The dual Meissner effect due to Abelian Dirac-type monopoles in QCD

Atsuki Hiraguchi (NYCU, Kochi Univ.)

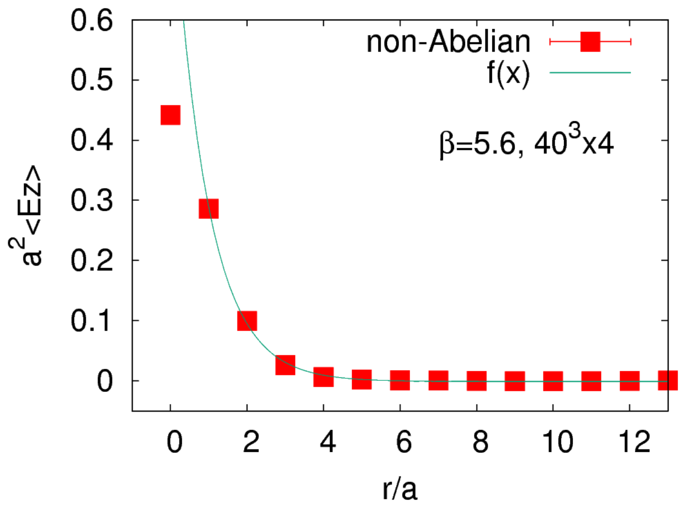
Katsuya Ishiguro (Kochi University)

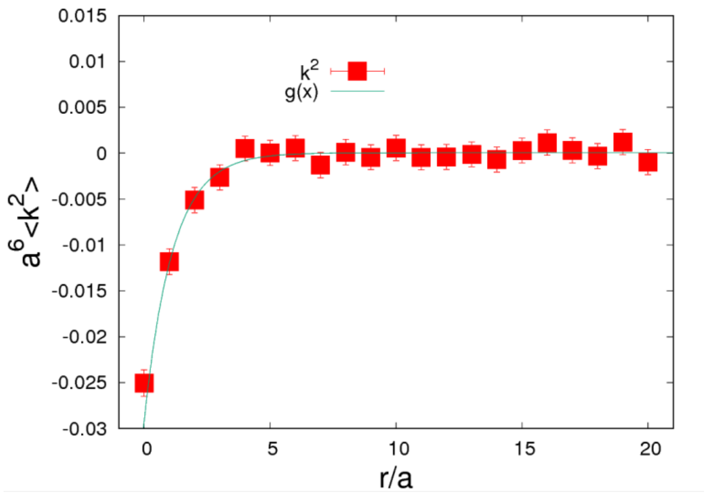
Tsuneo Suzuki (RCNP, Osaka University)

27, July 2021, The 38th International Symposium on Lattice Field Theory, online

Purpose

Evaluating the vacuum type of SU(3) gauge theory from Abelian monopole currents





The GL parameter

$$\kappa = \frac{\lambda}{\xi}$$

No.1

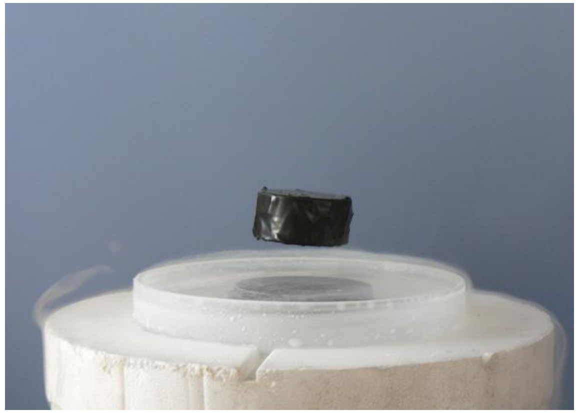
Introduction

The dual Meissner effect

G. ’t Hooft, in Proceedings of the EPS International, edited by A. Zichichi (Editrice Compositori, Bologna, 1976), p. 1225.

