The flux tube profile in full QCD

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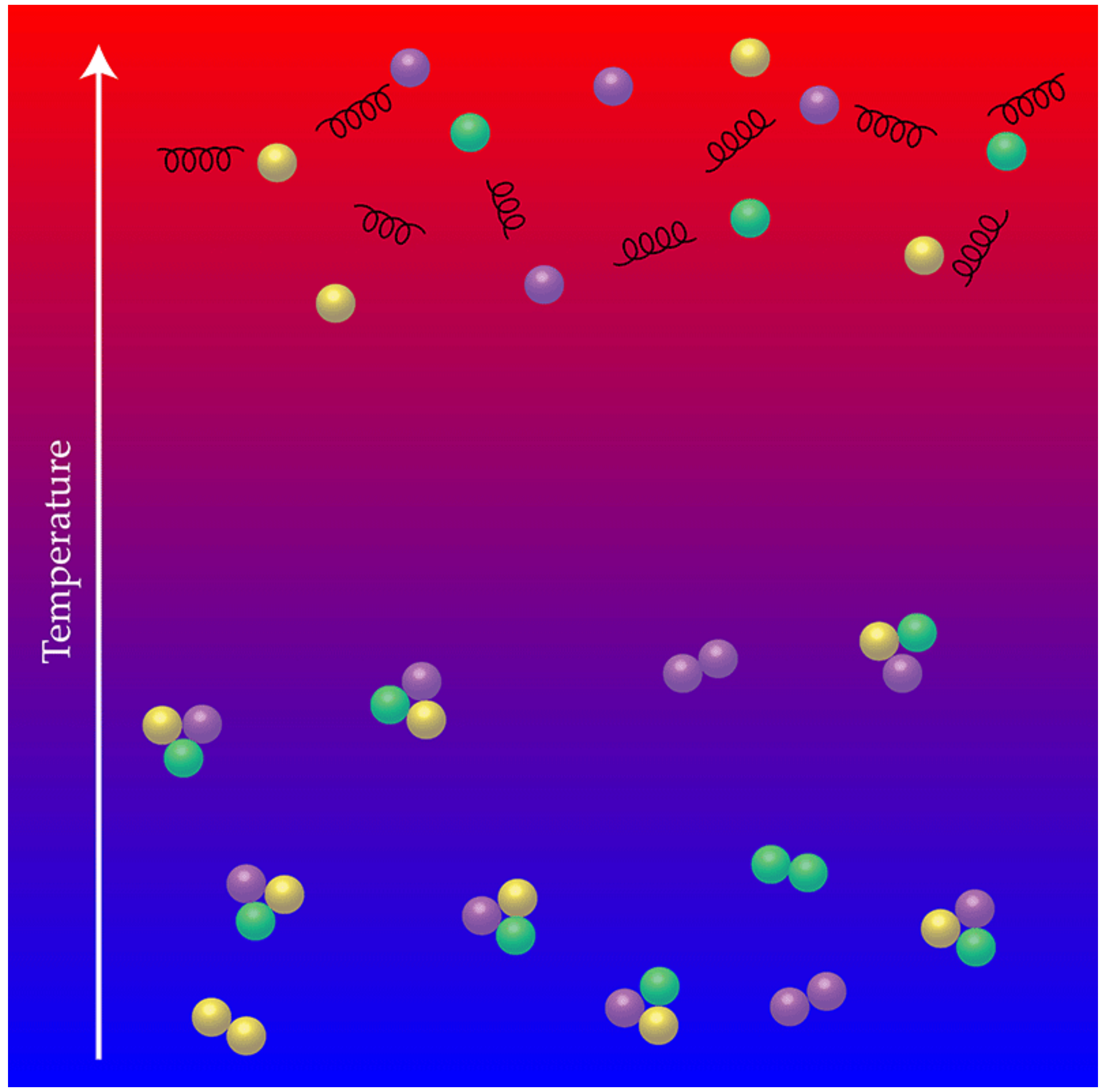
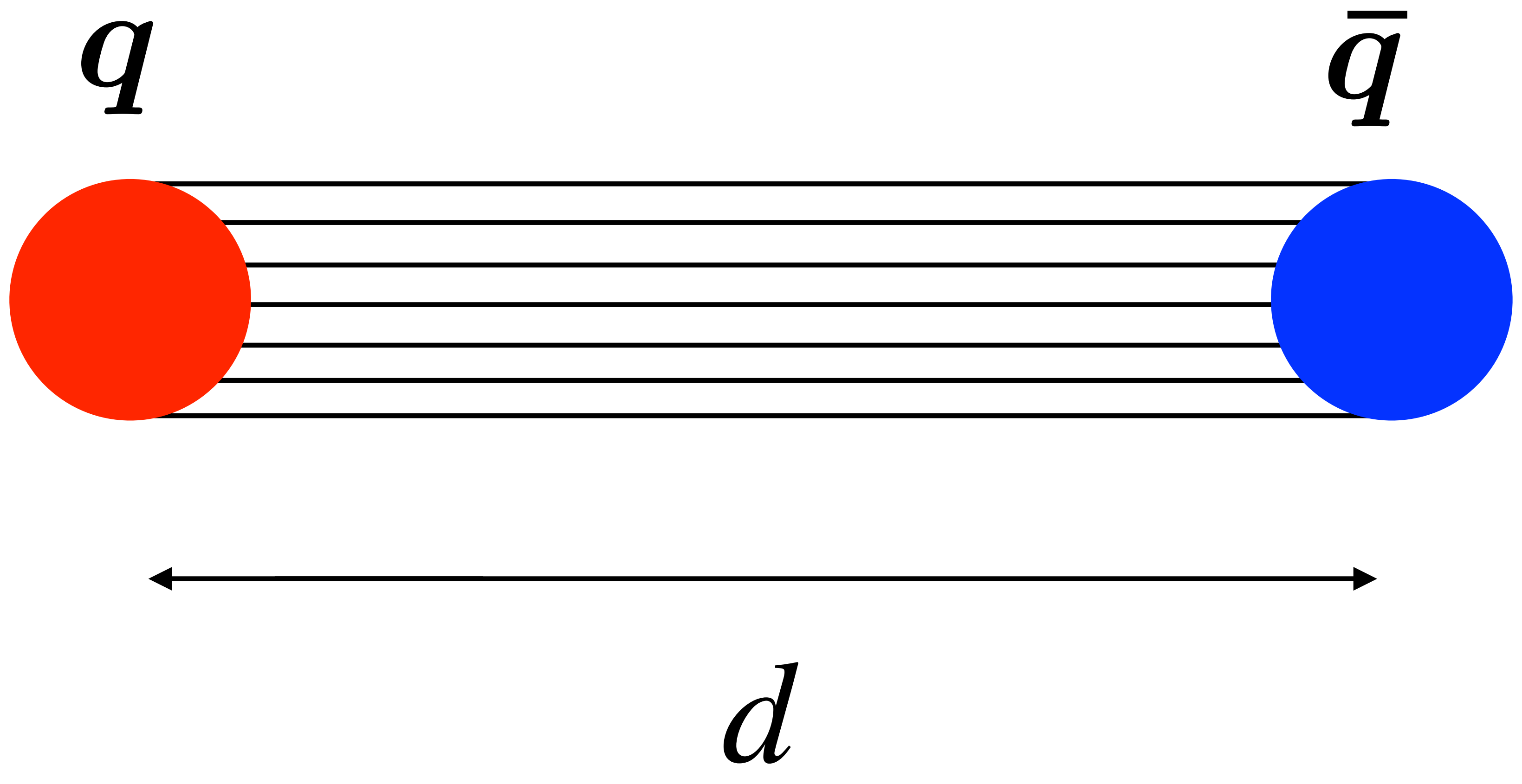
Flux Tubes and Confinement

