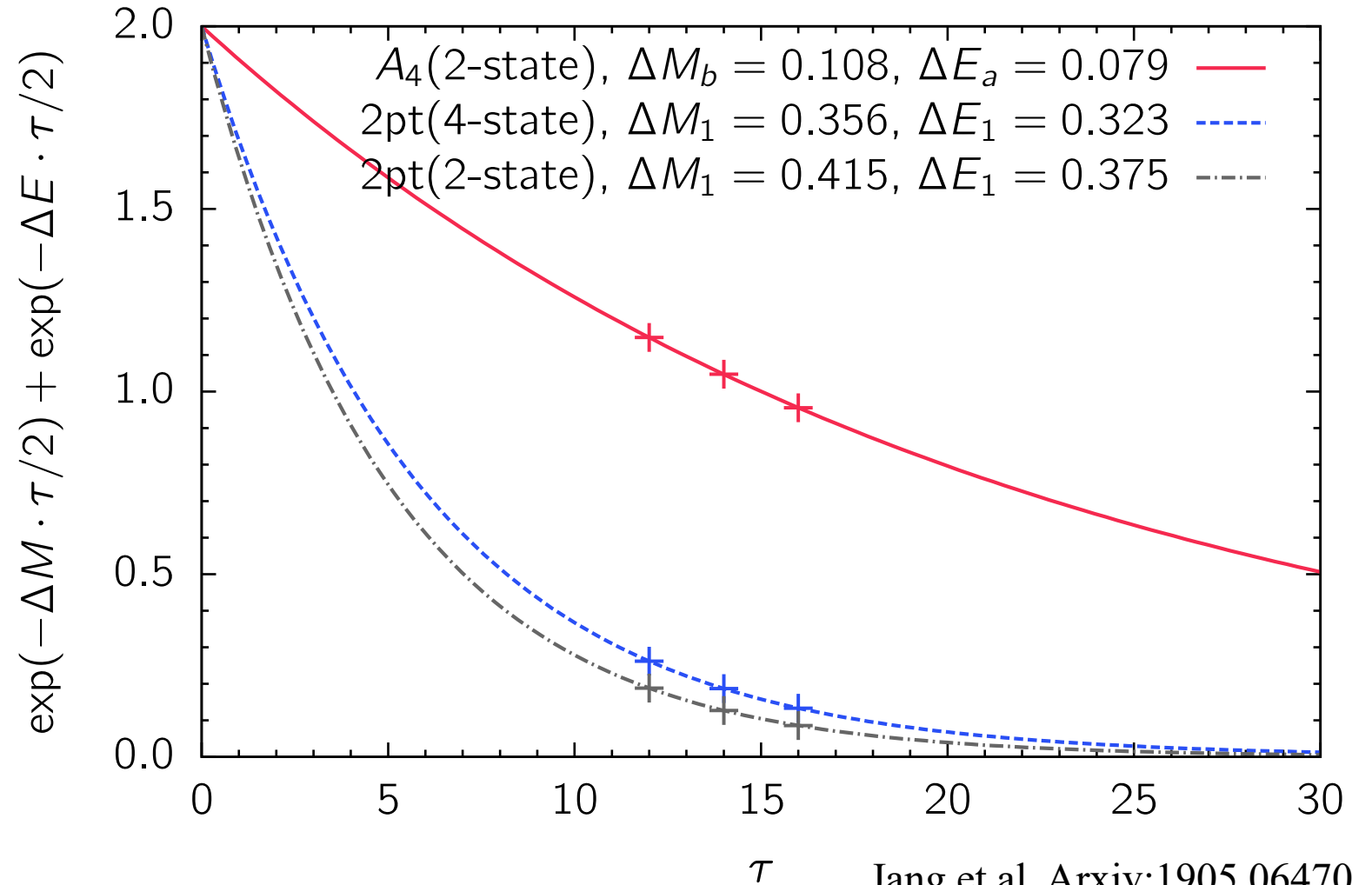
# Excited States in Nucleon Correlation Functions

- Statistical signal dies as  $e^{-(M_N - 1.5M_\pi)t}$
- Towers of multi-hadron excited states start at  $\sim 1200$  MeV
  - $N\pi$ ,  $N\pi\pi$ , ...
  - Arbitrarily dense as  $\vec{p} \rightarrow 0$
  - Difference in mass gap increases as  $\vec{p} \rightarrow 0$  and  $M_\pi \rightarrow 135$  MeV
  - Contribution increases as  $M_\pi \rightarrow 135$  MeV
- Which states contribute significantly?
- With current statistics, can fit the spectral decomposition with up to 3 states.  $\chi^2$  does not discriminate between fits!

