Instanton effects on chiral symmetry breaking and hadron spectroscopy

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Instantons and chiral symmetry breaking

- The spontaneous breaking of chiral symmetry causes interesting phenomena in the low energy of QCD [Nambu and Goldstone].
- Once chiral symmetry spontaneously breaks, a massless pion, which is the NG (Nambu-Goldstone) boson, appears, and the chiral condensate, which is an order parameter of chiral symmetry breaking, obtains non-zero values.
- The quarks obtain small masses from the non-zero values of the chiral condensate.
- The pion decay constant is defined as the strength of the coupling constant between the NG boson and the axial-vector current.
- The pion obtains mass by supposing a partially conserved axial current (PCAC) [Weinberg].
- These phenomena are well explained by models concerning the instanton [Belavi, Dyakonov, and Shuryak].
- The models demonstrate that the chiral condensate and the pion decay constant are estimated from the instanton vacuum and that instantons induce the breaking of the chiral symmetry [Dyakonov].

Our research results

- We add the monopole and anti-monopole into the QCD vacuum of the quenched SU(3)) by applying the monopole creation operator [Bonati, and et al. (2012)] and generate the configurations.
- We then calculate the eigenvalues and eigenvectors of the overlap Dirac operator using these configurations.
- We investigate the monopole and instanton effects on chiral symmetry breaking and hadrons.
- We have demonstrated the following results by increasing the number density of the instantons and anti-instantons [JHEP 09 (2020) 113].
  1) The chiral condensate decreases.
  2) The light quark masses ((u+d)/2, s) become heavy.
  3) The masses and decay constants of Pion and Kaon increase.
  4) The lifetime of charged Pion becomes shorter than the experiment.
- Lastly, we found the quantitative relations among the number density of the instantons and anti-instantons and these observables.

Purpose of research

- **To show indications that the effects of magnetic monopoles and instantons can be detected by experiments to reveal the existence of magnetic monopoles and instantons.**
- First, we demonstrate by conducting simulations of lattice QCD:

  the monopoles in the low energy of QCD induce the chiral symmetry breaking through instantons.
- Now, we investigate the effects of the finite lattice volume and discretization on the numerical results and evaluate the interpolated results at the continuum limit.

  1) Finite lattice volume effects: β = 6.000, V = 14^4, 14^3X28, 16^3X32.

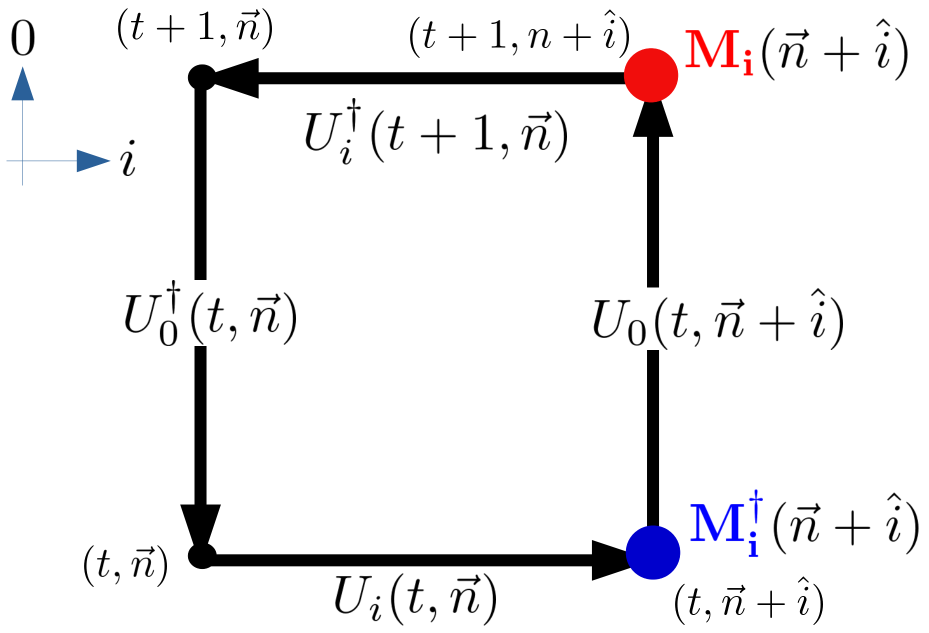
  2) Discretization effects and interpolation:

  V = 9.868 [fm^4], β = 5.8426, 5.9256, 6.000, 6.05217, 6.1366.

Monopole creation operator

- The monopole creation operator $\bar{\mu} = \exp\left(-\beta \overline{\Delta S}\right)$ acts on the vacuum.
- The plaquette action ***S*** is shifted $\mathbf{S} \rightarrow \mathbf{S} + \overline{\mathbf{\Delta S}}$; the pair of static Abelian monopole and its anti-monopole are created [PRD 85 (2012) 065001].
- We use the same creation operator as [PRD 91 (2015) 054512].
- These additional monopoles and ant-monopoles produce the instantons and anti-instantons.