- Reaching a detailed understanding of **color confinement** is still a central goal for nonperturbative studies of QCD.

- It is known since long that, in lattice numerical simulations, **tubelike structures** emerge by analyzing the chromoelectric fields between static quarks.
  Such tubelike structures naturally lead to a linear potential between static color charges and, consequently, to a ***direct numerical evidence of color confinement***.

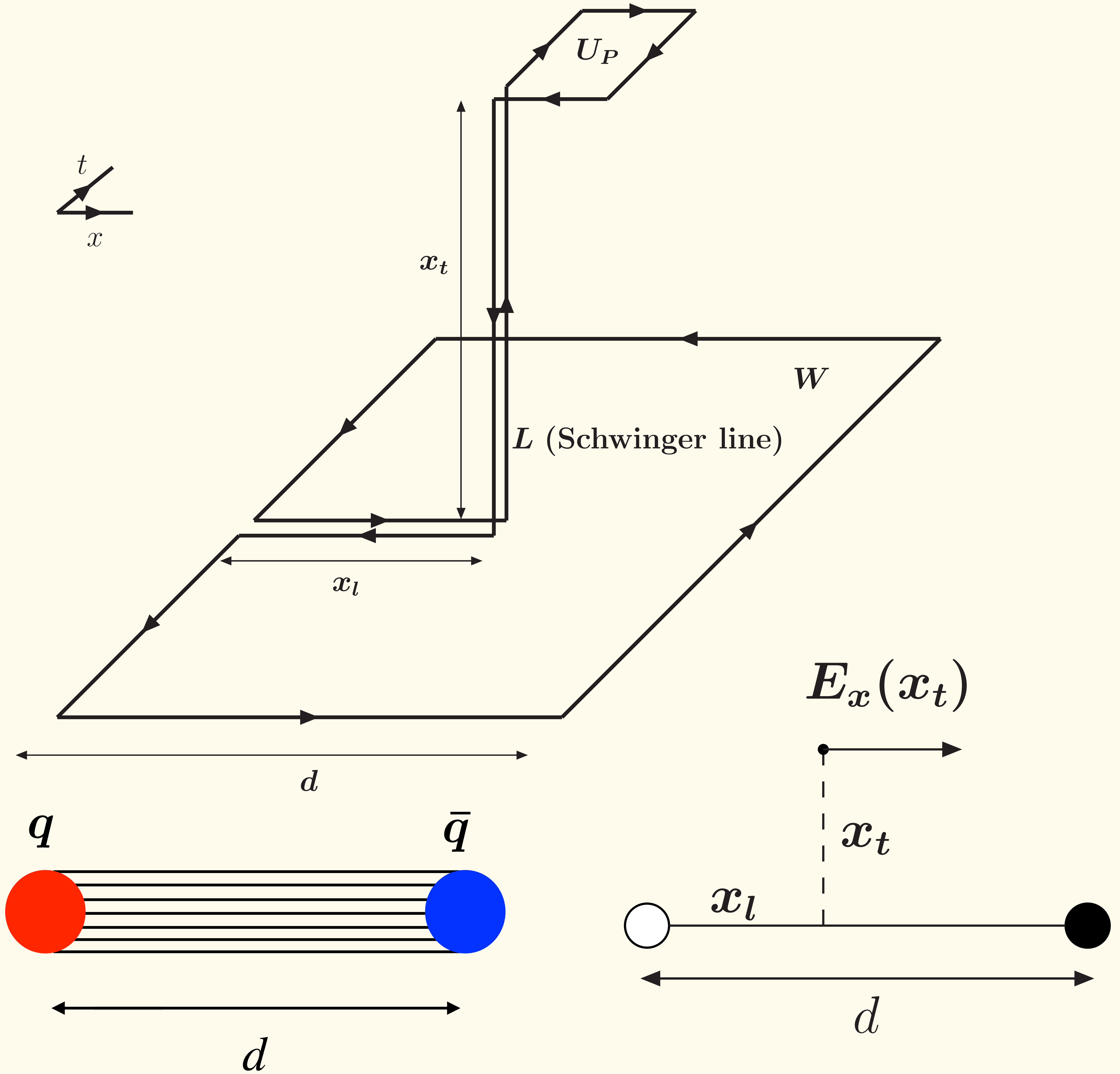
[Credit: APS/Joan Tycko]