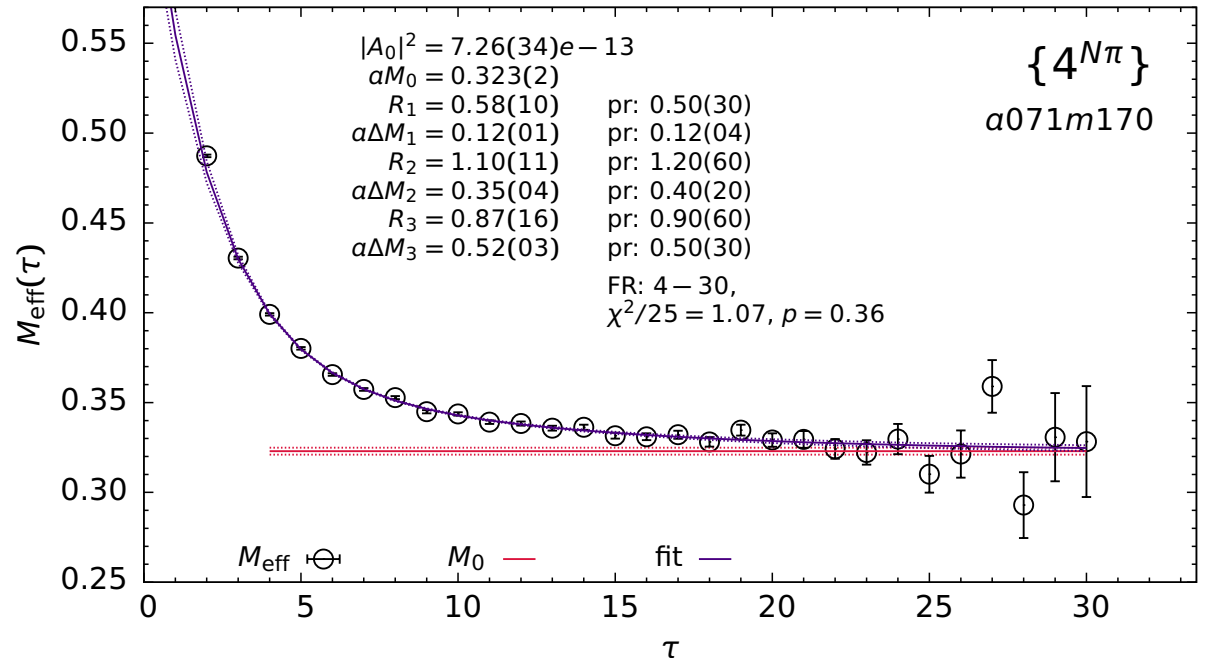
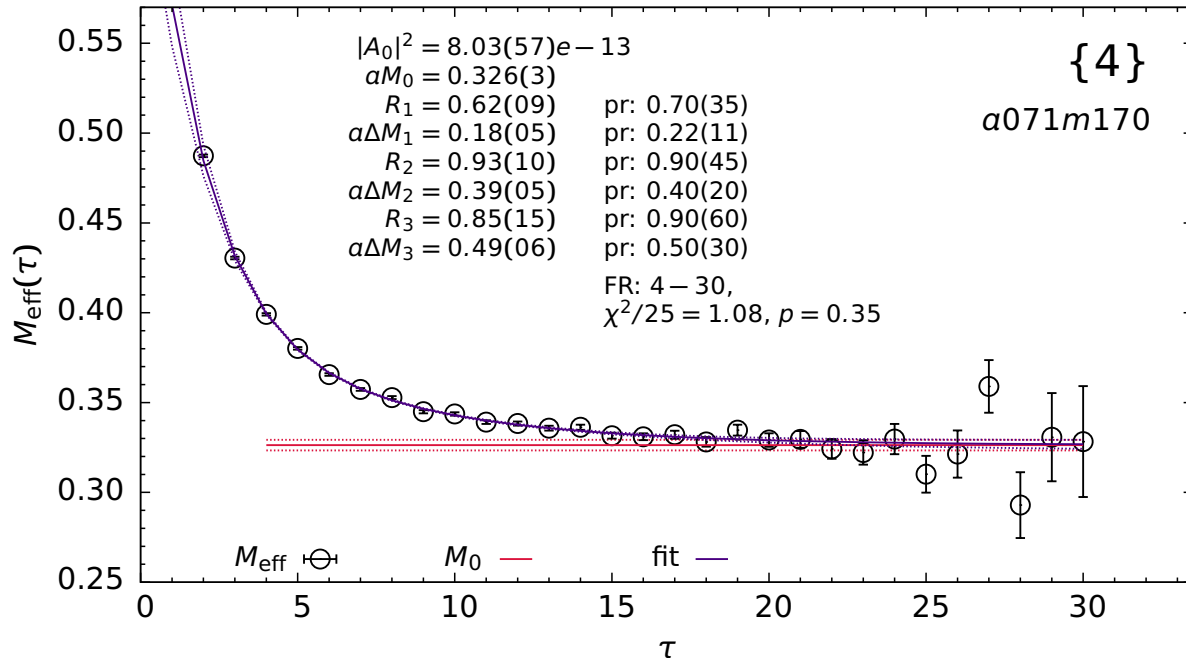
# Fit $\chi^2$ do not discriminate for $1 \lesssim \tau \lesssim 1.5$ fm

A cartoon of 1<sup>st</sup> ESC contribution assuming amplitude  $A_I = 1$

- $\tau = 12, 14, 16$
- The behavior is almost linear
- 3 curves differ in value
- At small  $\tau$  need more states



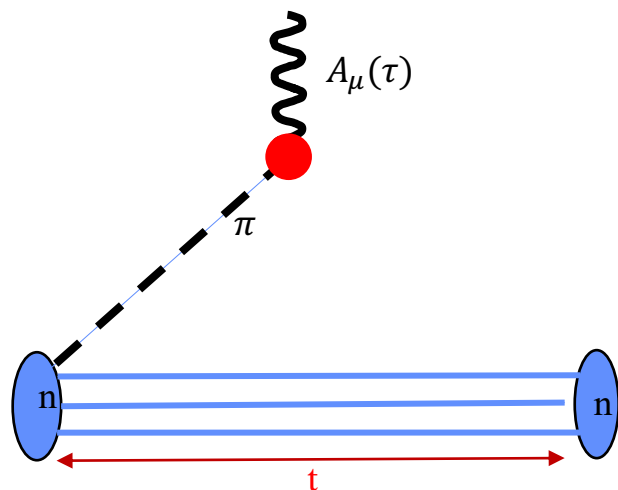
# Nucleon 2-point function



- Signal extends to  $\approx 2$  fm
- Even ground state mass  $M_0$  sensitive (few %) to including  $N\pi$  state
- A large region in  $M_i$  and  $A_i$  space that gives similar  $\chi^2$

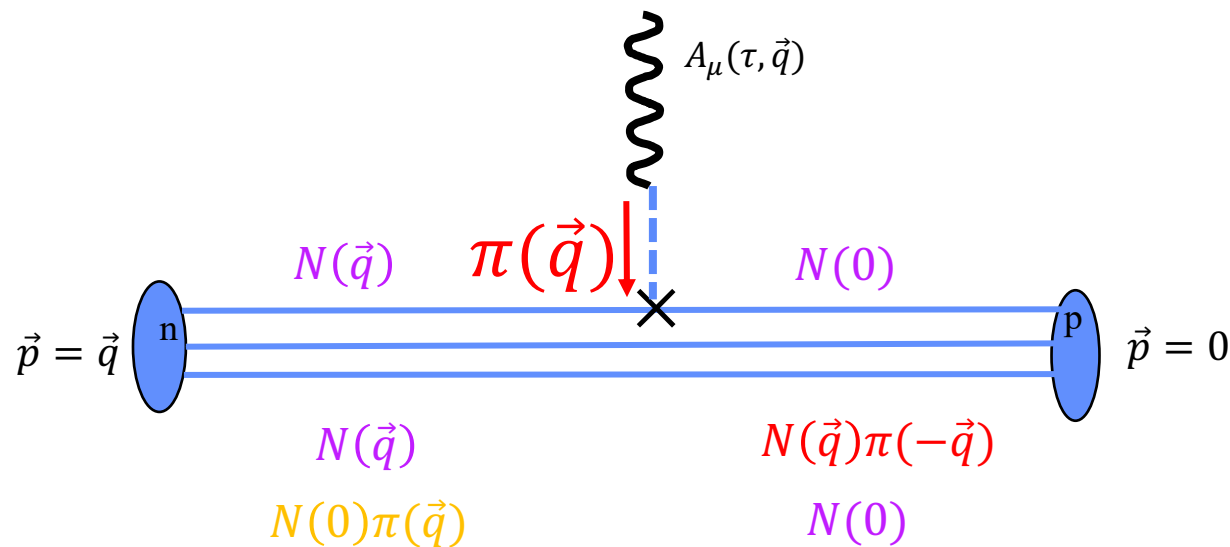
# Intuitive understanding of enhancement