S. Mandelstam, Phys. Rep. 23, 245 (1976).

Fig 1. Magnetic flux pinning



Wikipedia

QCD		Superconductivity
Color electric flux-tube	⟺	Magnetic flux-tube
Condensation of color magnetic monopoles	⟺	Condensation of pair of electrons (Cooper pairs)

No.2

The violation of non-Abelian Bianchi identities(VNABI)

T.Suzuki,K.Ishiguro,V.Bornyakov,Phys.Rev.D97:034501 (2018)
T.Suzuki,Phys.Rev.D97,034509 (2018)

$$D_\nu G^*_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} D_\nu G_{\rho\sigma}$$

$$D_\nu G^*_{\mu\nu} = \partial_\nu f^*_{\mu\nu} = k_\mu$$

$$f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)\frac{\lambda^a}{2}$$

(**Suppose a gauge field has a line singularity**)

$$\begin{aligned}[D_\rho, D_\sigma] &= [\partial_\rho - igA_\rho, \partial_\sigma - igA_\sigma] \\ &= -ig(\partial_\rho A_\sigma - \partial_\sigma A_\rho - ig[A_\rho, A_\sigma]) + [\partial_\rho, \partial_\sigma] \\ &= -igG_{\rho\sigma} + \underline{[\partial_\rho, \partial_\sigma]}\end{aligned}$$

On the continuum theory, the violation of non-Abelian Bianchi identity leads to Abelian monopole currents for each color.

We define these monopole currents on the lattice!

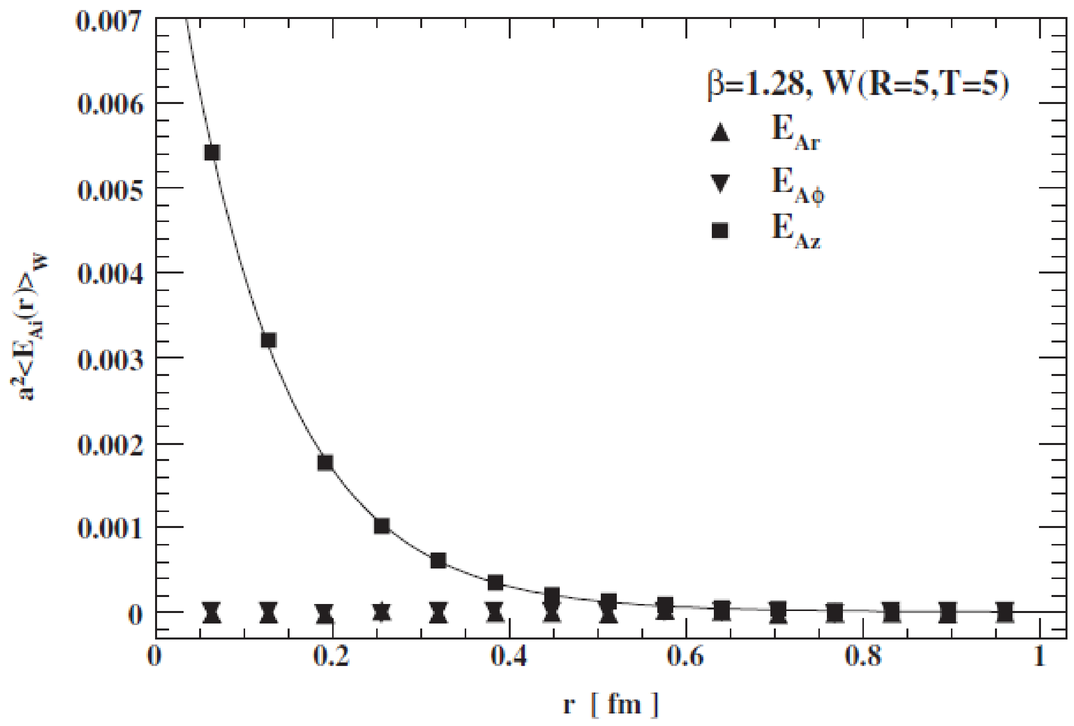
In SU(2) gauge theory, Abelian color electric fields and monopole currents can be evaluated without gauge fixing.