Flux Tubes on the lattice

To explore on the lattice the field configurations produced by a static quark-antiquark pair —> connected correlation function (*)



distance between sources

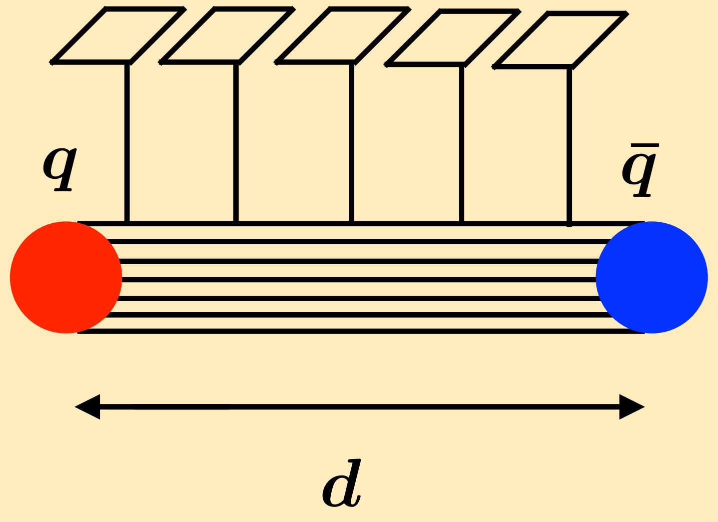
$$\rho^{\rm conn}_{W,\,\mu\nu} = \frac{\langle {\rm tr}\left(W L U_P L^\dagger\right)\rangle}{\langle {\rm tr}(W)\rangle} - \frac{1}{N}\frac{\langle {\rm tr}(U_P){\rm tr}(W)\rangle}{\langle {\rm tr}(W)\rangle}$$

$U_P = U_{\mu\nu}(x)$ plaquette in the (μ,ν) plane $\quad L$ Schwinger line $\quad N$ number of colors

$$\rho^{\rm conn}_{W,\,\mu\nu} \xrightarrow{a\to 0} a^2 g \left[\langle F_{\mu\nu}\rangle_{q\bar{q}} - \langle F_{\mu\nu}\rangle_0\right]$$

