Reconstructing QCD Spectral Functions with Gaussian Processes

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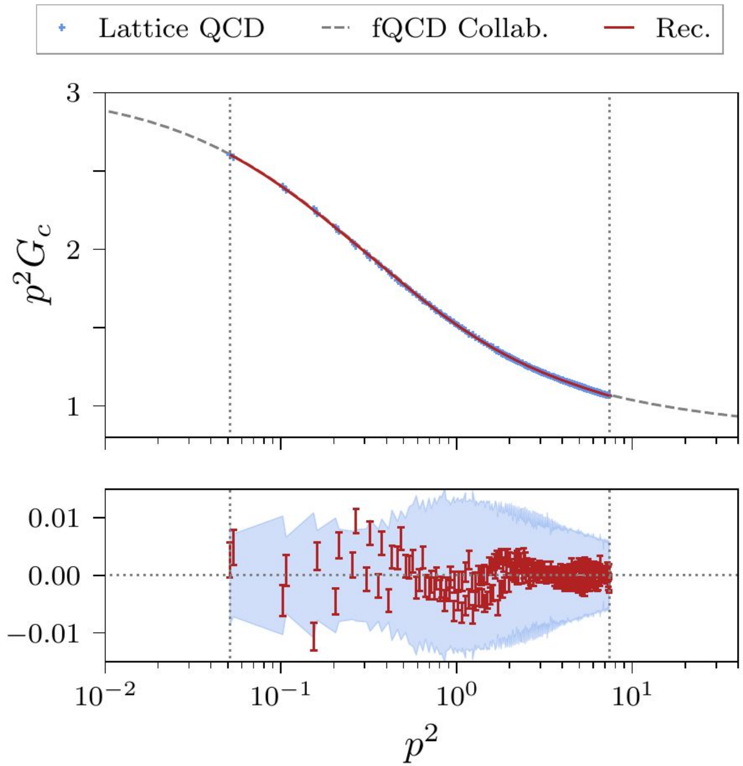
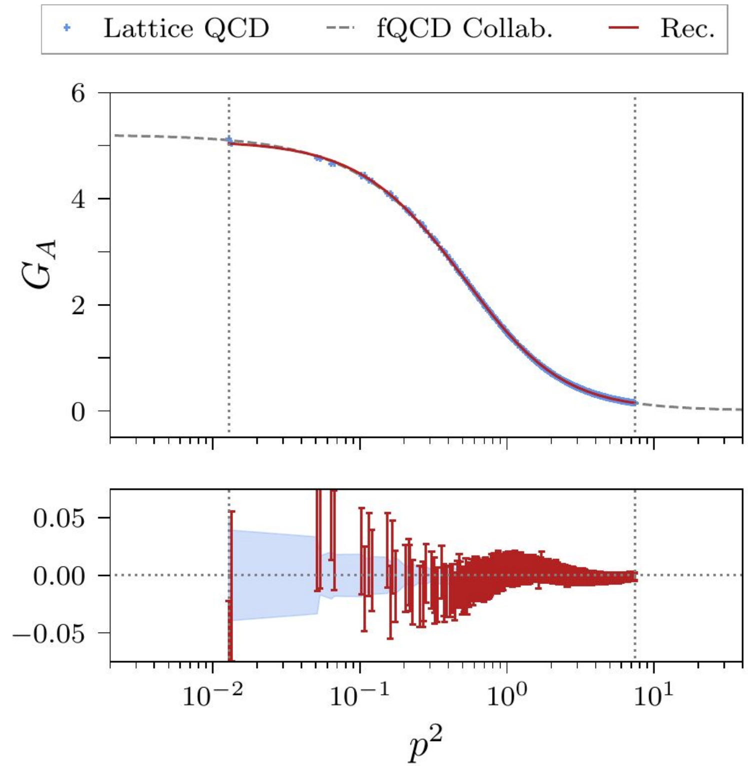
Results Teaser

Ghost



Gluon



Outline

- Motivation
- Källén-Lehmann spectral representation
- Spectral reconstruction with Gaussian process regression
- Results and discussion
- Summary and outlook

Motivation

- Imaginary-time accessible via lattice and functional methods
- Real-time properties still notoriously difficult
- Applications
  - Hadronic resonance spectrum
  - Scattering processes
  - Transport & non-equilibrium phenomena in heavy-ion collisions