T.Suzuki,M.Hasegawa,K.Ishiguro,Y.Koma,T.Sekido,Phys.Rev.D80:054504(2009)

TABLE III. Best fitted values of the string tension σa^2, the Coulombic coefficient c, and the constant μa for the potentials V_{NA}, V_A, V_{mon}, and V_{ph}.

$24^3 \times 4$	σa^2	c	μa	FR (R/a)	χ^2/N_{df}
V_{NA}	0.181(8)	0.25(15)	0.54(7)	3.9–8.5	1.00
V_A	0.183(8)	0.20(15)	0.98(7)	3.9–8.2	1.00
V_{mon}	0.183(6)	0.25(11)	1.31(5)	3.9–6.7	0.98
V_{ph}	$-2(1) \times 10^{-4}$	0.010(1)	0.48(1)	4.9–9.4	1.02
$24^3 \times 6$					
V_{NA}	0.072(3)	0.49(6)	0.53(3)	4.0–9.0	0.99
V_A	0.073(4)	0.41(7)	1.09(3)	3.7–10.9	1.00
V_{mon}	0.073(4)	0.44(10)	1.41(4)	3.9–9.3	1.00
V_{ph}	$-1.7(3) \times 10^{-4}$	0.0131(1)	0.4717(3)	5.1–9.4	0.99
$36^3 \times 6$					
V_{NA}	0.072(3)	0.48(9)	0.53(3)	4.6–12.1	1.03
V_A	0.073(2)	0.47(6)	1.10(2)	4.3–11.2	1.03
V_{mon}	0.073(3)	0.46(7)	1.43(3)	4.0–11.8	1.01
V_{ph}	$-1.0(1) \times 10^{-4}$	0.0132(1)	0.4770(2)	6.4–11.5	1.03
$24^3 \times 8$					
V_{NA}	0.0415(9)	0.47(2)	0.46(8)	4.1–7.8	0.99
V_A	0.041(2)	0.47(6)	1.10(3)	4.5–8.5	1.00
V_{mon}	0.043(3)	0.37(4)	1.39(2)	2.1–7.5	0.99
V_{ph}	$-6.0(3) \times 10^{-5}$	0.0059(3)	0.466 49(6)	7.7–11.5	1.02

Abelian color electric field



No.4

Abelian projection in SU(3) gauge theory

Previous research	Our research
G. 't Hooft, Nucl. Phys. B 190 (1981) 455. F. Brandstaeter et al., Phys. Lett. B 272 (1991) 631.	
Gauge fixing : Maximal Abelian	**Gauge fixing : None**
Projection : SU(3) → U(1) x U(1)	**Projection : SU(3) → U(1)**

Previous research

Gauge fixing : Maximal Abelian

$$R = \sum_{s,\mu,a} \mathrm{Tr}(U_\mu(s)\lambda_a U_\mu^\dagger(s)\lambda_a)$$

$$\lambda_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Projection : SU(3) → U(1) x U(1)

$$u(s,\mu) = \mathrm{diag}[u_1, u_2, u_3]$$

$$u_i(s,\mu) = \exp\{i\arg[U_{ii}^{MA}(s,\mu)] - \frac{1}{3}i\phi(s,\mu)\}$$

$$\phi(s,\mu) = \sum_i \arg[U_{ii}^{MA}(s,\mu)]|_{\mathrm{mod}2\pi}$$

Our research

Gauge fixing : None

Projection : SU(3) → U(1)

$$R_1 = \sum_{s,\mu} \mathrm{Re}\,\mathrm{Tr}[e^{i\theta_\mu^1(s)\lambda^1} U_\mu^\dagger(s)]$$

$$\Rightarrow \quad \theta_\mu^1(s) = \tan^{-1}\frac{\{\mathrm{Im}(U_{12}) + \mathrm{Im}(U_{21})\}}{\{\mathrm{Re}(U_{11}) + \mathrm{Re}(U_{22})\}}$$