## Axial Channel

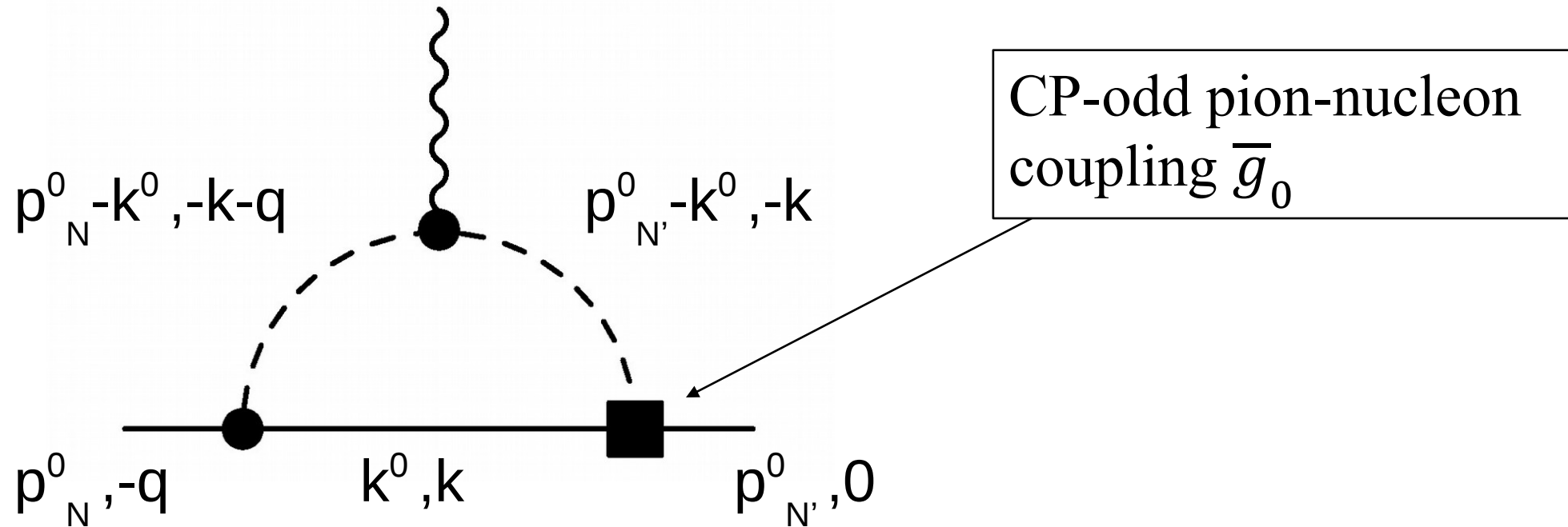


Since the pion is light,  
the vertex ● can be  
anywhere in the 3-volume

## Pion Pole Dominance



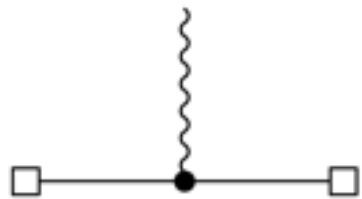
# $\Theta$ -term contribution to neutron EDM



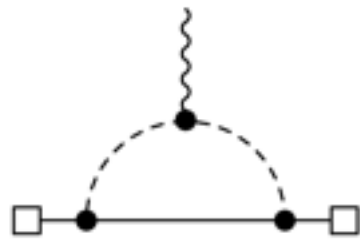
This loop term ( $N\pi$  contribution) is not suppressed w.r.t the ground state



Nucleon  $\sigma$ -term  $\sigma_{ud} = m_{ud} g_S^{u+d}$

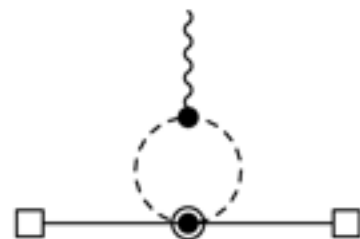


LO



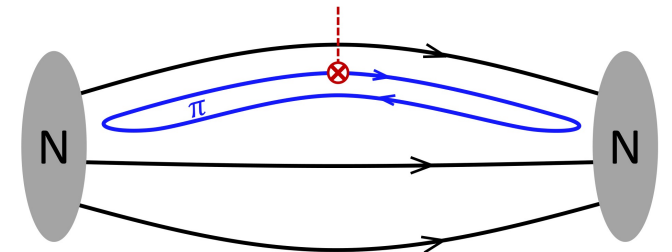
NLO

Large coupling  
of  $O_S$  to 2 pions



NNLO

Large LEC  
 $c_1, c_2, c_3$



arXiv:2105.12095

# 3 Strategies

## S1: Ward Identities

- AFF (PCAC/PPD)
- VFF (VMD)