➔ From Euclidean to Minkowski via spectral reconstruction

Spectral Functions

- Spectral representation of two-point correlators

$$G(p_0) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \frac{\omega\rho(\omega)}{\omega^2 + p_0^2} = \int_0^\infty \mathrm{d}\omega\, K(p_0, \omega)\, \rho(\omega)$$

$$\rho(\omega) = 2\,\mathrm{Im}\, G(-\mathrm{i}(\omega + \mathrm{i}0^+))$$

- Properties of the spectral function
  - Vanishing spectral weight: $\int_0^\infty \frac{\mathrm{d}\omega}{\pi} \omega\rho_{A/c}(\omega) = 0$
  - Decomposition into resonance peaks and continuous part:

$$\rho_c(\omega) = \frac{\pi}{Z_c}\frac{\delta(\omega)}{\omega} + \tilde{\rho}_c(\omega)\ , \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\, \omega\, \tilde{\rho}_c(\omega) = -\frac{1}{Z_c}$$

Reconstruction with GPR

- Gaussian process for spectral function

$$\rho(\omega) \sim \mathcal{GP}(\mu(\omega), C(\omega, \omega'))$$

- Gaussian distribution for correlator via KL integral

$$G_i \sim \mathcal{N}\Big(\int \mathrm{d}\omega\ K(p_i, \omega)\mu(\omega),$$
$$\int \mathrm{d}\omega\, \mathrm{d}\omega' K(p_i, \omega) C(\omega, \omega') K(p_j, \omega')\Big)$$
$$\equiv \mathcal{N}(\tilde{\mu}_i, \tilde{C}_{ij})$$

- Inverse inference step

$$\rho(\omega) \,|\, G_i \sim \mathcal{GP}\Big(\mu(\omega) +$$
$$\sum_{i,j=1}^{N_G} \int \mathrm{d}\omega'\, K(p_i, \omega') C(\omega', \omega) \left(\tilde{C} + \sigma_n^2 \cdot \mathbf{1}\right)^{-1}_{ij} (G_j - \tilde{\mu}_j)$$
$$C(\omega, \omega') - \sum_{i,j=1}^{N_G} \int \mathrm{d}\omega' \mathrm{d}\omega''\, K(p_i, \omega') C(\omega', \omega)$$
$$\left(\tilde{C} + \sigma_n^2 \cdot \mathbf{1}\right)^{-1}_{ij} K(p_j, \omega'') C(\omega'', \omega)\Big)$$

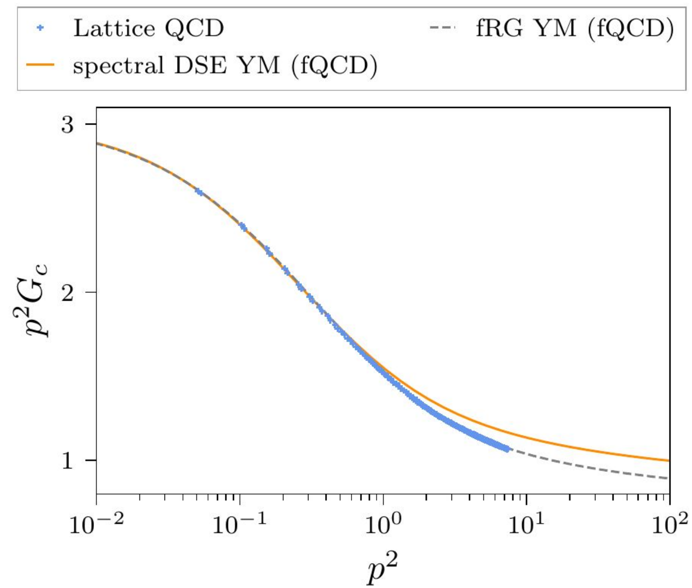
- Kernel parametrization, e.g. radial basis function