Method

Methods

Wilson action 40^3 x Nt

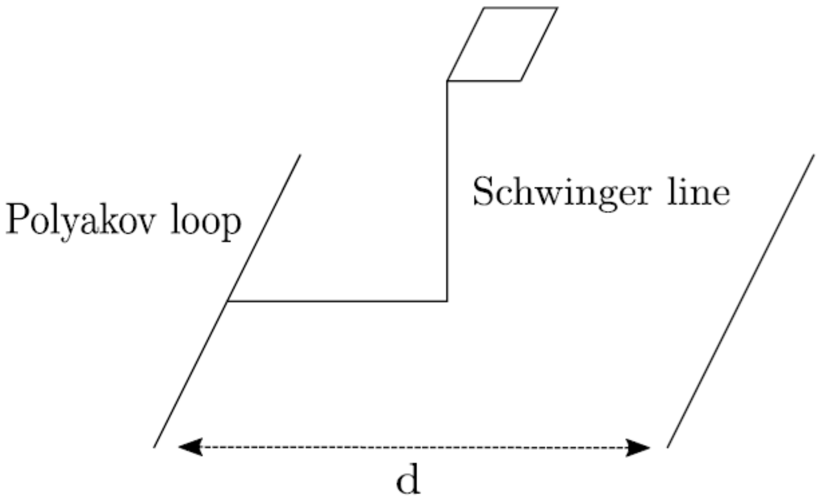
- β =5.6(Nt=4), β =5.75(Nt=6) at 0.8Tc
- 1000~16000conf
- APE and HYP smearings, random gauge transformations
- No gauge fixing

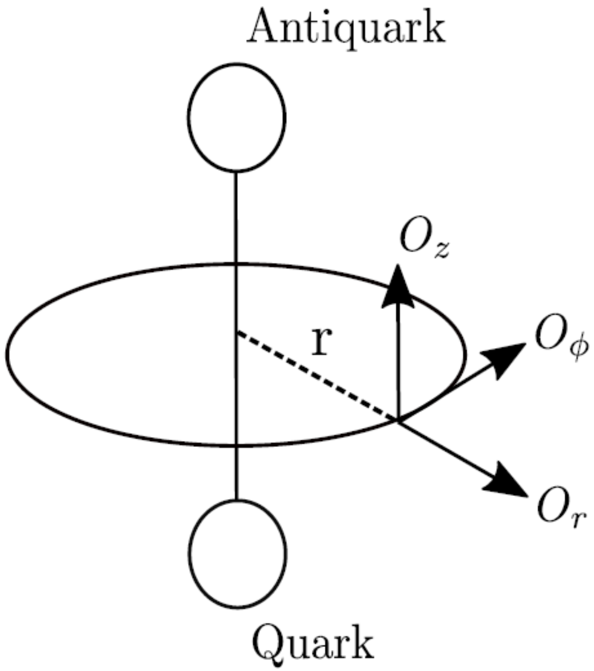
The connected correlation P.Cea, L.Cosmai, F.Cuteri, A.Papa, EPJ Web Conf. 175 (2018) 12006

$$\rho_{conn}(O(r)) = \frac{\langle \mathrm{Tr}(PLO(r)L^{\dagger})\,\mathrm{Tr}\,P^{\dagger}\rangle}{\langle \mathrm{Tr}\,P\,\mathrm{Tr}\,P^{\dagger}\rangle} - \frac{1}{3}\frac{\langle \mathrm{Tr}\,P\,\mathrm{Tr}\,P^{\dagger}\,\mathrm{Tr}\,O(r)\rangle}{\langle \mathrm{Tr}\,P\,\mathrm{Tr}\,P^{\dagger}\rangle}$$