- **The shifted plaquette action**



$$\mathbf{M_i}(\vec{n}) = \exp(iA_i^0(\vec{n} - \vec{x}_1))$$

$$\mathbf{M_i^\dagger}(\vec{n}) = \exp(-iA_i^0(\vec{n} - \vec{x}_2))$$

Simulation parameters

We generate the SU(3) normal configurations and configurations adding the pair of monopole and anti-monopole as follows:

- The magnetic charge of the monopole m_c (**positive**) are from **0** to **6** and anti-monopole **$-m_c$** (**negative**) are from **0** to **-6**. The magnetic charge m_c indicates both are added. The electric charge ***g*** is added.
- The distances between monopole and anti-monopole are about 1.1 [fm].
- We perform standard Monte Carlo simulations in which the gauge links are updated using the heat bath and over-relaxation methods.
- We set the scale of the lattice [NPB622 (2002) 328].
- The eigenvalues and eigenvectors of overlap Dirac operators are computed.

β	a [fm]	V	m_c	Nconf
5.8457	0.1242	$12^3\times24$	0-4	$1.0\times10^3 \sim 1.2\times10^3$
5.9256	0.1065	$14^3\times28$	0-5	$8\times10^2 \sim 9\times10^2$
6.0000	9.3150×10^{-2}	14^4	0-4	$2.0\times10^3 \sim 2.1\times10^3$
		$14^3\times28$	0-4	$1.7\times10^3 \sim 1.8\times10^3$
		$16^3\times32$	0-5	$8\times10^2 \sim 9\times10^2$
6.0522	8.5274×10^{-2}	$18^3\times32$	0-6	8×10^2
6.1366	7.4520×10^{-2}	$20^3\times40$	4-5	4×10^2

Zero modes and instantons

- **Fermion zero modes in the eigenvalues.**

  The number of **zero modes** of the **positive chirality is n_+.**

  The number of **zero modes** of the **negative chirality is n_-.**

- The exact zero modes λ_{Zero} are $|\lambda_{\mathbf{Zero}}| \leq \mathbf{O}(\mathbf{10^{-8}})$.
- We suppose that the Atiyah-Singer index theorem.

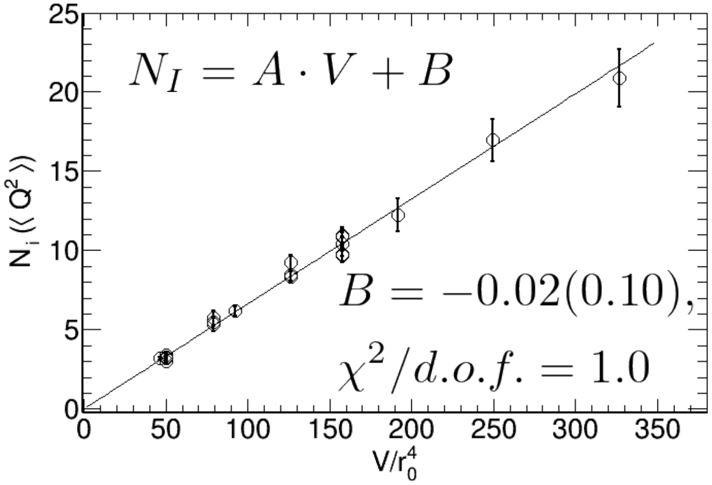
  The number of **instantons** of the **positive charge is n_+.**

  The number of **anti-instantons** of the **negative charge is n_-.**

- We have shown that the number of instantons and anti-instantons N_I can be calculated from the average square of the topological charges Q^2 [PRD 91 (2015) 054512]:

$$\mathbf{N_I} = \langle \mathbf{Q^2} \rangle, \ \mathbf{Q} = \mathbf{n_+} - \mathbf{n_-}$$

- **Topological susceptibility is** $\chi = \dfrac{\langle Q^2 \rangle}{V}$



- **Instanton (anti-instanton) density**

$$\frac{Ar_0^4}{2} = \frac{N_I r_0^4}{2V} = \mathbf{8.09(17)} \times \mathbf{10^{-4}} \ [\mathrm{GeV}^4]$$

- **E. V. Shuryak [NPB 203 (1982) 93]:**

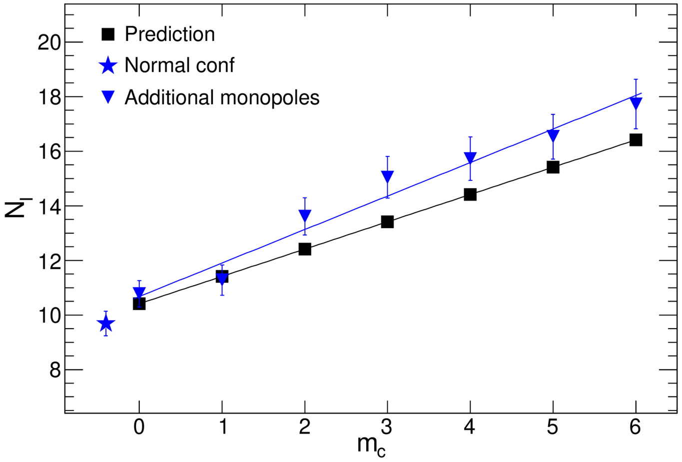
$$\mathrm{n}_c = \mathbf{8} \times \mathbf{10^{-4}} \ [\mathrm{GeV}^4]$$

