

Transversity PDF of nucleon using pseudo-distribution approach

Lattice 2021 @ MIT

July 28, 2021

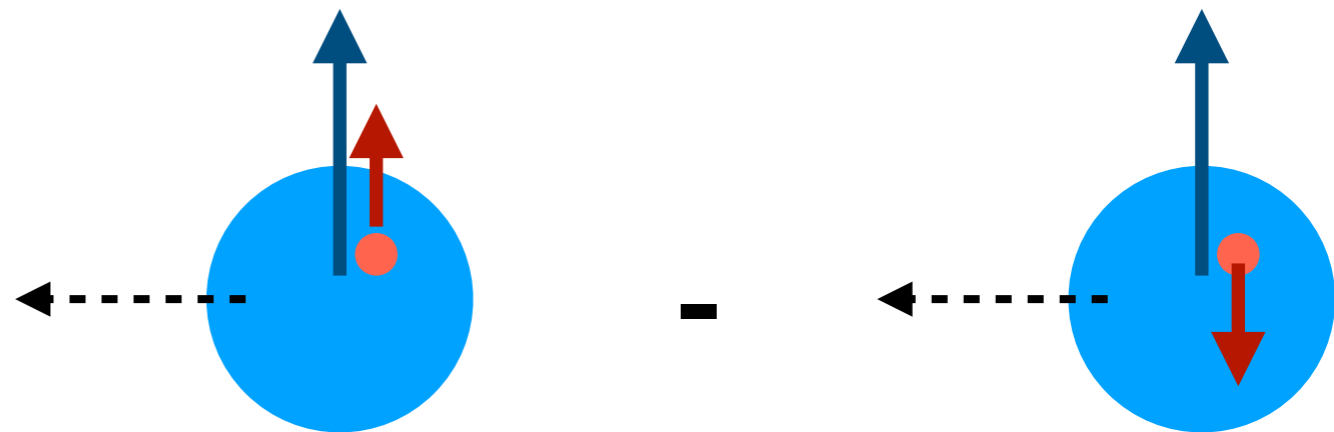
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William & Mary - Jefferson Lab

On behalf of HadStruc Collaboration

Transversity PDF

$$h_1(x) = f_{\uparrow}(x) - f_{\downarrow}(x)$$



Chiral-odd

Need to couple to chiral-odd process

Accessible via semi-inclusive DIS

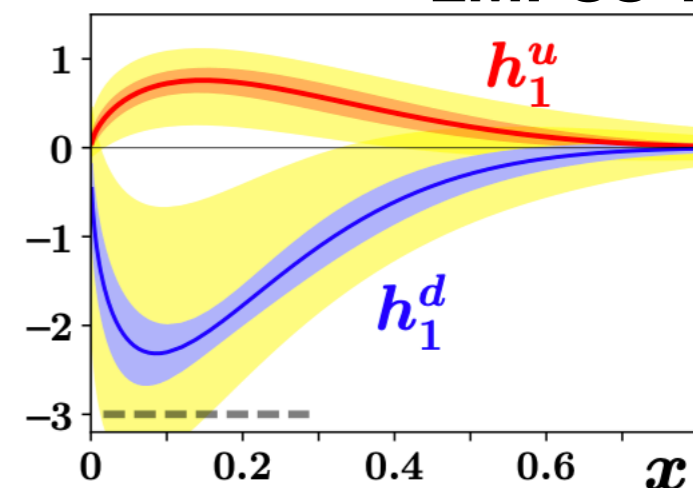
$$\ell + N \rightarrow \pi^+ + \pi^- + \ell + X$$

Important goal of JLab 12 GeV upgrade

The least pheno-constrained twist-2 quark PDF → Opportunity for lattice!

LMPSS'17

This work: Obtain x -dependent transversity PDF using perturbative matching of Euclidean M.E. (pseudo-distribution) to light-cone transversity PDF



- A good choice of kinematic variables and directions for pseudo-PDF approach:

$$\left\langle N; P_z, S_T \left| \bar{\psi}(z) \gamma_5 \gamma_t \gamma_T W(z, 0) \tau_3 \psi(0) \right| N; P_z, S_T \right\rangle = 2ES_T \mathcal{M}(zP_z, z^2)$$

- Renormalize by ratio: $\mathfrak{M}(zP_z, z^2) = \frac{\mathcal{M}(z, P_z)}{\mathcal{M}(z, 0)} \rightarrow \langle x^0 \rangle = \int_0^1 h_1(x, \mu) dx = 1$
K. Orginos et al, '17
(A trivial overall normalization g_T deferred)

- Capture the zP_z and z^2 dependence via leading-twist factorization / lead-twist OPE