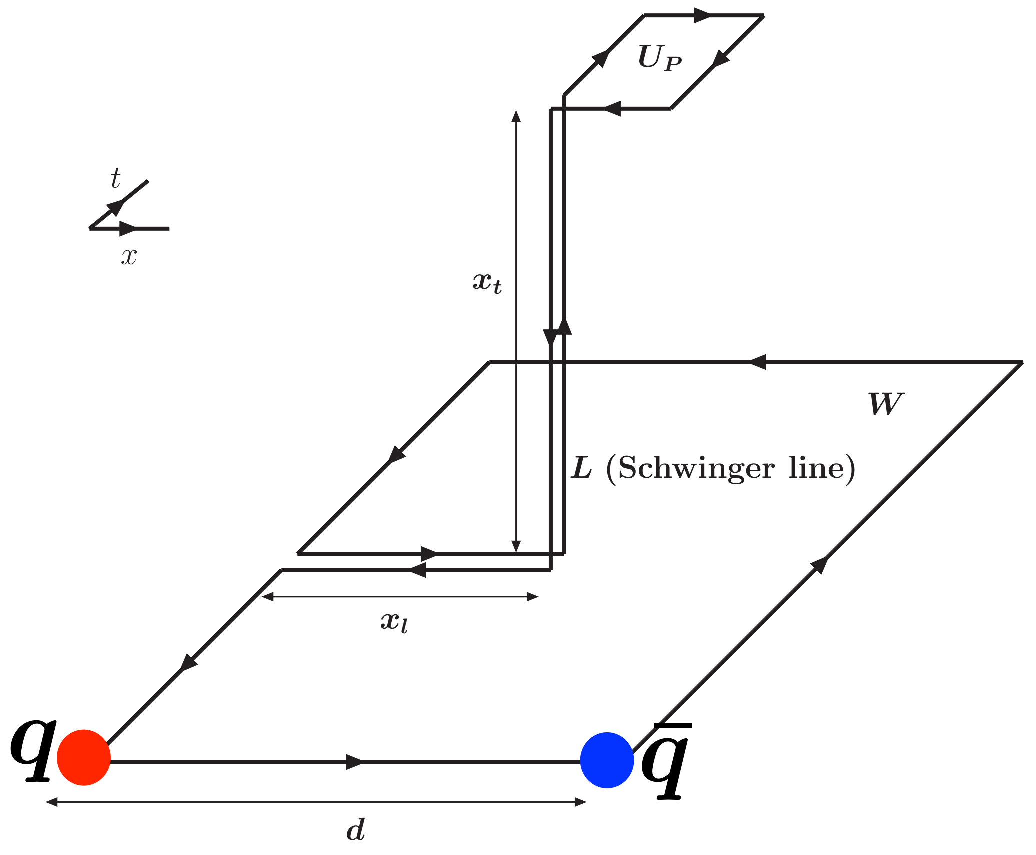
$\langle\ \rangle_{q\bar{q}}$ average in the presence of a static quark-antiquark pair $\quad \langle\ \rangle_0$ vacuum average (expected to vanish)

Full profile of the flux tube by varying the plaquette orientation and the position along the line connecting the status sources



(*) Di Giacomo, Maggiore, Oleínik , NPB347(1990)441 — Kuzmenko, Simonov, PLB494(2000)81
Skala, Faber, Zach, NPB494(1997)293 — Di Giacomo, Dosch, Shevchenko, Simonov, Phys.Rept.372(2002)319

The quark-antiquark field strength tensor on the lattice



$$a^2 \langle F_{\mu\nu} \rangle_{q\bar{q}} = \sqrt{\frac{1}{g^2} \rho^{\text{conn}}_{W,\,\mu\nu}}$$

SYMMETRY: The color fields takes on the same values at spatial points connected by rotations around the axis on which the sources are located

for example, if the Wilson Loop lies on the plane $\hat{\mu} = 4$ and $\hat{\nu} = 1$:

- plaquette U_P in the plane $(\hat{\mu} = 4, \hat{\nu} = 1) \longrightarrow E_x$
- plaquette U_P in the plane $(\hat{\mu} = 4, \hat{\nu} = 2) \longrightarrow E_y$
- plaquette U_P in the plane $(\hat{\mu} = 4, \hat{\nu} = 3) \longrightarrow E_z$
- plaquette U_P in the plane $(\hat{\mu} = 2, \hat{\nu} = 3) \longrightarrow B_x$
- plaquette U_P in the plane $(\hat{\mu} = 3, \hat{\nu} = 1) \longrightarrow B_y$
- plaquette U_P in the plane $(\hat{\mu} = 4, \hat{\nu} = 2) \longrightarrow B_z$

- Results for SU(3): [M. Baker, P. Cea, V. Chelnolov, L.C., F. Cuteri, A. Papa, arXiv:1810.07133, arXiv:1912.04739]

SU(3) Wilson action

Measurements at several values of the distance d between the static sources ($0.37\text{ fm} \leq d \leq 1.19\text{ fm}$)

Lattice scale:

[S. Necco, R. Sommer, arXiv:hep-lat/0108008]

$$a(\beta) = r_0 \times \exp\left[c_0 + c_1(\beta-6) + c_2(\beta-6)^2 + c_3(\beta-6)^3\right]$$
$$r_0 = 0.5\text{ fm}$$
$$c_0 = -1.6804\,,\ c_1 = -1.7331$$
$$c_2 = 0.7849\,,\ c_3 = -0.4428$$

Smoothing of the gauge configurations: 1 HYP t + n APE 3d with $60 \leq n \leq 120$