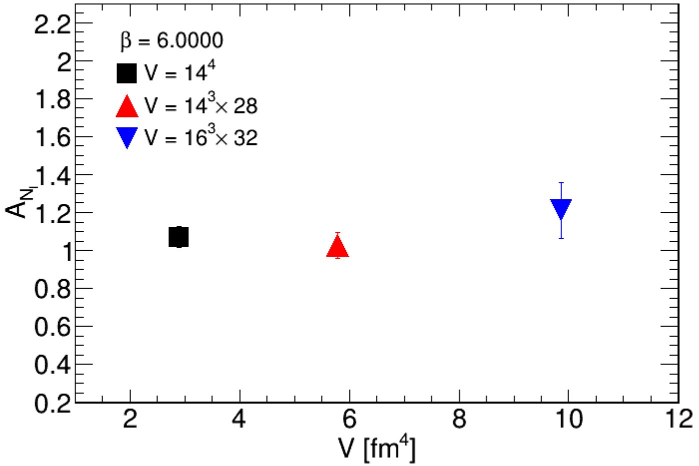
Monopoles and instantons

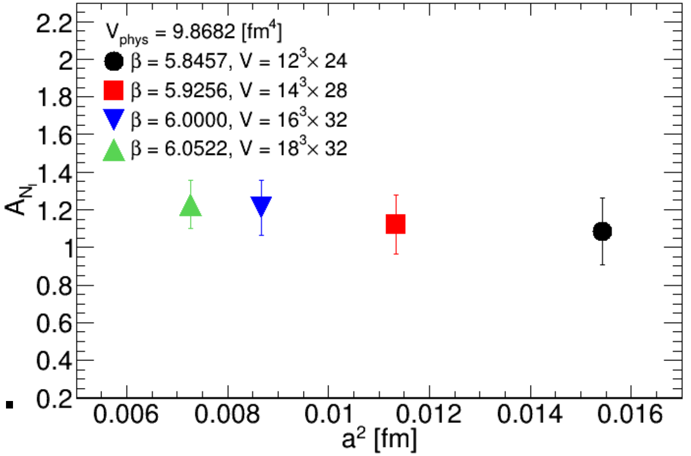
The quantitative relation between monopoles and instantons.

- We fit the following function.

$$N_I = Am_c + B$$







- **The monopole of a magnetic charge +1 and the anti-monopole of a magnetic charge -1 makes one instanton or one anti-instanton** [PRD 91 (2015) 054512].

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Making predictions

We make the predictions to demonstrate that the chiral condensate decreases and the decay constant increases.

- **Instanton vacuum**
  [Diakonov and Petrov, JTEP 62 (1985) 204]:

$$\langle\bar{\psi}\psi\rangle = -\frac{1}{\bar{\rho}}\left(\frac{\pi N_c}{13.2}\right)^{\frac{1}{2}}\left(\frac{N_I}{V}\right)^{\frac{1}{2}} = -(272.7\ [\mathrm{MeV}])^3.$$

- **The inverse of the average size of instanton**
  [NPB 203 (1982) 93]:

$$\frac{1}{\bar{\rho}} = 6.00\times10^2\quad[\mathrm{MeV}].$$

- **Gell-Mann-Oakes-Renner (GMOR) relation:**

$$\langle\bar{\psi}\psi\rangle = -\lim_{\bar{m}_q\to0}\frac{(m_\pi F_\pi)^2}{2\bar{m}_q} = -(274^{+18}_{-8}\ [\mathrm{MeV}])^3.$$

- **Prediction of the chiral condensate:**

$$\langle\bar{\psi}\psi\rangle^{Pre} = -\frac{1}{\bar{\rho}}\left(\frac{\pi N_c}{13.2}\right)^{\frac{1}{2}}\left(\frac{N_I^{Pre}}{V}\right)^{\frac{1}{2}},$$

$$N_I^{Pre} = m_c + N_I^{Nor}.$$

- **Prediction of the decay constant at the chiral limit:**

$$F_0^{Pre} = \frac{1}{m_\pi}\left(\frac{2\bar{m}_q}{\bar{\rho}}\right)^{\frac{1}{2}}\left(\frac{\pi N_c}{13.2}\right)^{\frac{1}{4}}\left(\frac{N_I^{Pre}}{V}\right)^{\frac{1}{4}}.$$

- **For m_c = 0** ($V = 18^3\times32$):

$$F_0^{Pre} = 85^{+9}_{-4}\ [\mathrm{MeV}]$$

- **Chiral perturbation theory**
  [EPJC 33 (2004) 543]:

$$F_0^{\chi PT} = 86.2(5)\ [\mathrm{MeV}]$$

Operators and correlation functions

- **Quark propagator:**

$$G(\vec{y}, y^0; \vec{x}, x^0) \equiv \sum_i \frac{\psi_i(\vec{x}, x^0)\psi_i^\dagger(\vec{y}, y^0)}{\lambda_i^{mass}}$$