## S2: $\chi$ PT

- AFF (PCAC)
- $\Theta$ -term
- Nucleon  $\sigma$ -term

## S3: What if

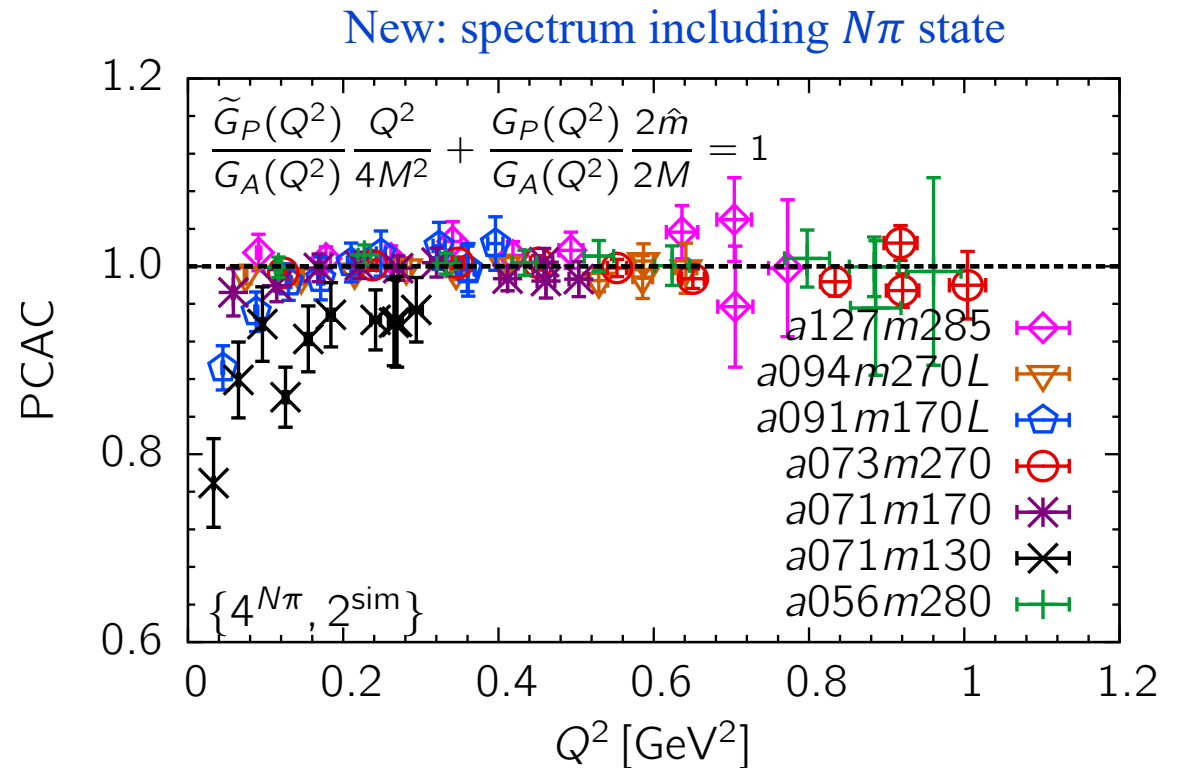
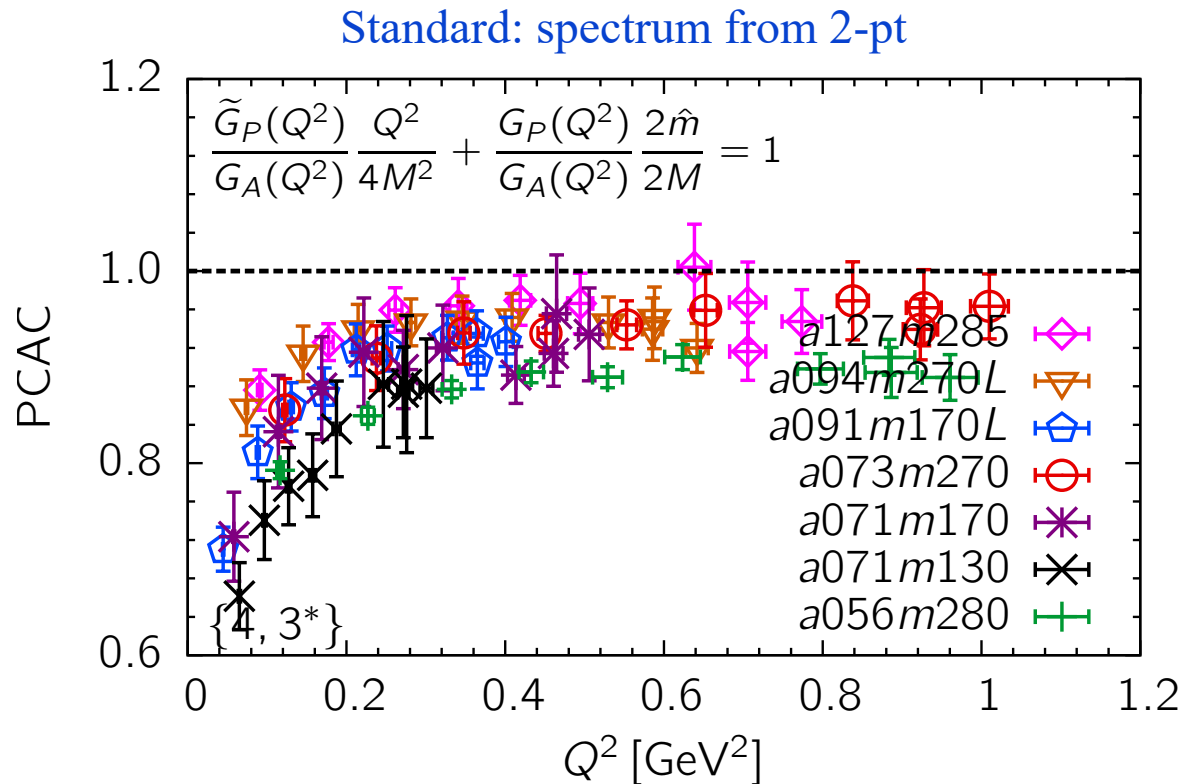
### $N\pi$ contributes

- All NME

**S3 recognizes that  $N\pi$ ,  $N\pi\pi$ , .... states are present at loop level in all NME and provides sensitivity to their contribution**