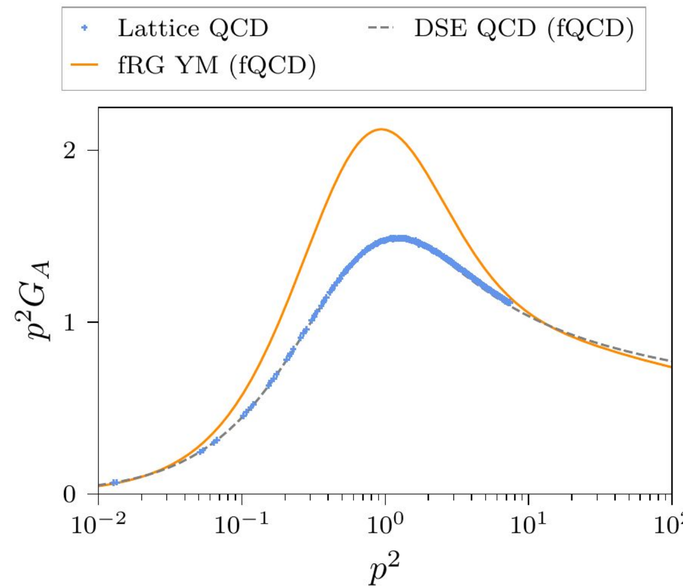
$$C(\omega, \omega') = \sigma_C^2 \exp\!\left(-\frac{(\omega - \omega')^2}{2l^2}\right)$$