- λ_i^{mass} **of massive Dirac operator:**

$$\lambda_i^{mass} = \left(1 - \frac{a\bar{m}_q}{2\rho}\right)\lambda_i + \bar{m}_q$$

Light quark masses

- **Pion:** $\bar{m}_{ud} \equiv \dfrac{m_u + m_d}{2}$

- **Kaon:** $\bar{m}_{sud} \equiv \dfrac{m_s + \bar{m}_{ud}}{2}$

- **Scalar:** $\mathcal{O}_S = \bar{\psi}_1 \left(1 - \frac{a}{2\rho} D\right) \psi_2$

- **Pseudoscalar:** $\mathcal{O}_{PS} = \bar{\psi}_1 \gamma_5 \left(1 - \frac{a}{2\rho} D\right) \psi_2$

- **Scalar density:**

$$C_{SS}(\Delta t) = \frac{a^3}{V} \sum_{\vec{x}_1} \sum_{\vec{x}_2,\ t} \langle \mathcal{O}_S^C(\vec{x}_2, t)\mathcal{O}_S(\vec{x}_1, t + \Delta t)\rangle$$

- **Pseudoscalar density:**

$$C_{PS}(\Delta t) = \frac{a^3}{V} \sum_{\vec{x}_1} \sum_{\vec{x}_2,\ t} \langle \mathcal{O}_{PS}^C(\vec{x}_2, t)\mathcal{O}_{PS}(\vec{x}_1, t + \Delta t)\rangle$$

- **Correlation function** [PRD 69 (2004) 074502]**:**

$$C_{PS-SS}(\Delta t) \equiv C_{PS}(\Delta t) - C_{SS}(\Delta t)$$

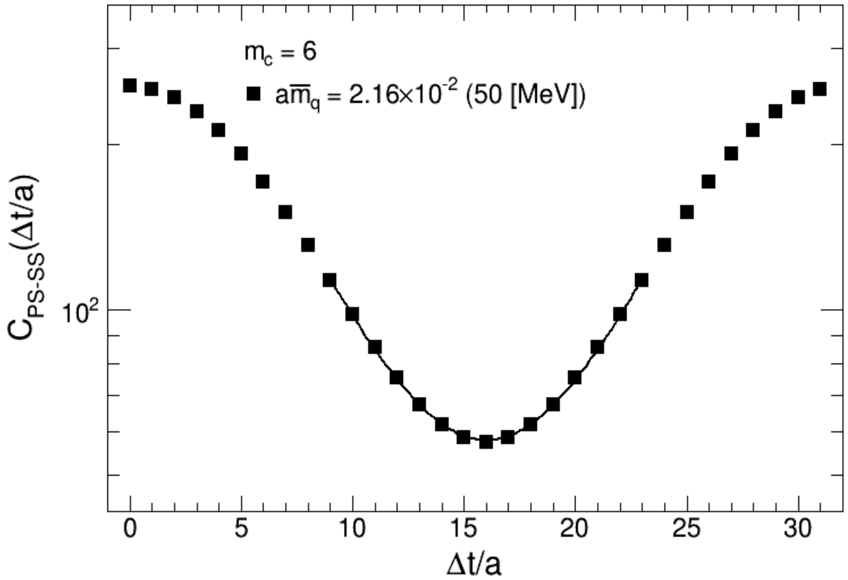
Fitting to the correlations

- We compute the correlations $C_{PS\text{-}SS}$ and aC_{AP}.
- The mass range of the bar quark is $\mathbf{a\bar{m}_q} = 30 - 150$ [MeV].
- We determine two parameters $a^4G_{PS\text{-}SS}$ and am_{PS}, by fitting the following function:

$$C_{PS-SS}(t) = \frac{a^4 G_{PS-SS}}{am_{PS}} \exp\left(-\frac{m_{PS}}{2}T\right) \cosh\left[m_{PS}\left(\frac{T}{2} - t\right)\right].$$

- We set the fitting range so that the fitting result of χ^2/d.o.f. becomes approximately 1.
- We then calculate chiral condensate, light quark masses, meson masses, and decay constants using the fitting results $a^4G_{PS\text{-}SS}$ and am_{PS}.