# Axial Form Factors: Satisfying PCAC

Preliminary data from 2+1-flavor clover ensembles

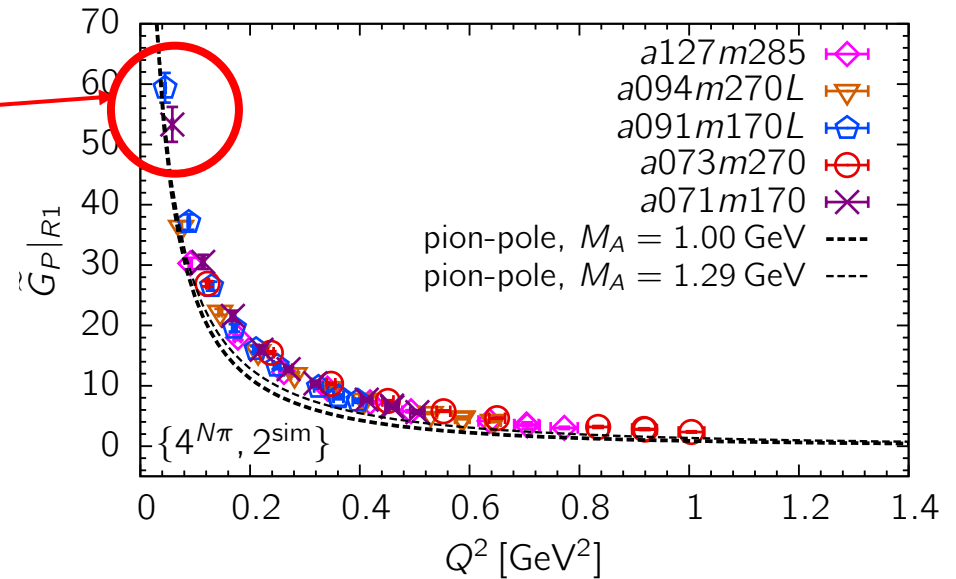
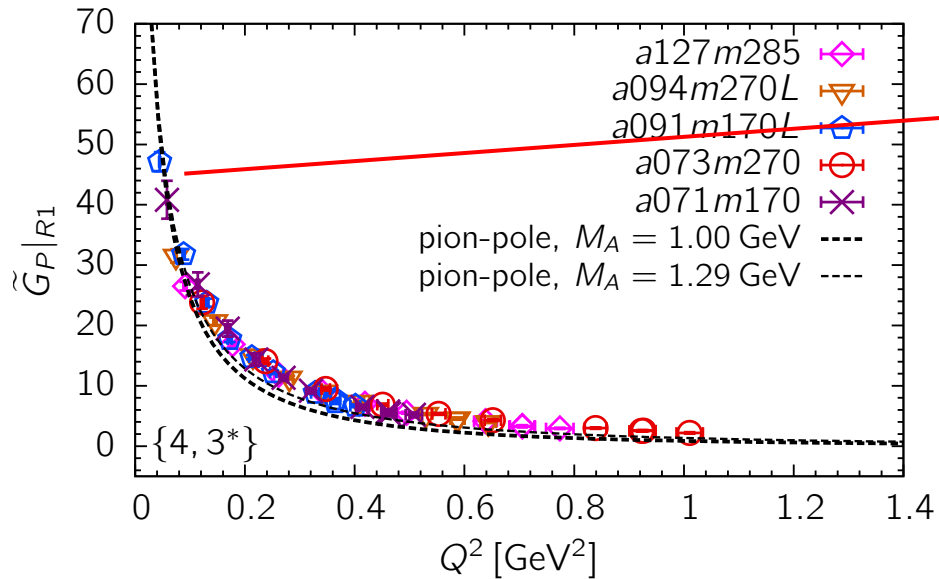
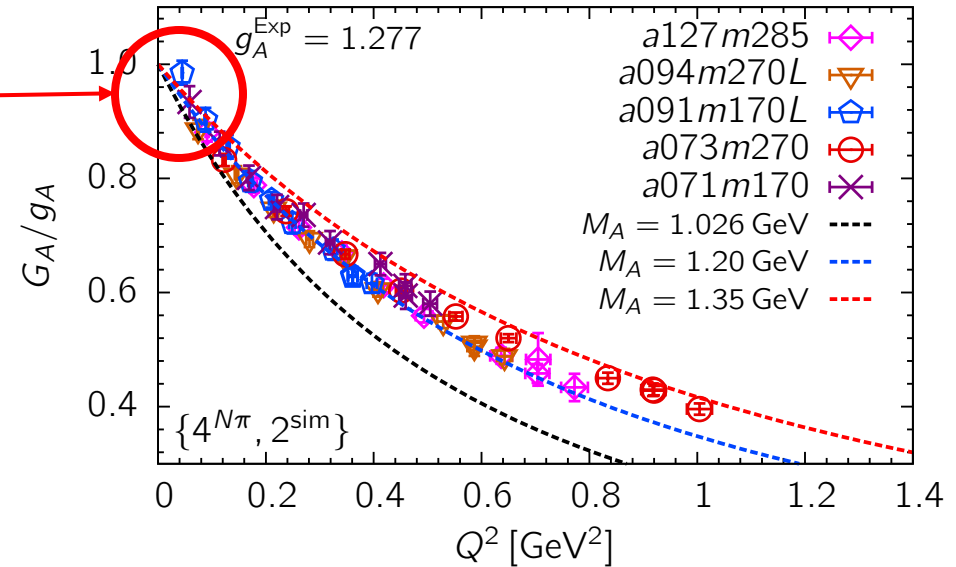
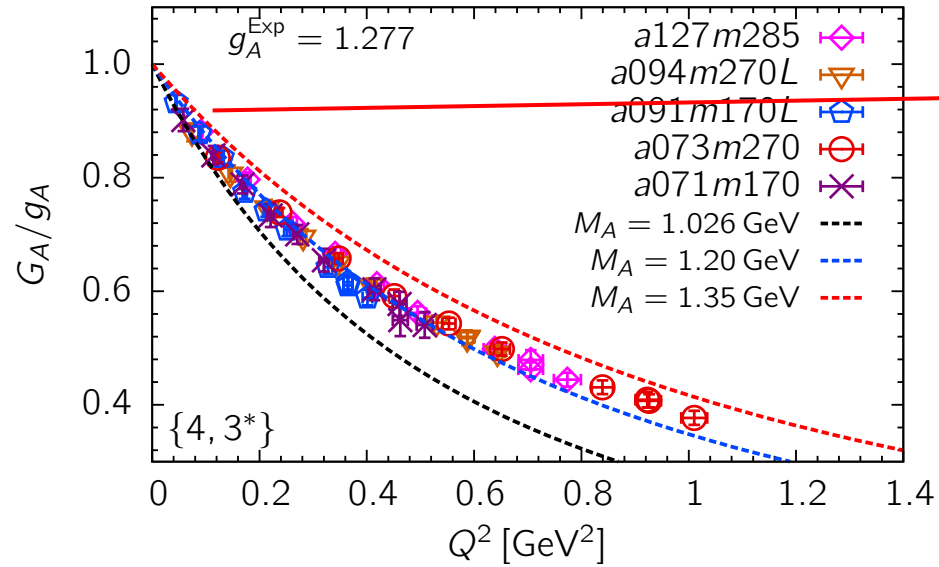