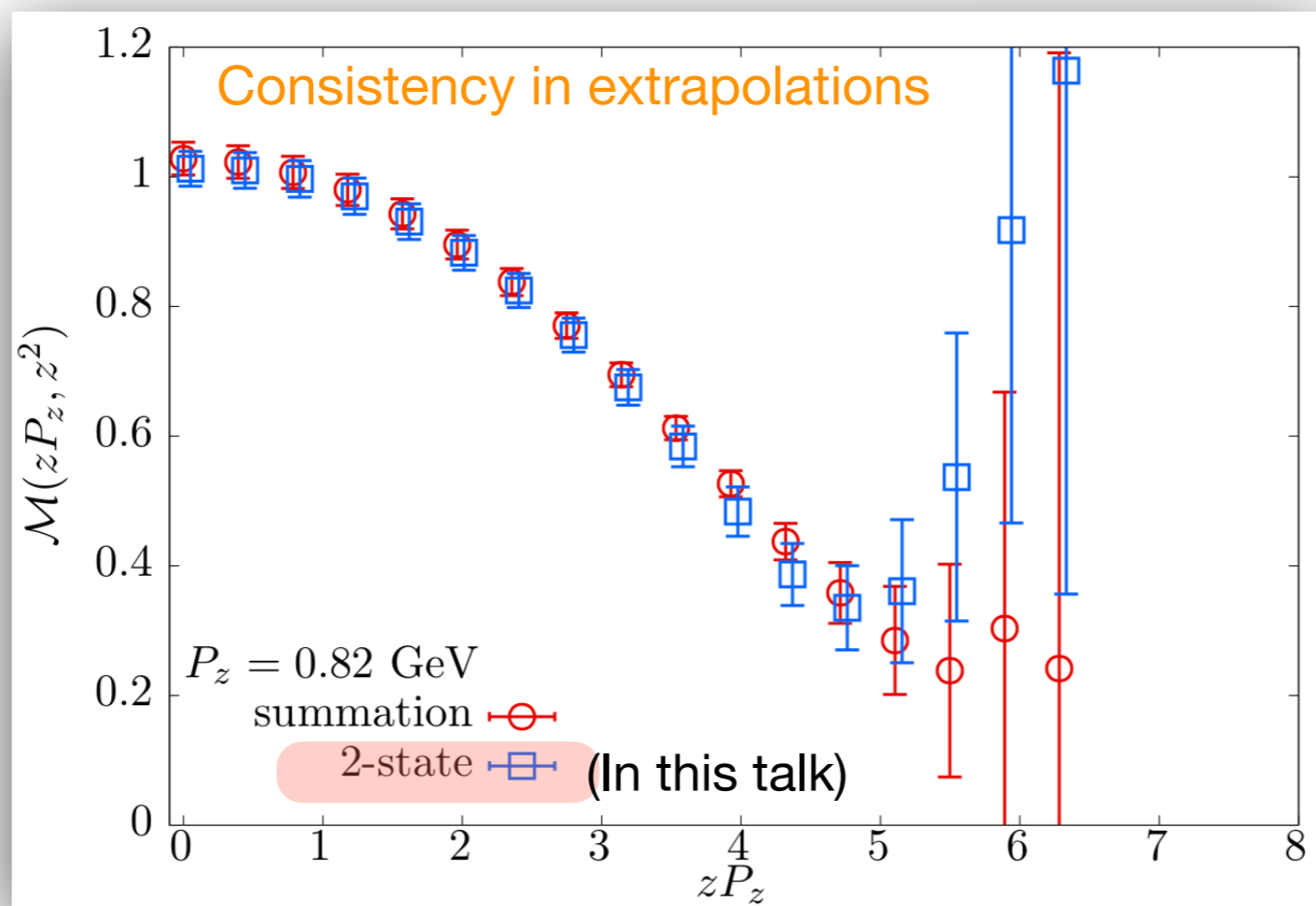
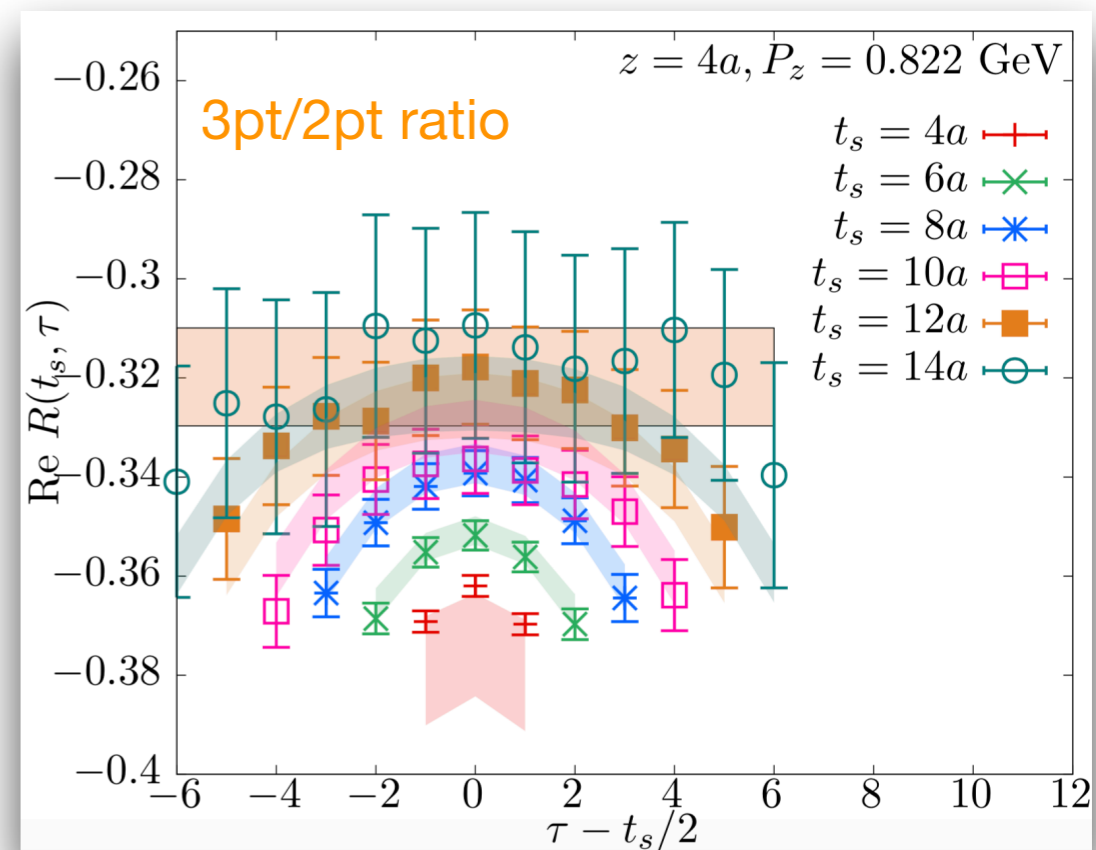
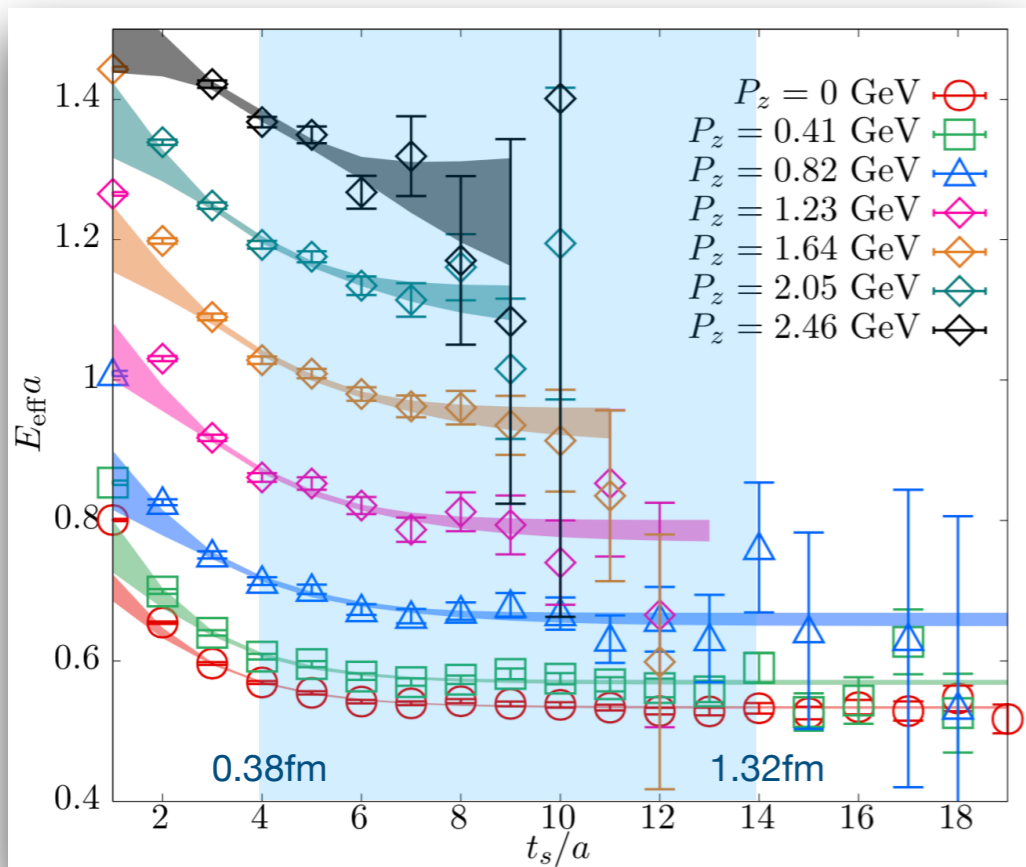
$$\mathfrak{M}(zP_z, z^2) = \sum_{n=0} C_n(\mu^2 z^2) \frac{(-izP_z)^n}{n!} \langle x^n \rangle(\mu)$$