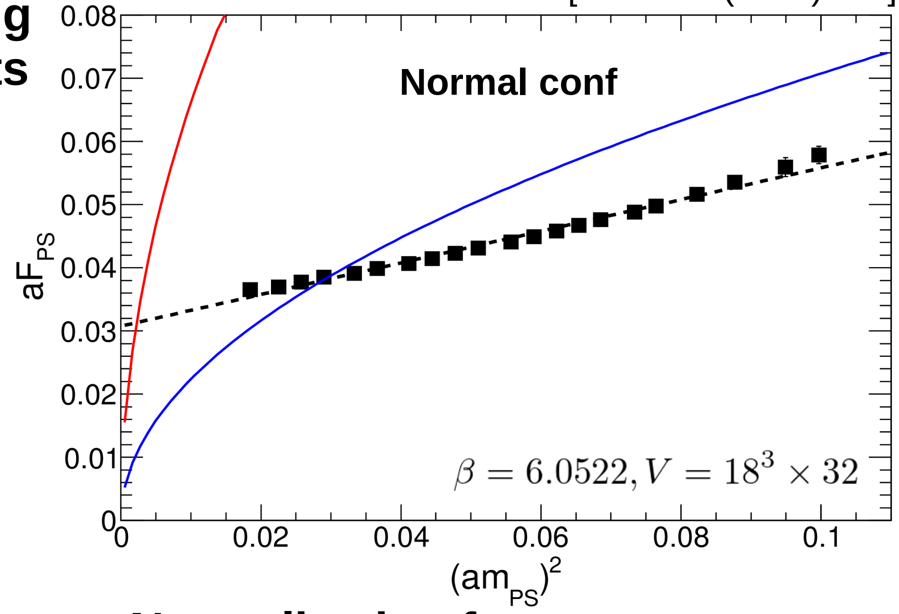
$\beta = 6.0522, V = 18^3 \times 32$



Fitting result:

χ^2/d.o.f. ≈ 0.6 (FR Δt/a = 9-23).

Matching with experiments



Determining the normalization factors by matching the numerical results with the experimental results of Pion and Kaon [NPB 489 (1997) 427, PRD 64 (2001) 114508]**.**

- Fitting function: $aF_{PS} = a^{-1}A(am_{PS})^2 + aB.$
- Making two functions:
  - **Pion**:
    $$aF_{PS} = C_{\pi}^{Exp.} am_{PS}, \; C_{\pi}^{Exp.} = \frac{F_{\pi^-}^{Exp.}}{\sqrt{2}m_{\pi^\pm}^{Exp.}} = \frac{92.277}{139.570}.$$
  - **Kaon**:
    $$aF_{PS} = C_{K}^{Exp.} am_{PS}, \; C_{K}^{Exp.} = \frac{F_{K^-}^{Exp.}}{\sqrt{2}m_{K^\pm}^{Exp.}} = \frac{110.11}{493.677}.$$
- Calculating the intersections between these functions (normal conf).
  - **Pion**: $(aF_{PS}^{\pi}, am_{PS}^{\pi}) = (3.13(6), 4.74(8) \times 10^{-2}).$
  - **Kaon**: $(aF_{PS}^{K}, am_{PS}^{K}) = (3.80(10), 0.171(4)).$

- **Normalization factors:**

$$Z_{\pi} = \frac{F_{\pi^-}^{Exp.}}{\sqrt{2}F_{PS}^{\pi}} = \frac{m_{\pi^\pm}^{Exp.}}{m_{PS}^{\pi}} = 1.27(2)$$

$$Z_{K} = \frac{F_{K^-}^{Exp.}}{\sqrt{2}F_{PS}^{K}} = \frac{m_{K^\pm}^{Exp.}}{m_{PS}^{K}} = 1.25(3)$$

Instanton effects on chiral condensate

- The chiral condensate is derived using the slope ***aA***$^{(2)}$ of the PCAC relation and the decay constant $\boldsymbol{F_0^Z}$.

$$a^3\langle\bar{\psi}\psi\rangle^Z = -\lim_{a\bar{m}_q\to 0}\frac{(Z_\pi a m_{PS})^2(Z_\pi aF_{PS})^2}{2a\bar{m}_q^Z}$$
$$= -\frac{aA^{(2)}}{2}(aF_0^Z)^2,\ (F_0^Z = Z_\pi F_0).$$

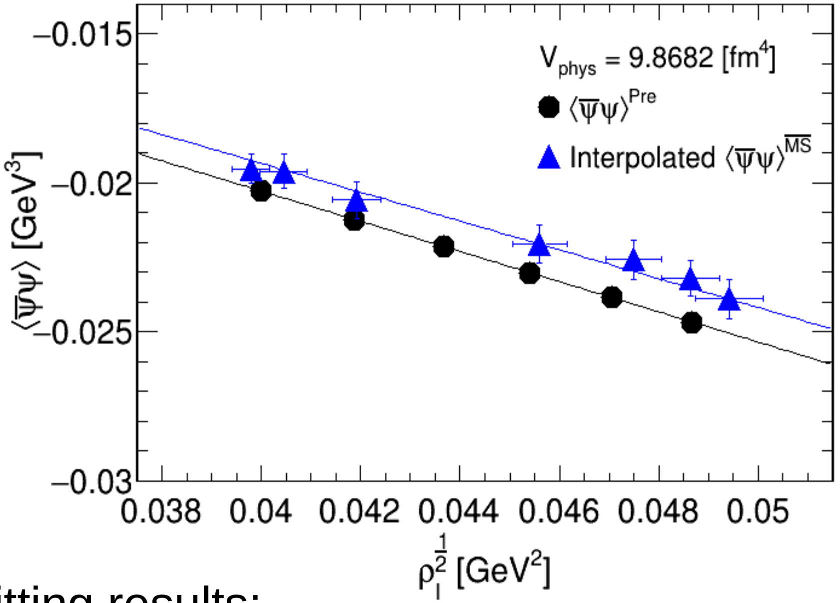
- The renormalized chiral condensate into the MS-bar scheme at 2 [GeV]:

$$\langle\bar{\psi}\psi\rangle^Z_{\overline{MS}} = \frac{Z_S}{0.72076}\langle\bar{\psi}\psi\rangle^Z$$