Update of plots in arXiv:2021.05599 with 2 additional ensembles

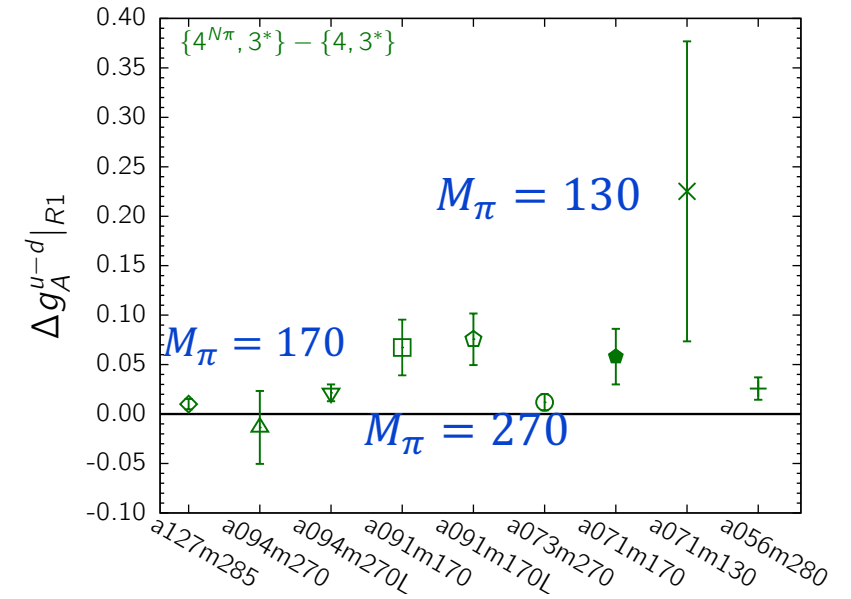
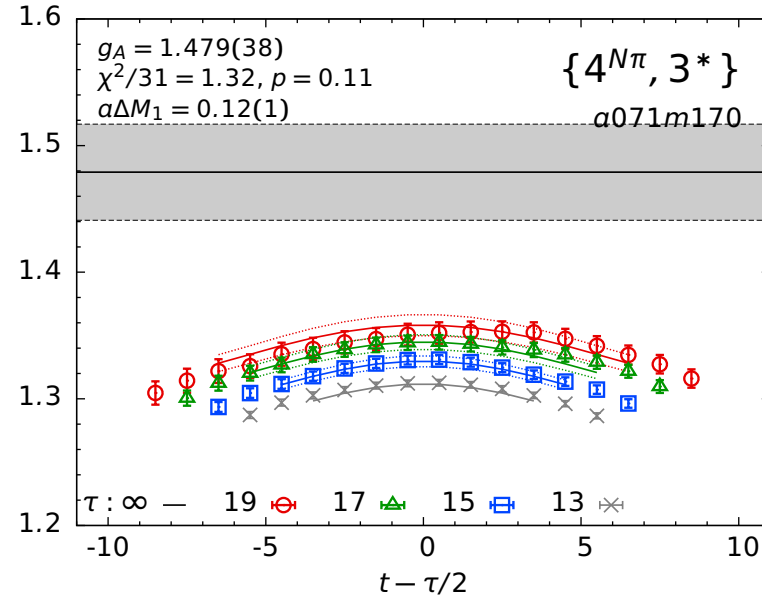
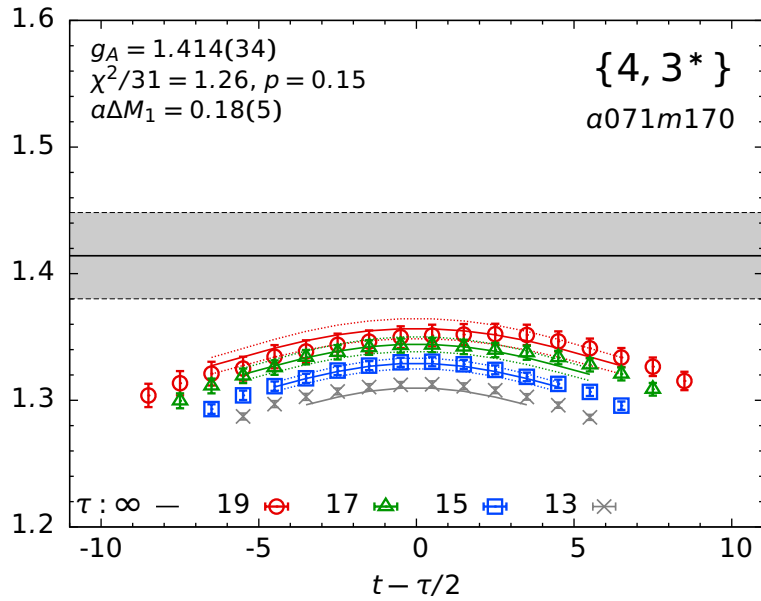
# Change on removing $N\pi$ contamination: $G_A$ ( $\sim 5\%$ ) $\tilde{G}_P$ ( $\sim 35\%$ )