Further details: `10.1093/gji/ggz520`

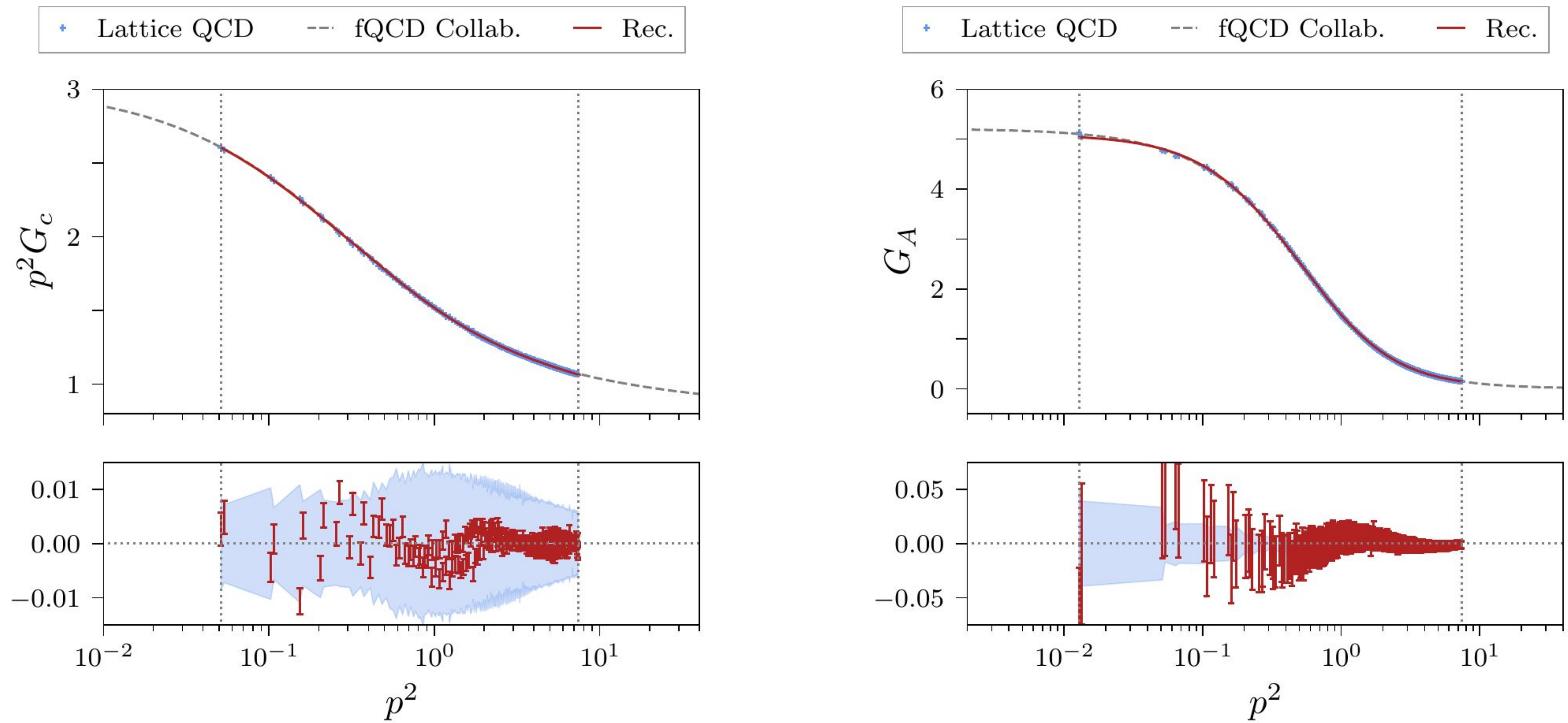
Input Data & Benchmarks

- Lattice: 2+1 flavor, Iwasaki + DWF, m_π = 139 MeV, courtesy of RBC/UKQCD collab.
  1902.08148,1912.08232
- Functional: 2+1 flavor & Yang-Mills, DSE & fRG, courtesy of fQCD collab.
  1605.01856,1706.06326,1804.00945,1909.02991,2002.07500,2102.13053,2103.16175