- The interpolated numerical result is

$$\langle\bar{\psi}\psi\rangle^Z_{\overline{MS}}\ (2\ [\text{GeV}]) = -(269(2)\ [\text{MeV}])^3.$$

- The fitting curve: $\langle\bar{\psi}\psi\rangle = -A_\chi\left(\frac{N_I}{V}\right)^{\frac{1}{2}}$



- Fitting results:

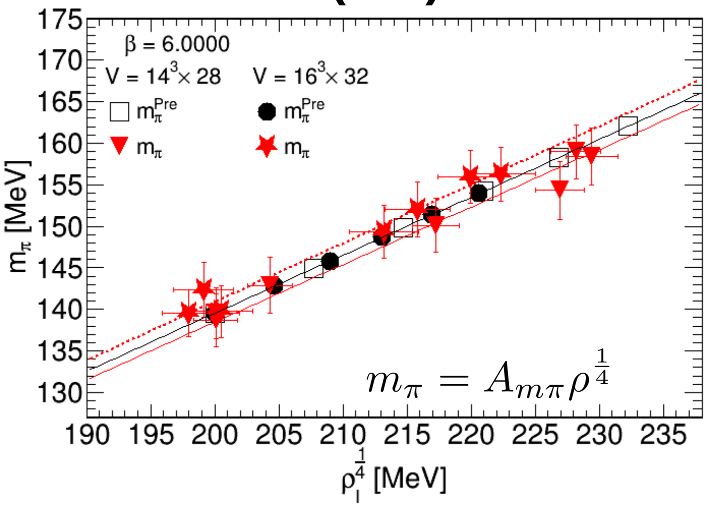
$$A_\chi = 0.484(6)\ [\text{GeV}], \chi^2/d.o.f. = 1.0/6.0.$$
$$A_\chi^{Pre} = 0.5070\ [\text{GeV}]$$

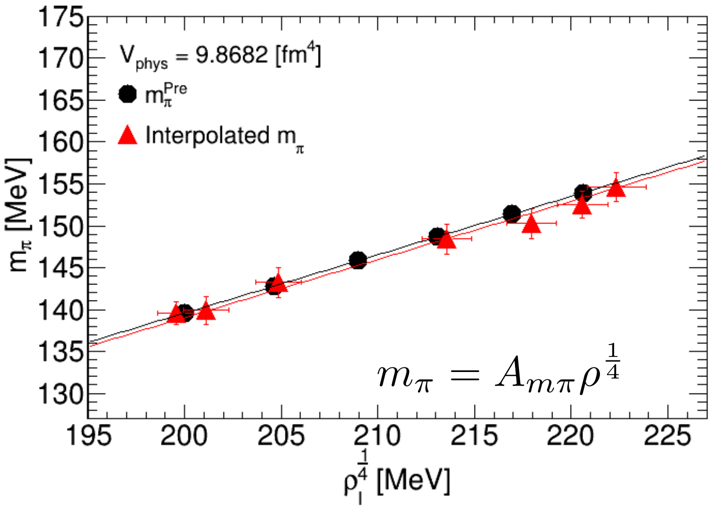
- Inverse of the average size of the instanton:

$$\frac{1}{\bar{\rho}} = 5.72(6)\times 10^2\ [\text{MeV}].$$

Instanton effects on m_π

- From the PCAC relation, the pion (and kaon) masses increase in direct proportion to the one-fourth root of the number density of the instantons and anti-instantons.
- Making the predictions about the pion (and kaon) masses using the experimental results and the ratio $(R^{Pre})^{1/2}$.