Lattice set-up 

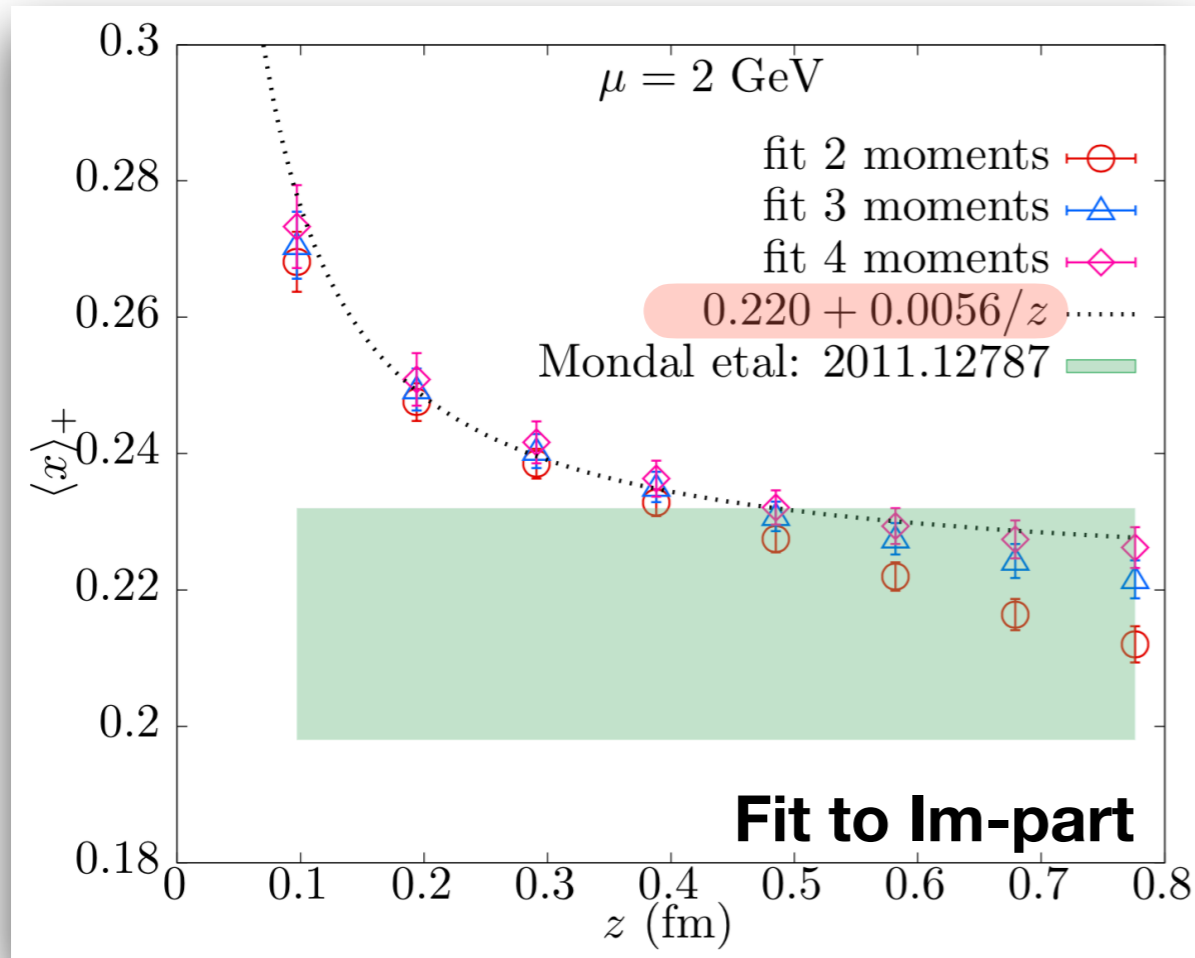
- Isotropic clover sea and valence
- $32^3 \times 64$ lattice, 358 cfg X 4src
- $M_\pi = 358$ MeV
- $a = 0.094$ fm
- Usage of distillation (rank 64)