Flux-tube profiles at finite temperature

The connected correlation P.Cea, L.Cosmai, F.Cuteri, A.Papa, EPJ Web Conf. 175 (2018) 12006



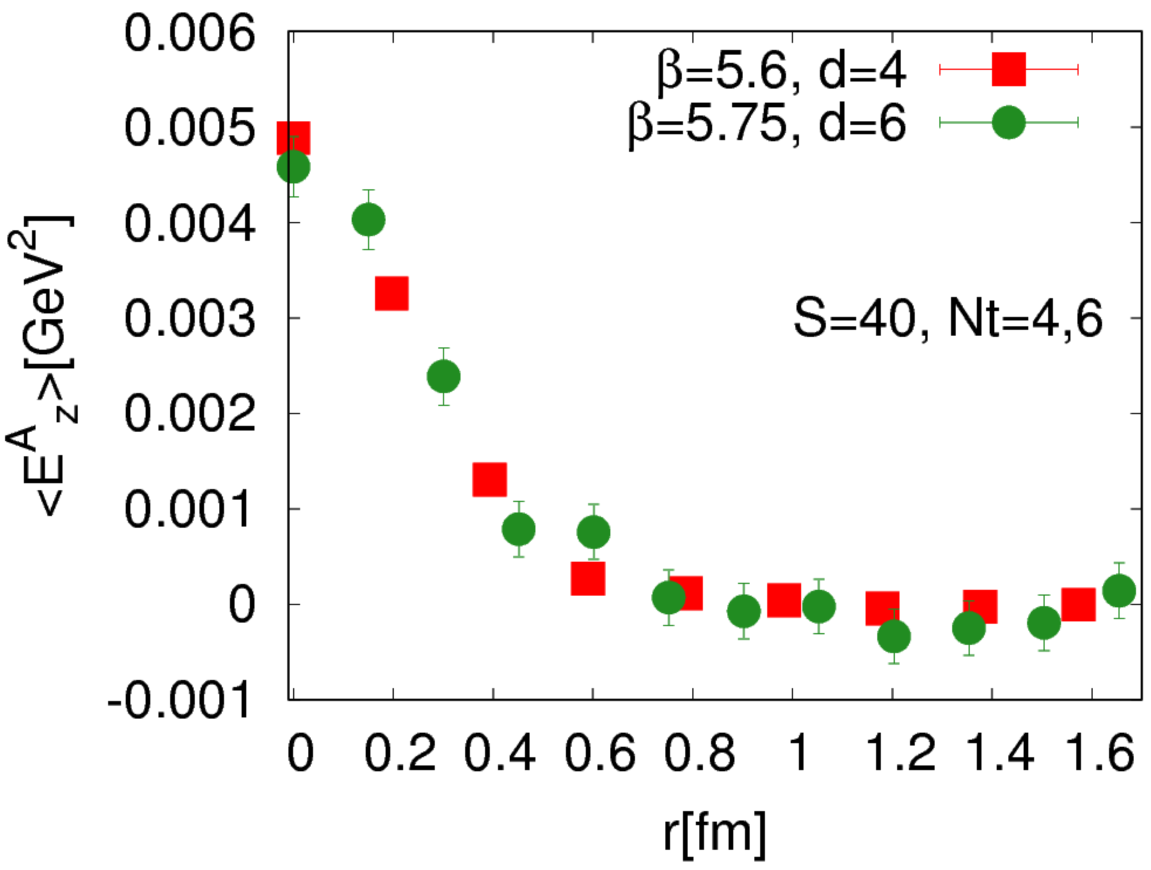


Operators:
- non-Abelian plaquette
- Abelian plaquette
- Monopole currents

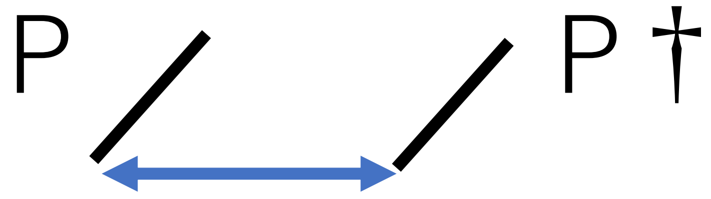
Results

Preliminary

Abelian Color electric fields