Standard: spectrum from 2-pt

New: spectrum including  $N\pi$  state



# Axial charge $g_A^{u-d}$ (Include $N\pi$ or not?): S3



The effect increases as  $M_\pi \rightarrow 135$  MeV

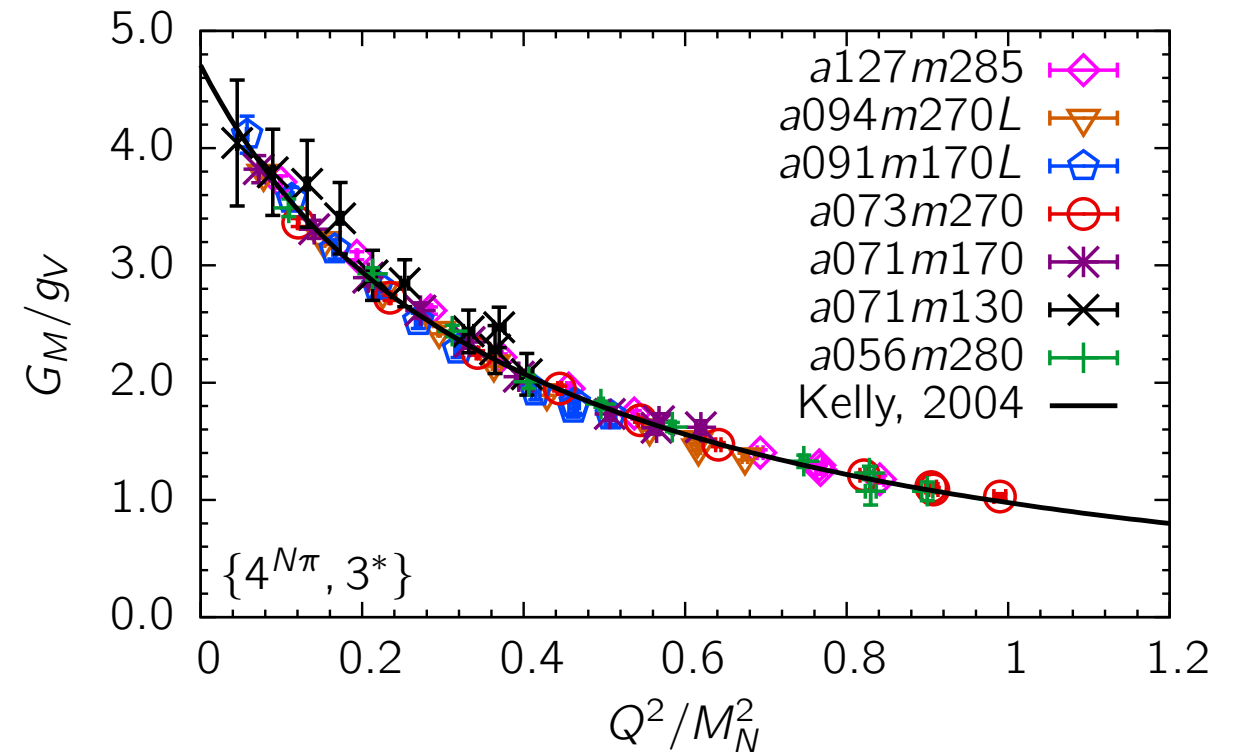
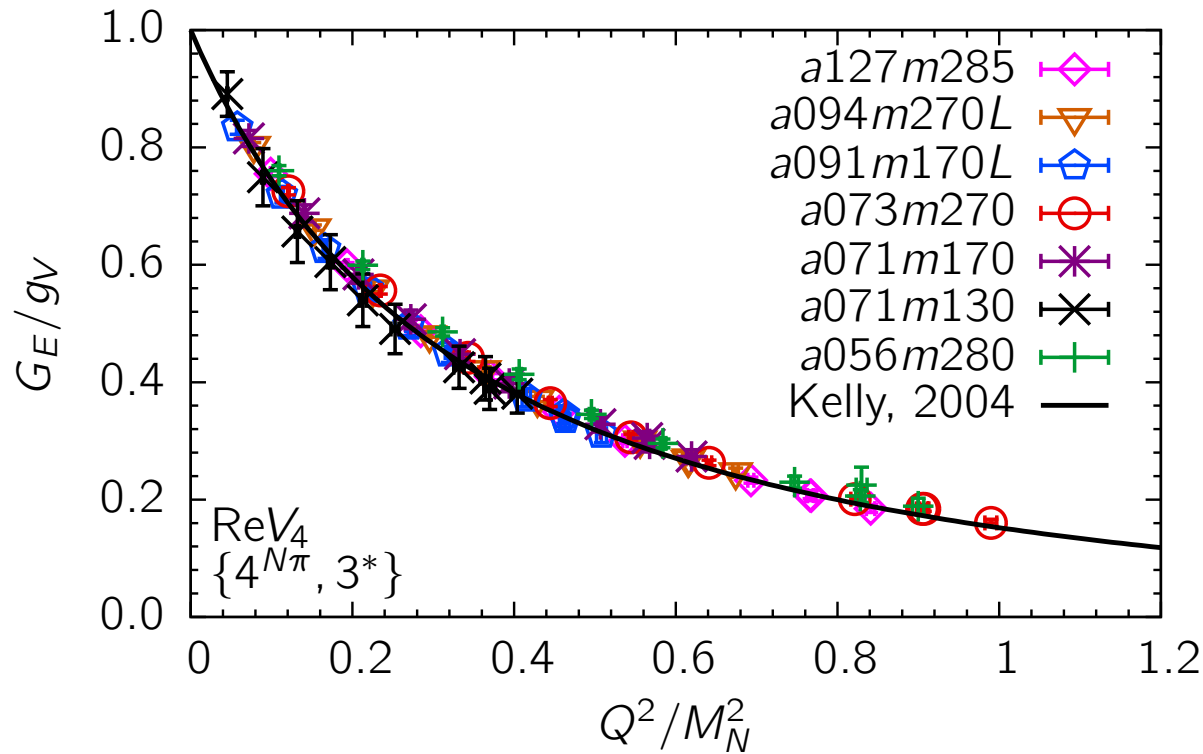
Including  $N\pi$  state in the analysis increases  $g_A^{u-d}$  by  $\sim 8\%$

$g_A^{u-d}$  ( $N\pi$ ) agrees with extrapolation of  $G_A(Q^2 \rightarrow 0)$

Need to increase statistics on the  $M_\pi = 130$  MeV ensemble

# Electric and Magnetic Form Factors

## Preliminary data from 2+1-flavor clover ensembles

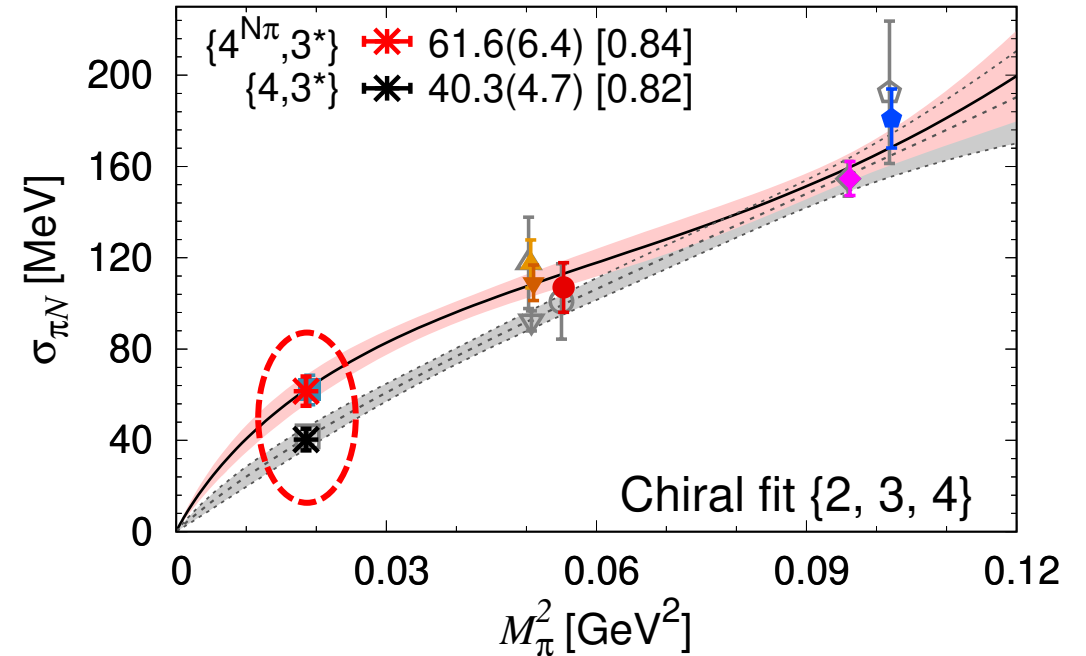


Update of arXiv:2021.05599 with 2 additional ensembles

# Scalar matrix elements: nucleon $\sigma$ -term $\sigma_{ud} = m_{ud} g_S^{u+d}$

arXiv:2105:12945

$\chi$ PT analysis shows large contributions from  $N\pi$  and  $N\pi\pi$  excited states



Without  $N\pi$ :  $\sigma_{ud} \approx 40$  MeV

With  $N\pi$ :  $\sigma_{ud} \approx 60$  MeV.