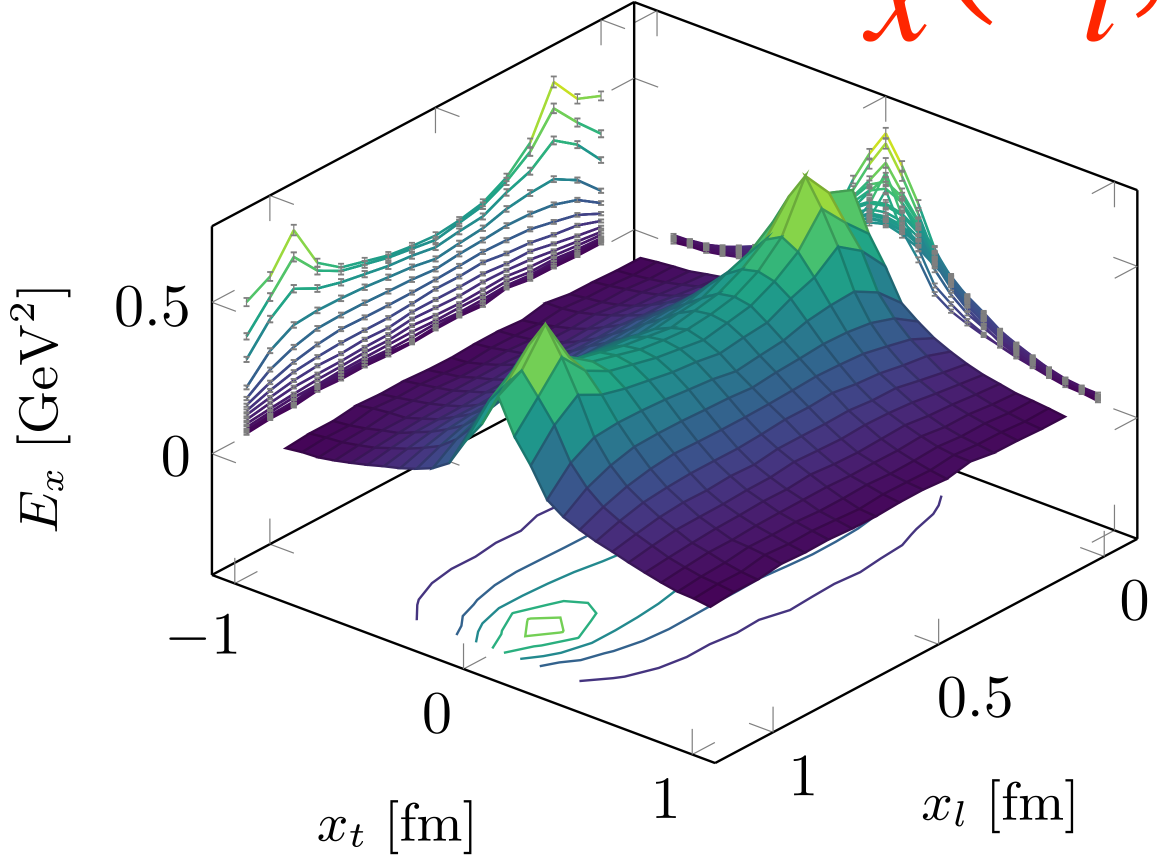
$\text{HYP1t} \rightarrow (\alpha_1 = 1.0,\ \alpha_2 = 1.0,\ \alpha_3 = 0.5)$ $\text{APE} \rightarrow \alpha_{\text{APE}} = 0.25$

- Preliminary results for QCD with (2+1) HISQ flavors for SU(3)