$\beta = 5.6$



d=4

$\beta = 5.75$, d=6

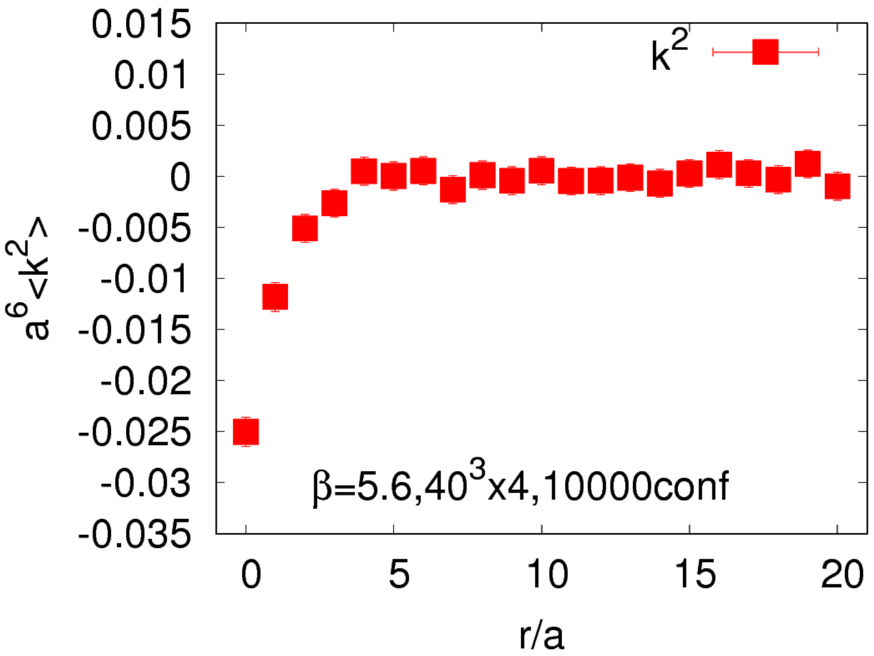
Preliminary

Coherence length

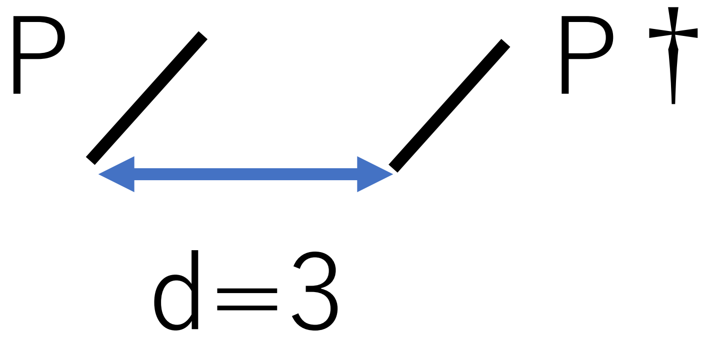
M. N. Chernodub, Katsuya Ishiguro, Yoshihiro Mori, Yoshifumi Nakamura, M. I. Polikarpov, Toru Sekido, Tsuneo Suzuki, and V. I. Zakharov. *Phys. Rev. D*, Vol. 72, p. 074505, Oct 2005.