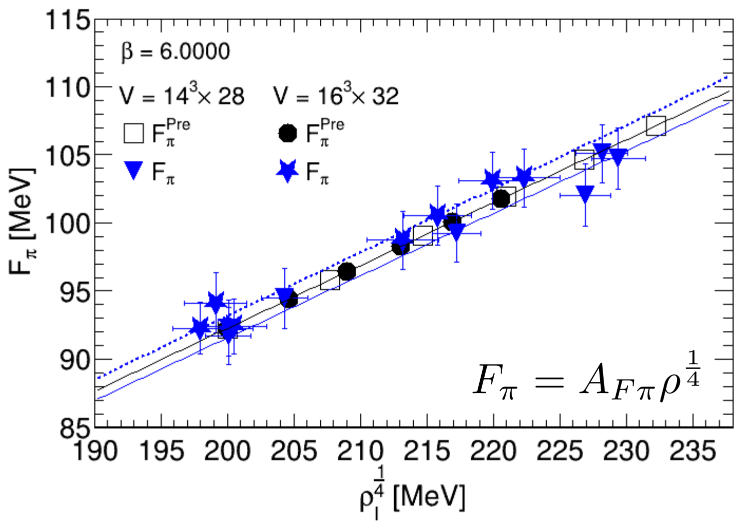
Fitting results

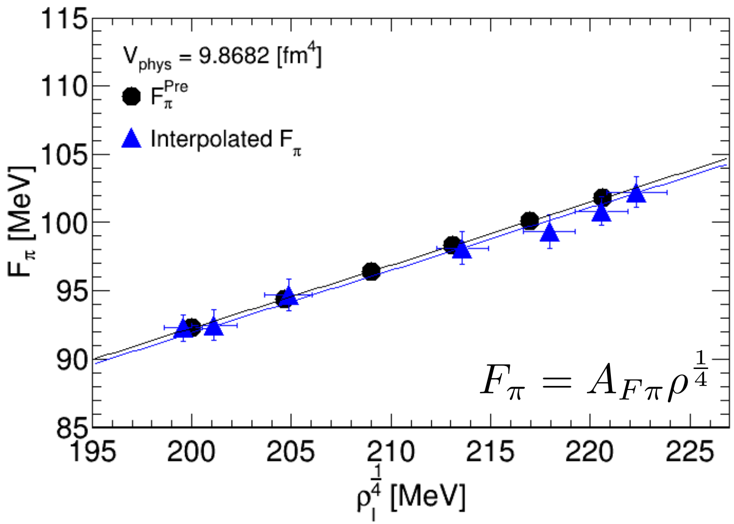
Pre: A = 0.697853(4),
χ^2/d.o.f. = 0/5.
$14^3 28$: A = 0.693(6),
χ^2/d.o.f. = 1/6.
$16^3 32$: A = 0.705(6),
χ^2/d.o.f. = 1/6.
Int.: A = 0.695(3),
χ^2/d.o.f. = 1.1/6.0.

- The fitting results agree with the predictions.
- The pion (and kaon) masses increase in **direct proportion** to the **one-fourth root** of the number density of the instantons and anti-instantons.

Instanton effects on F_π

- We have confirmed that the formula in the quenched chiral perturbation theory holds.
- The decay constants of the pion (and kaon) are in direct proportion to the one-fourth root of the number density of the instantons and anti-instantons.





Fitting results

Pre: A = 0.46140(17),
χ^2/d.o.f. = 0/5.

$14^3 28$: A = 0.458(4),
χ^2/d.o.f. = 1/6.

$16^3 32$: A = 0.466(4),
χ^2/d.o.f. = 1/6.

Int.: A = 0.460(2),
χ^2/d.o.f. = 1/6.

- The fitting results are consistent with the predictions.
- The decay constants of the pion (and kaon) increase in **direct proportion** to the **one-fourth root** of the number density of the instantons and anti-instantons.

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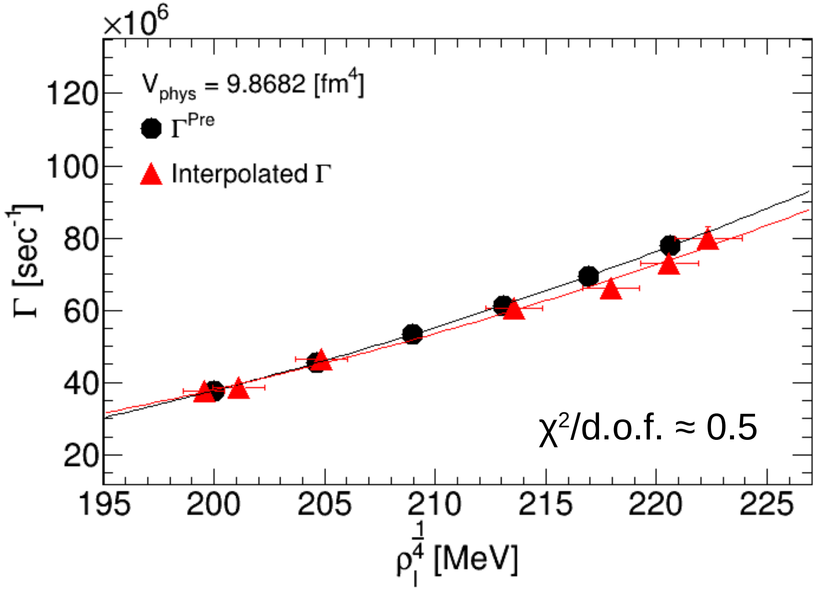
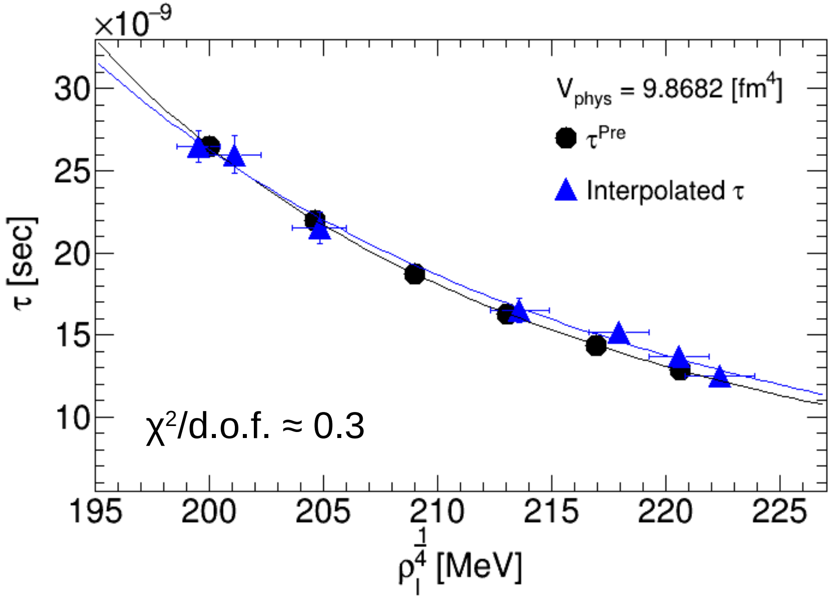
Catalytic effects on the charged pion