 **Talk by Colin Egerer**

Extraction of matrix elements



Deducing the corrections to twist-2 OPE

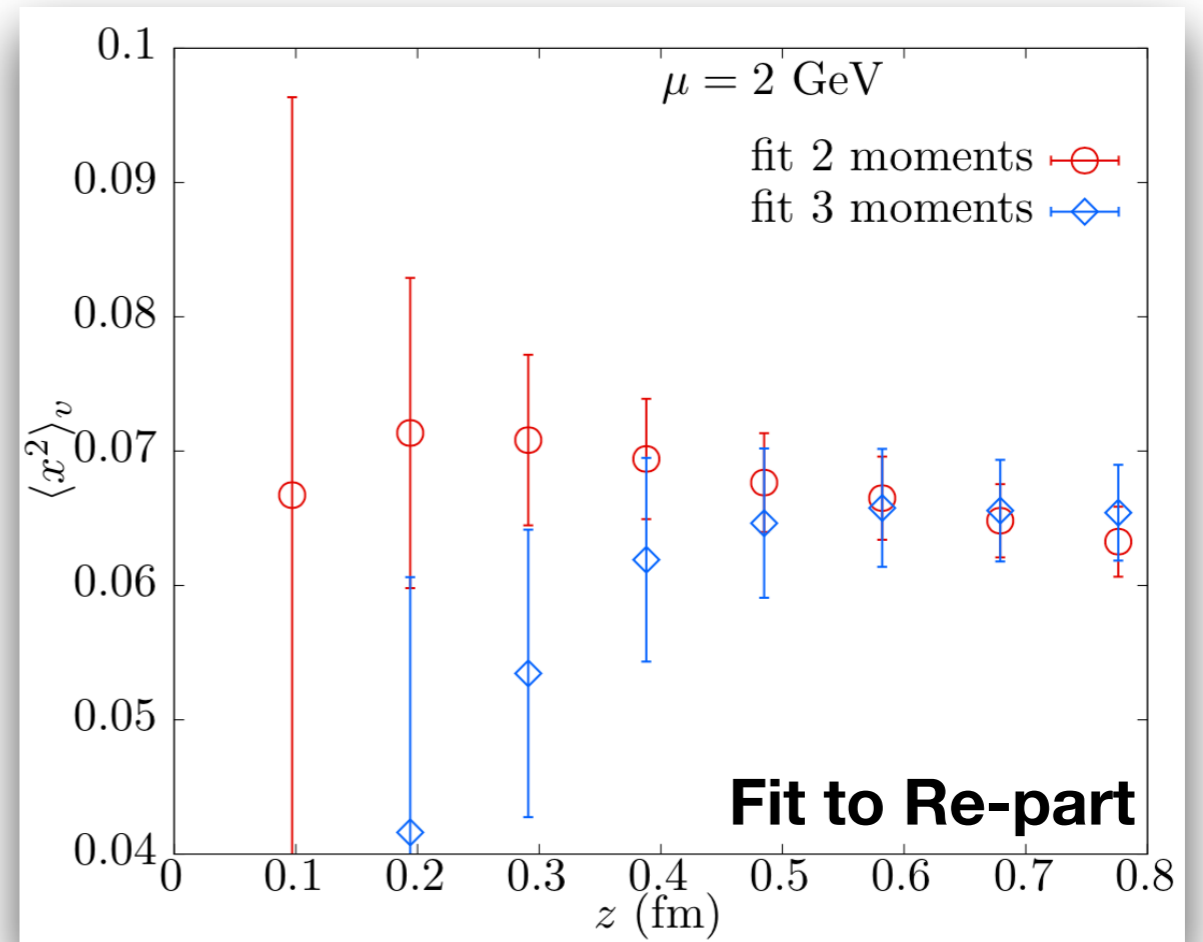


Leading-twist \downarrow Visible evidence in data \downarrow

$$\text{Im } \mathcal{M}_{\text{twist}-2}(\nu, z^2) + d_i \frac{a}{z} \nu + b_r z^2 \nu$$

Leading H.T. correction \uparrow

$$\text{Re } \mathcal{M}_{\text{twist}-2}(\nu, z^2) + d_r \frac{a}{|z|} \nu^2 + b_r z^2 \nu^2$$



Methodology of Fits : towards achieving model independence

$$h_1(x) = x^\alpha (1-x)^\beta \mathcal{G}(x) \longleftarrow \text{Some regular slowly varying function}$$

A common choice affecting small-x is JAM-type parametrization that empirically works:

$$\mathcal{G}(x) = 1 + \gamma\sqrt{x} + \delta x + \dots$$

Achieve model independence — expand in complete basis — A good choice is Jacobi Polynomials:

[J. Karpie et al, 2105.13313](#)

$$\mathcal{G}(x) = \sum_n^{N_{\max}} s_n P_n^{(\alpha, \beta)}(1-2x) \longleftarrow \text{Ortho-normal w.r.t } x^\alpha (1-x)^\beta$$

Methodology of Fits : towards achieving model independence

Fit pseudo-ITD using $x^\alpha (1 - x)^\beta (1 + \gamma\sqrt{x} + \delta x)$

Expand best fit PDF in $P_n^{(\alpha, \beta)}$

s_n

Feed as prior values of s_n to full Jacobi Polynomial fit

$$\chi^2 \rightarrow \chi^2 + \sum_n \frac{(s_n - s_n^{\text{prior}})^2}{\sigma_{\text{prior}}^2(s_n)}$$

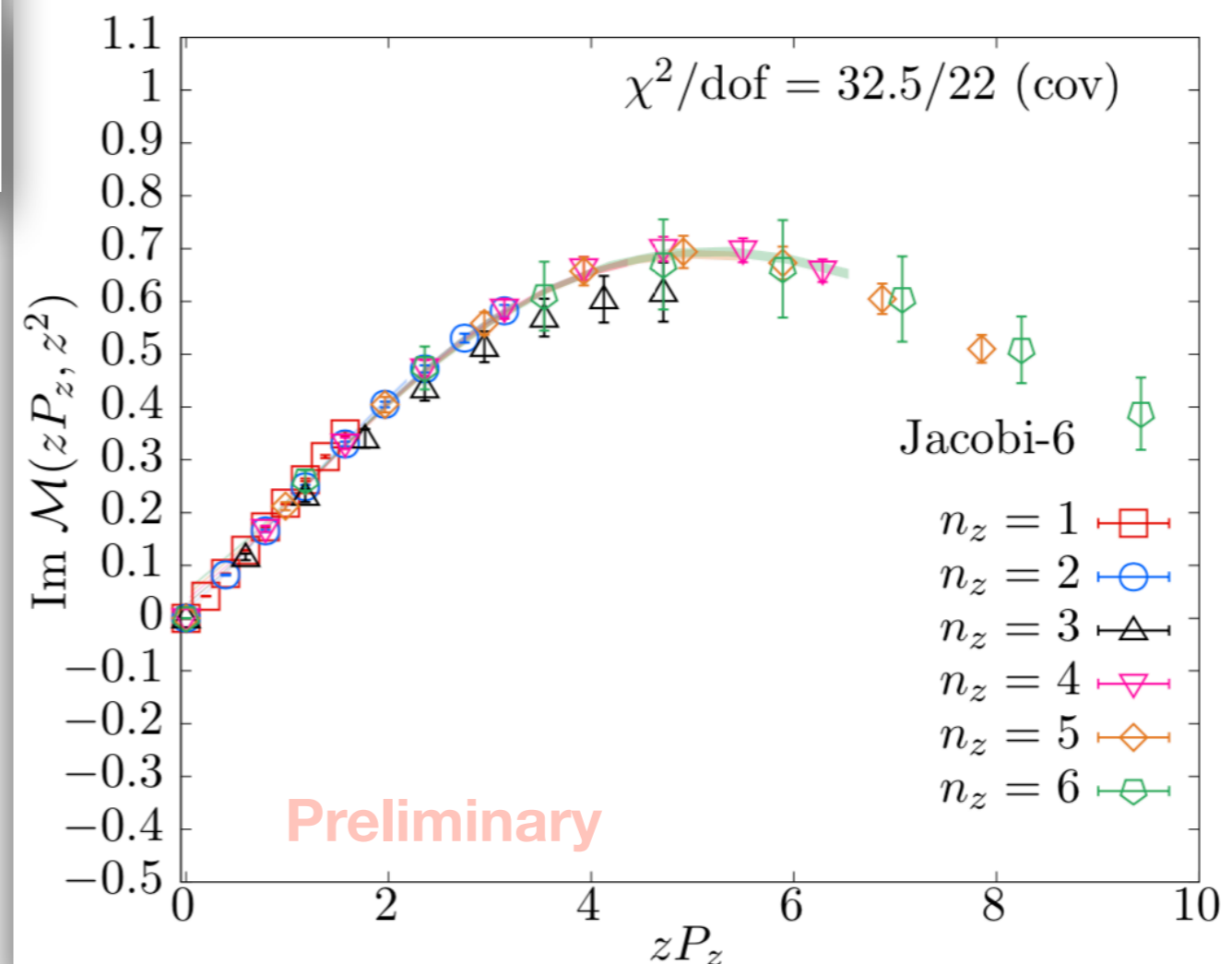
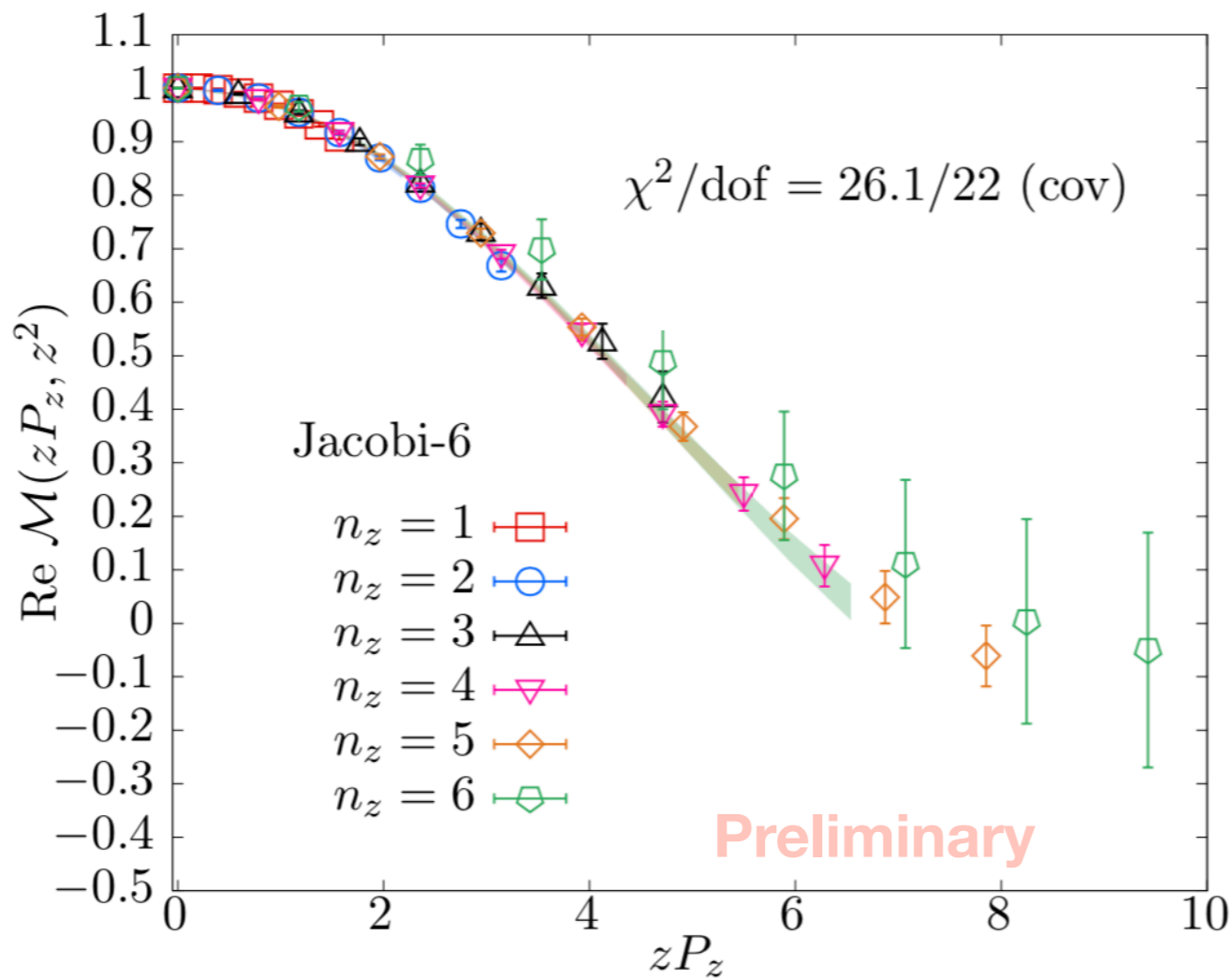
$h_1(x)$