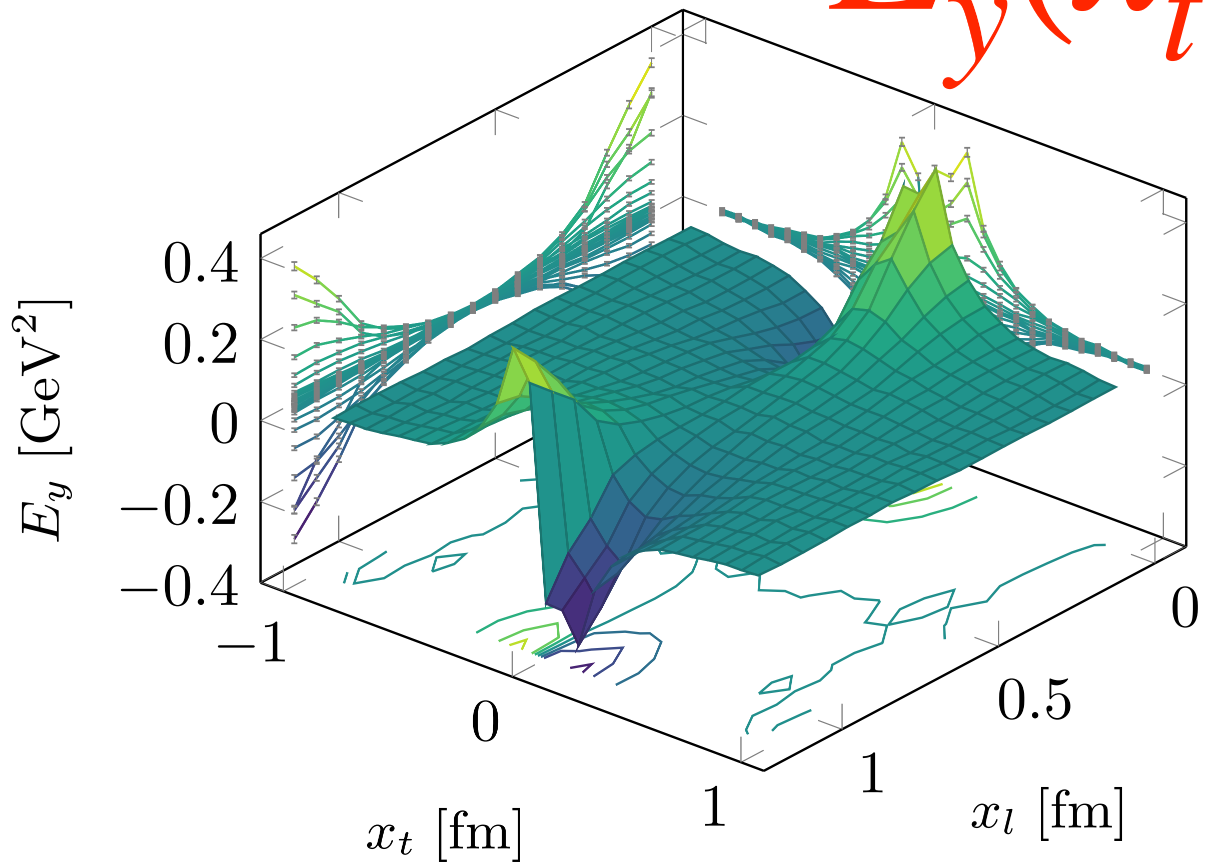
The case of SU(3) pure gauge theory

[M. Baker, P. Cea, V. Chelnolov, L.C., F. Cuteri, A. Papa, arXiv:1810.07133, arXiv:1912.04739]