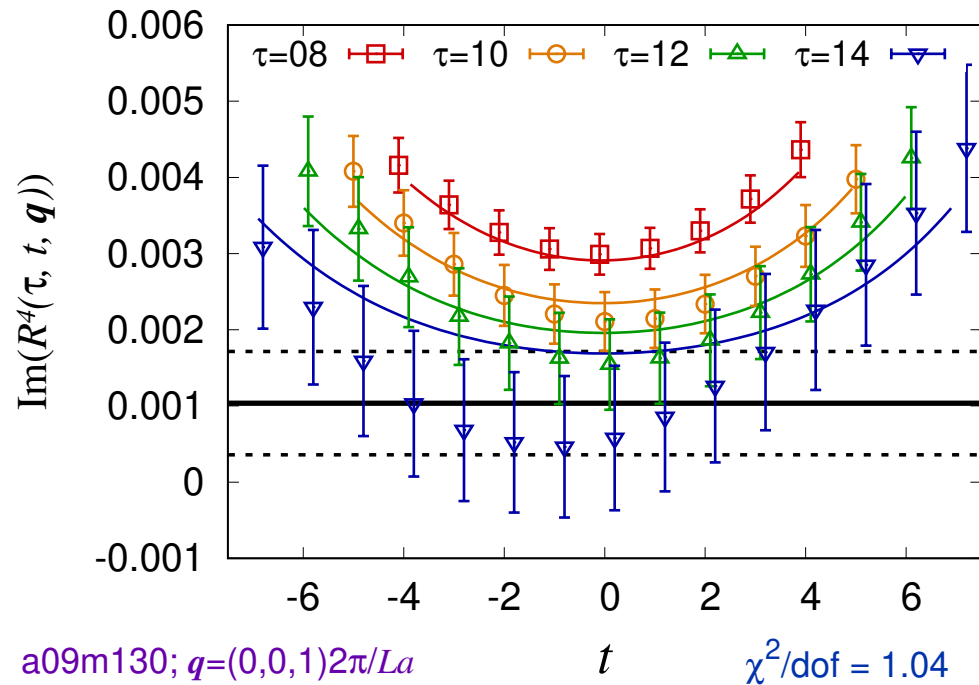
Consistent with results using dispersion theory

See Sungwoo Park's talk

# $\Theta$ -term contribution to nEDM: $\chi$ PT

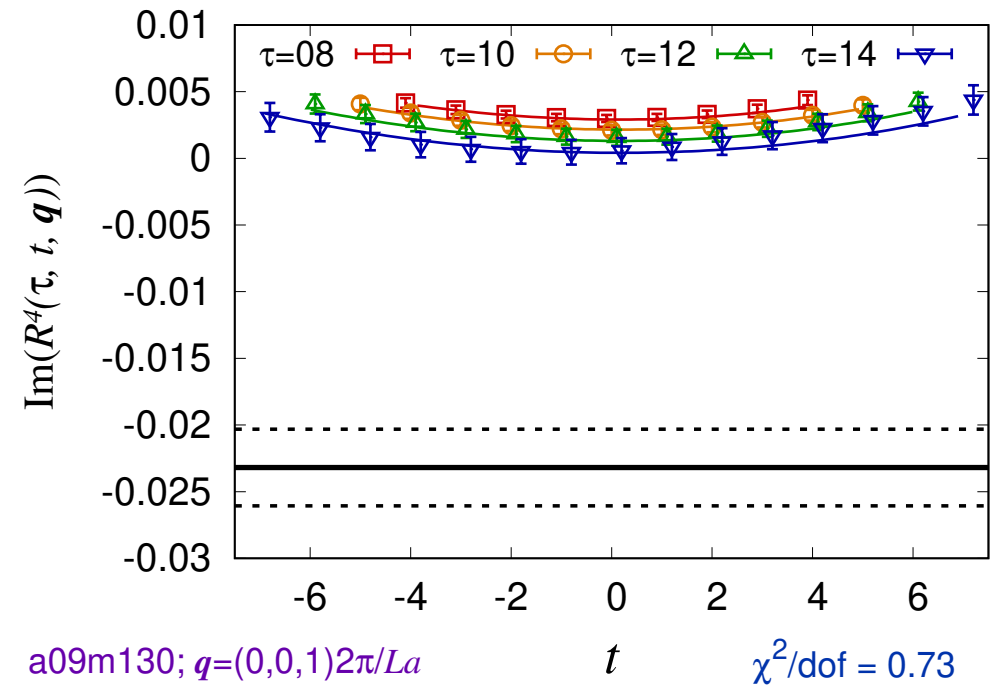
Phys Rev D103 (2021) 114507

Standard: spectrum from 2-pt



$$d_n = -0.003(7)(30) \bar{\Theta} \text{ e fm}$$

New: spectrum including  $N\pi$  state



$$d_n = -0.028(18)(54) \bar{\Theta} \text{ e fm}$$

See Tanmoy Bhattacharya's talk



# Summary

- $N\pi, N\pi\pi, \dots$  excited states contribute to all NME
- $\chi$ PT: these contributions are enhanced for a number of NME
  - Axial Form Factors & PCAC
  - $g_A^{u-d}$  ( $\approx 8\%$ ?)
  - Nucleon  $\sigma$ -term
  - $\Theta$ -term contribution to nEDM
  - ???
- Contributions becomes more significant as  $M_\pi \rightarrow 135$  MeV.
- $\sigma_{\pi N}$  changes from  $\sim 40$  MeV to  $\sim 60$  MeV [arXiv:2105.12095]
- Increase statistics, especially on the  $a071m130$  ensemble
- Increase  $\{a, M_\pi\}$  points