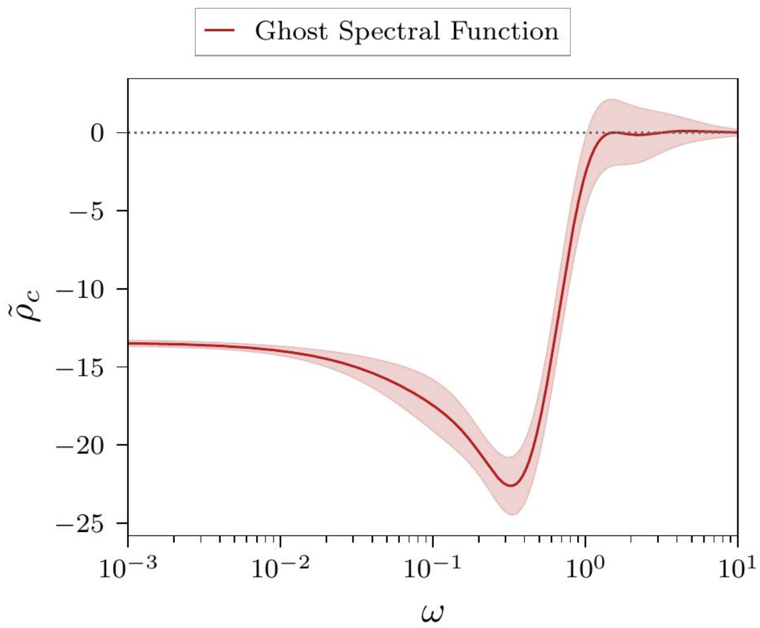
Results



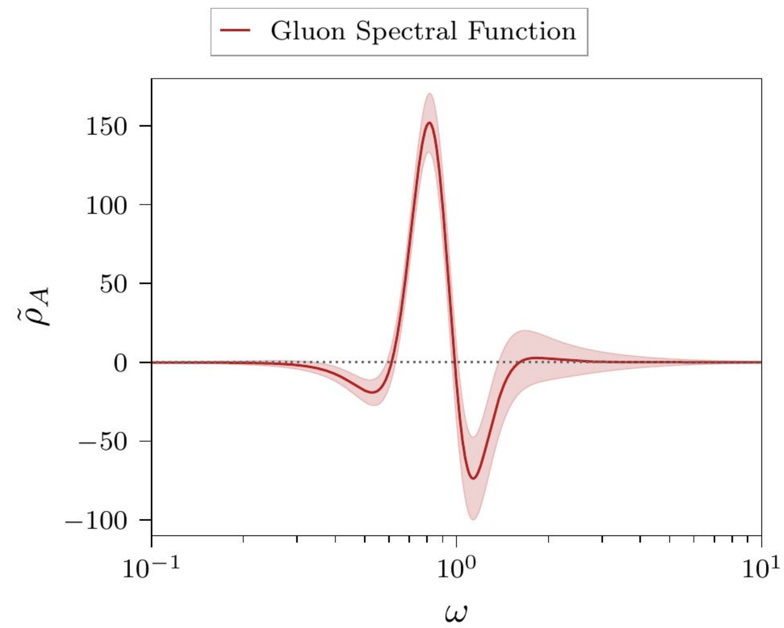
MSE: ~5e-6

MSE: ~4e-5

Results

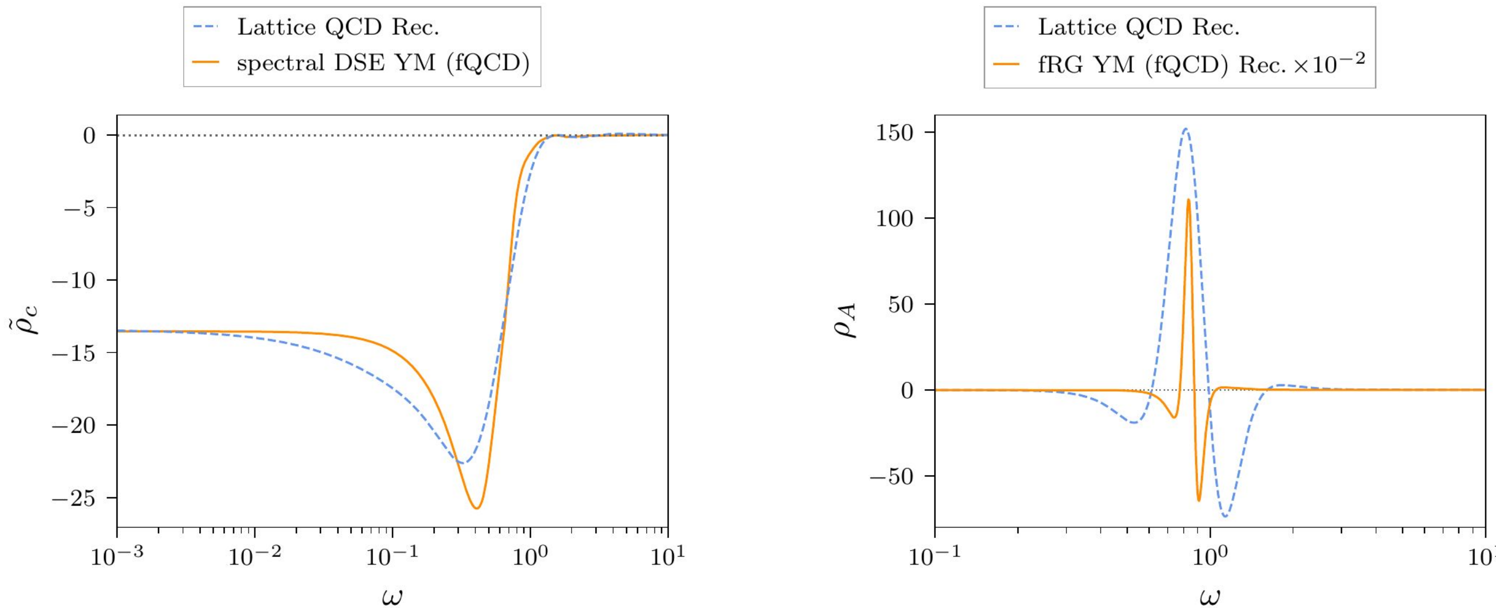


Ghost Spectral Function
$\tilde{\rho}_c$
0
−5
−10
−15
−20
−25
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
ω



Gluon Spectral Function
$\tilde{\rho}_A$
150
100
50
0
−50
−100
10^{-1}
10^{0}
10^{1}
ω

Comparison to Yang-Mills Results



- Leading peak positions of the gluon spectral function:

  QCD: ω ~ 0.818, Yang-Mills: ω ~ 0.835

Summary

- Reconstruction of ghost and gluon spectral functions in 2+1 flavor QCD with Gaussian process regression
- Lattice correlators accurately reproduced within statistical uncertainties
- Qualitative agreement with Yang-Mills spectral functions from functional computations
- Pronounced quasi-particle peak in the gluon spectral function supporting phenomenological transport approaches

Outlook

- Building blocks of diagrammatic representations for transport coefficients, bound state equations (Bethe-Salpeter, Faddeev)
- First-principle QCD inputs for phenomenological approaches to transport processes
- Improvements to reconstruction approach: custom kernels, hyperkernels, deep kernel learning, hyperpriors, derivatives, ...

Next steps: quark spectral functions & finite temperature

Thank you!