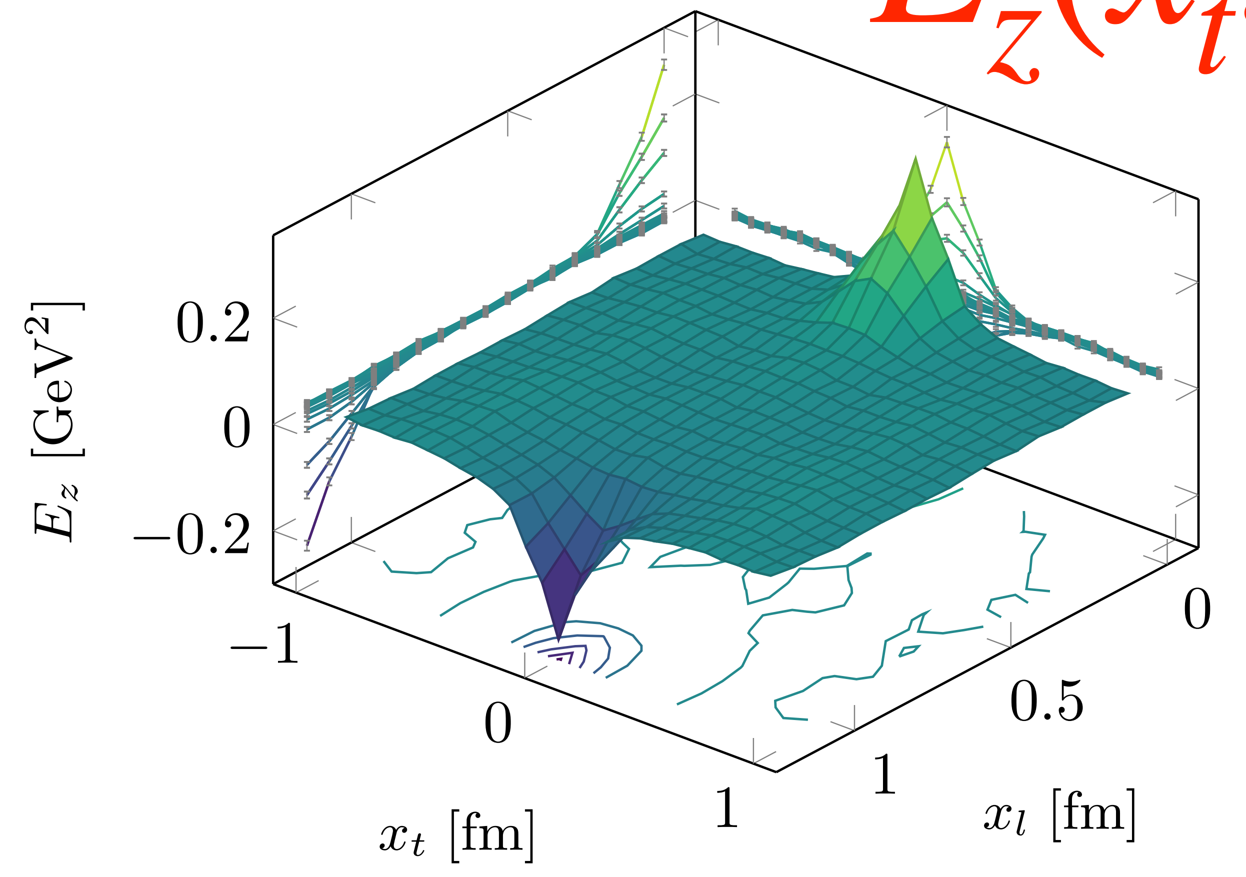
$E_x(x_t, x_l)$



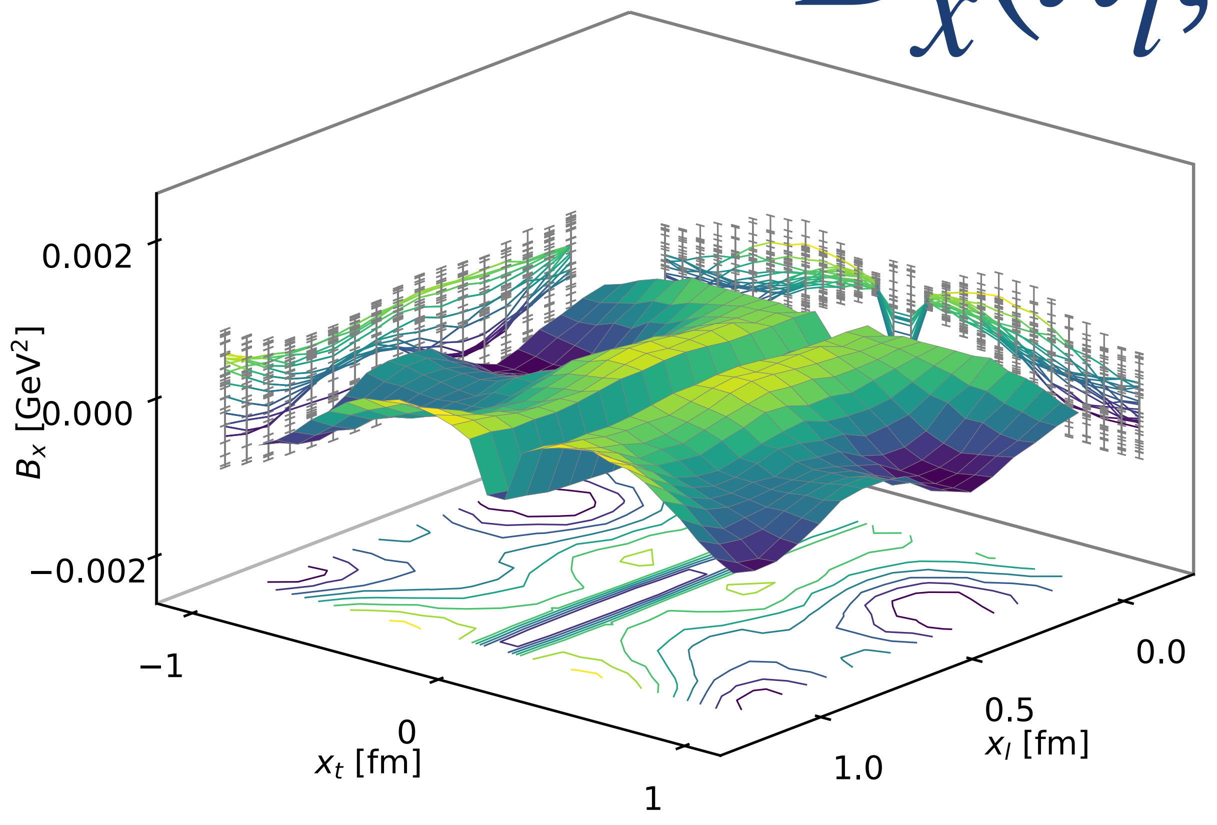
$E_y(x_t, x_l)$