Sample-by-sample AICc average
over Polynomial order N_{max} and other
fit variations such as $Z_{\text{min}}, Z_{\text{max}}$

$h_1(x) \pm \text{stat. error} \pm \text{sys. error}$

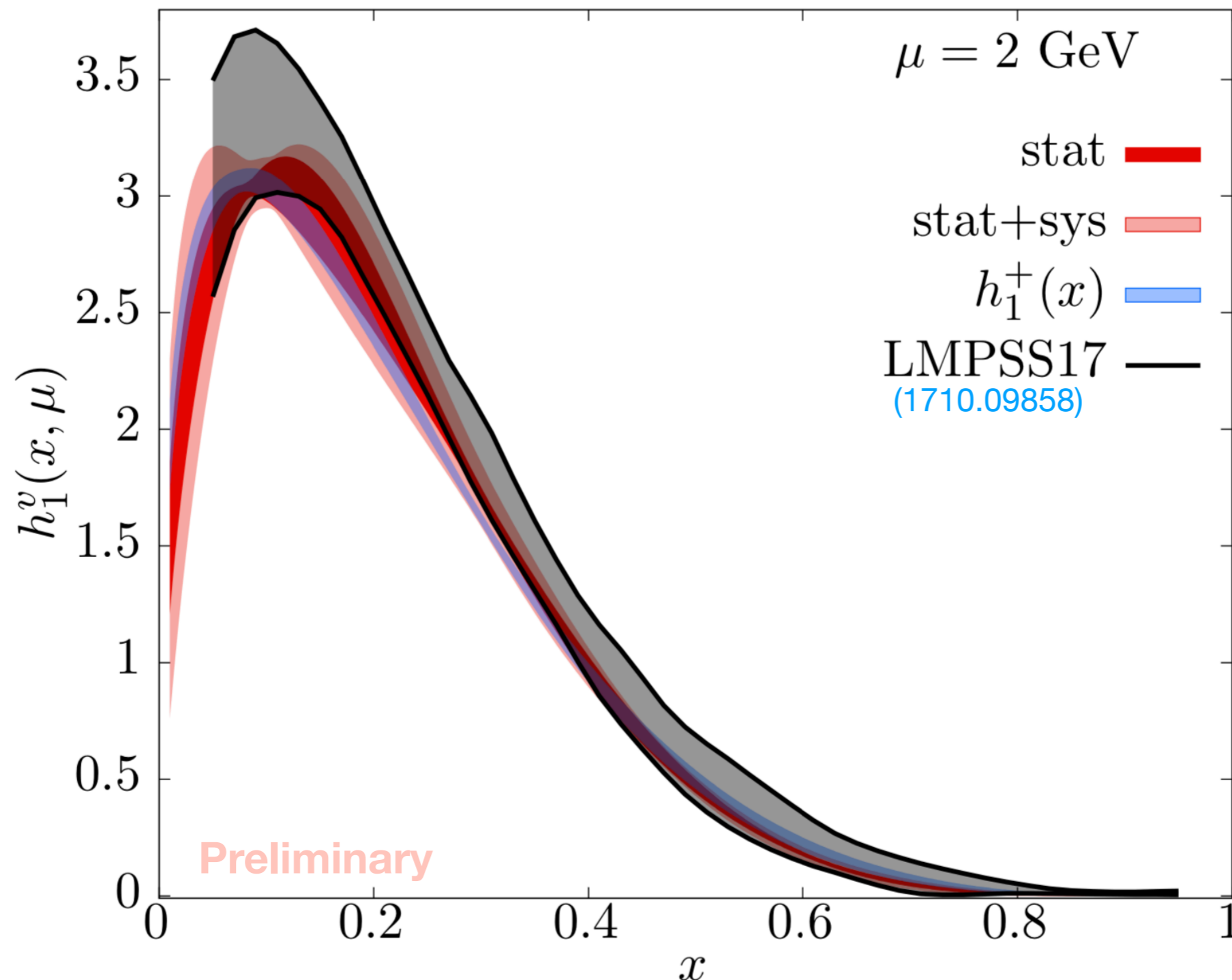
Jacobi polynomial fit to transversity pseudoITD

- Twist-2 OPE (+corrections) fit over a range $z \in [2a, 0.56 \text{ fm}]$ $P_z \in [0.41, 2.47] \text{ GeV}$



- **Systematic Error?** vary fit ranges, Jacobi polynomial order, H.T. corrections
- **Higher-loop?** Add 20% Gaussian noise to α_s . Very little effect was seen

Reconstructed x-dependent transversity PDF



- * A good agreement with LMPSS17 (pheno+lattice g_T) result
- * Systematic error comparable to statistical error — important to quantify in all pseudoPDF based works
- * $h_1^+(x)$ consistent with $h_1^v(x)$ — sea quark transversity is consistent with 0

Summary

- Presented a computation of transversity PDF of nucleon at 2 GeV using the pseudo-ITD approach at 358 MeV pion mass.
- Completely unitary calculation with isotropic Wilson-Clover fermion valence and sea quark action at $a=0.094$ fm spacing.
- Corrections to leading-twist empirically determined using “OPE without OPE” method.
- Matching performed in real space by fits to PDF ansatz.
- Model-independent perturbations around the JAM-type PDF default model achieved by using complete basis spanned by the Jacobi Polynomials.
- Statistical and systematic errors resulting from fitting choices folded into final results using model averaging.
- Good agreement with previous phenomenological determinations is very encouraging. Need to check if this persists as physical point is approached.