- One charged pion decays to a lepton (an electron or a muon) and a neutrino as follows: $\pi^+ \to l^+ + \nu_l, \quad \pi^- \to l^- + \bar{\nu}_l$





- The decay width becomes wider by increasing the number density of the instantons and anti-instantons. The lifetime becomes shorter by increasing the number density of the instantons and anti-instantons.

Instanton effects on Eta-prime

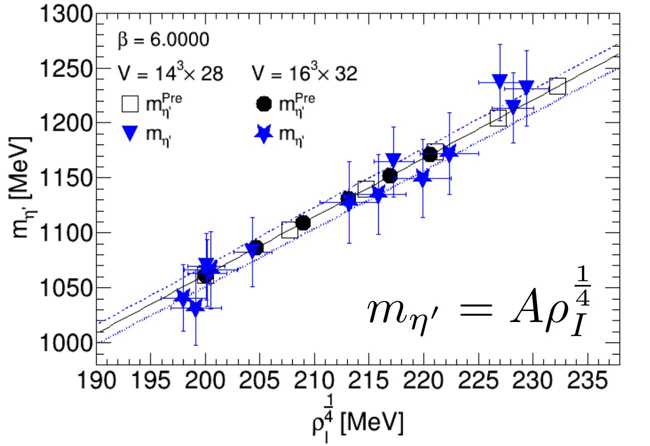
- **Eta-prime mass** [NPB 628 (2002) 234, PRD 64 (2002) 114501]
- The hairpin diagram of the pseudoscalar density.

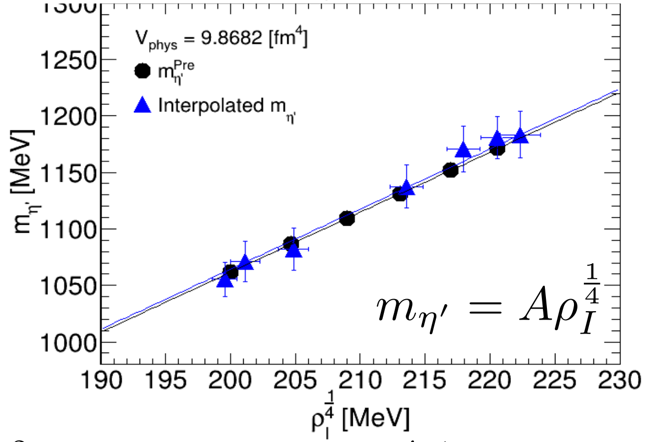
$$\frac{F_\pi^2}{2N_F} m_{\eta'}^2|_{m_u=0} = \int d^4x \left\langle \frac{1}{2} Tr[\gamma_5 D(x,x)] Tr[\gamma_5 D(y,y)] \right\rangle$$

- Witten-Veneziano relation:

$$m_{\eta'}^2 = \frac{2N_F}{F_\pi^2}\chi$$

- The eta-prime mass increases in **direct proportion** to the one-fourth root of the number density of the instantons and anti-instantons (preliminary result).





$\chi^2/d.o.f. = 0.7/6.0, \ A^{int} = 5.32(3)$

$A^{Pre} = 5.309(2)$

Conclusions

- The monopole with magnetic charge $\boldsymbol{m_c}$ **= 1** and the anti-monopole with magnetic charge $\boldsymbol{m_c}$ **= -1** produce one instanton or one anti-instanton.
- The values of the chiral condensate decrease in direct proportion to the **square root** of the number density of the instantons and anti-instantons.
- The mass and decay constant of the pion (and kaon) increase in direct proportion to the **one-fourth root** of the number density of the instantons and anti-instantons.
- The decay width of the charged pion becomes wider than the experiment by increasing the number density of the instantons and anti-instantons.
- The lifetime of the charged pion becomes shorter than the experiment by increasing the number density of the instantons and anti-instantons.
- The eta-prime mass increases in direct proportion to the **one-fourth root** of the number density of the instantons and anti-instantons (preliminary result).

These are the monopole and instanton effects in QCD.

Acknowledgments

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