T.Suzuki,M.Hasegawa,K.Ishiguro,Y.Koma,T.Sekido,Phys.Rev.D80:054504(2009)

$$\langle k^2(r)\rangle_{q\bar{q}} = \frac{\langle \mathrm{Tr}\, P(0)\, \mathrm{Tr}\, P^\dagger(d) \sum_{\mu,a} k_\mu^a(r) k_\mu^a(r)\rangle}{\langle \mathrm{Tr}\, P(0)\, \mathrm{Tr}\, P^\dagger(d)\rangle} - \langle \sum_{\mu,a} k_\mu^a(r) k_\mu^a(r)\rangle$$



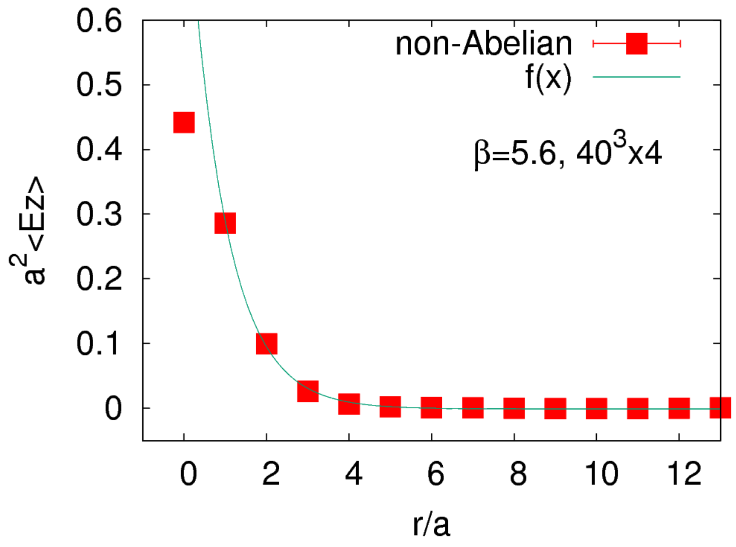
$\beta = 5.6$

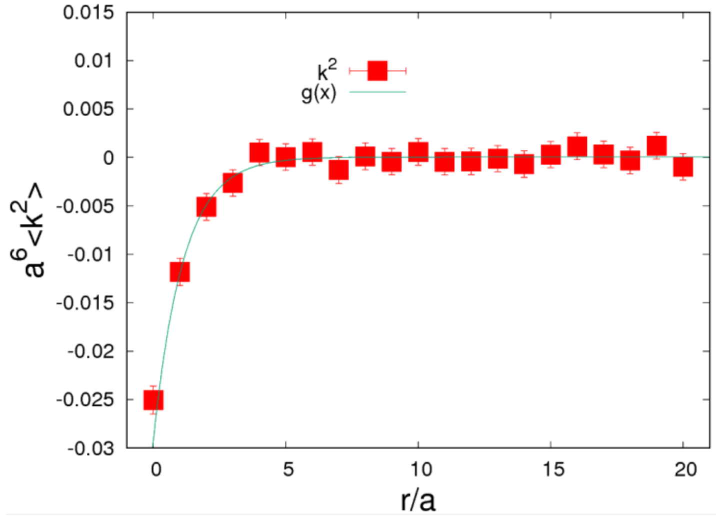


We need a huge number of statistics at large β.

Preliminary

The type of vacuum in SU(3) gauge theory





Fitting function :

$$f(r) = c_1 \exp(-\frac{r}{\lambda}) + c_0$$

$$g(r) = c_1' \exp(-\frac{\sqrt{2}r}{\xi}) + c_0'$$

λ: Penetration length

ξ: Coherence length

β	λ	$\xi/\sqrt{2}$	$\sqrt{2}\kappa$
5.6	0.93(3)	1.1(1)	0.83(8)

$\sqrt{2}\kappa < 1$ **Type I or the border**

Summary

- In the idea of VNABI, the Abelian monopole currents can be defined each colors and these are defined on the lattice.

- Abelian color electric fields show the scaling behavior.

- We calculated the vacuum type of SU(3) gauge theory at one coupling constant. It suggests the type is Type 1 or the border. To determine the vacuum type at other coupling constants, we need a huge number of statistics.