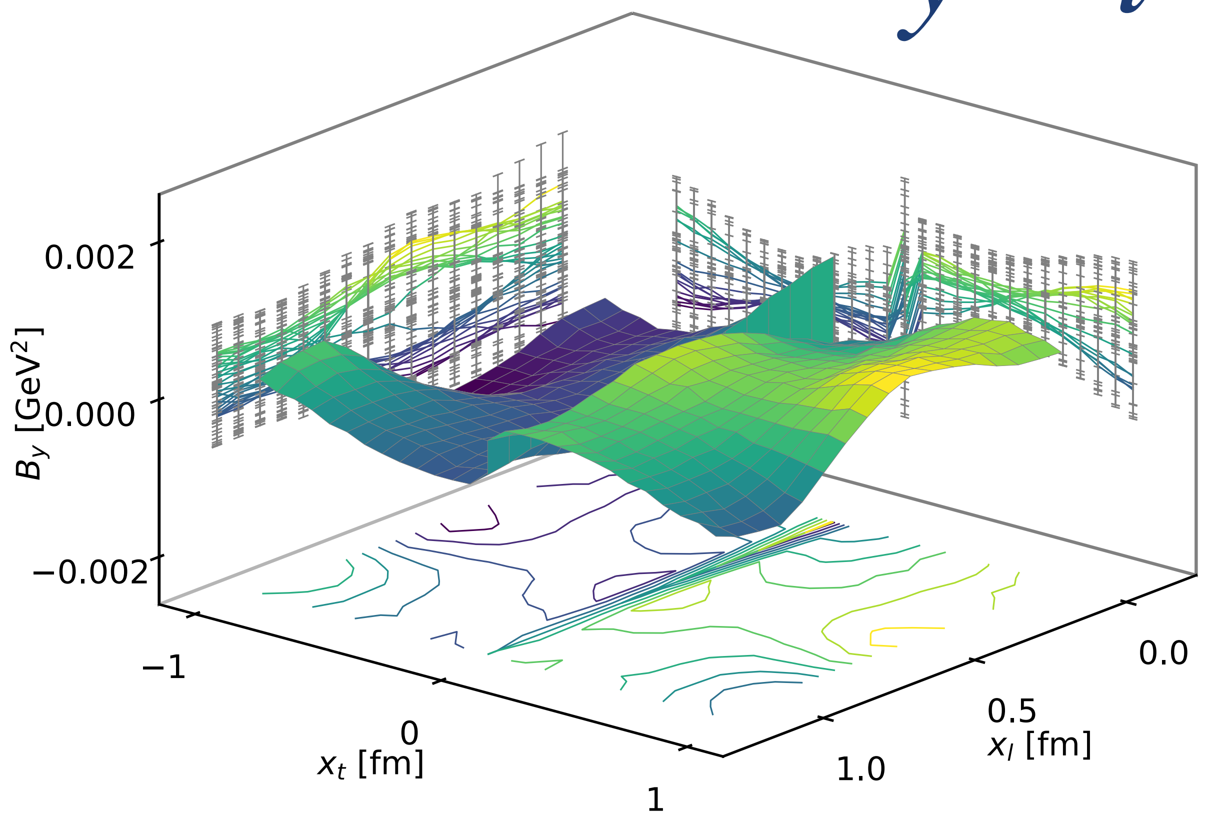
$E_z(x_t, x_l)$



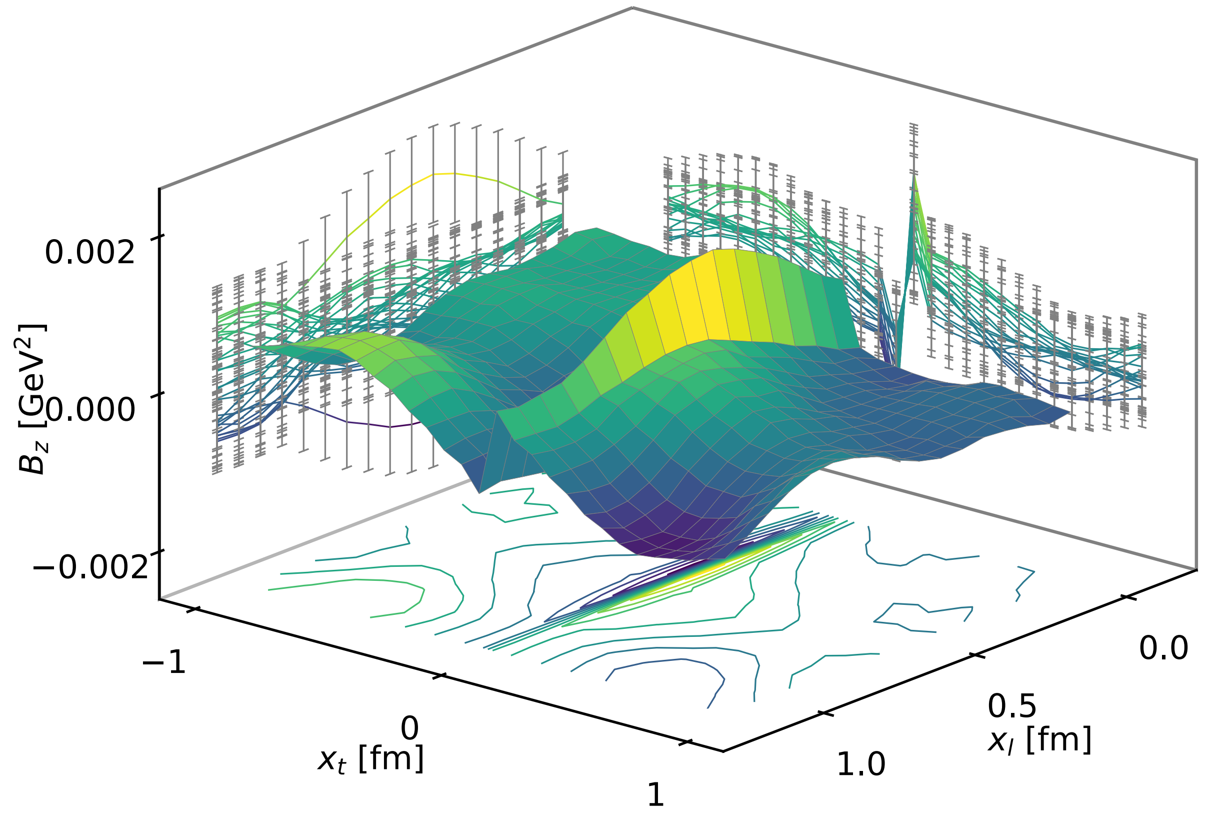
$B_x(x_t, x_l)$



$B_y(x_t, x_l)$



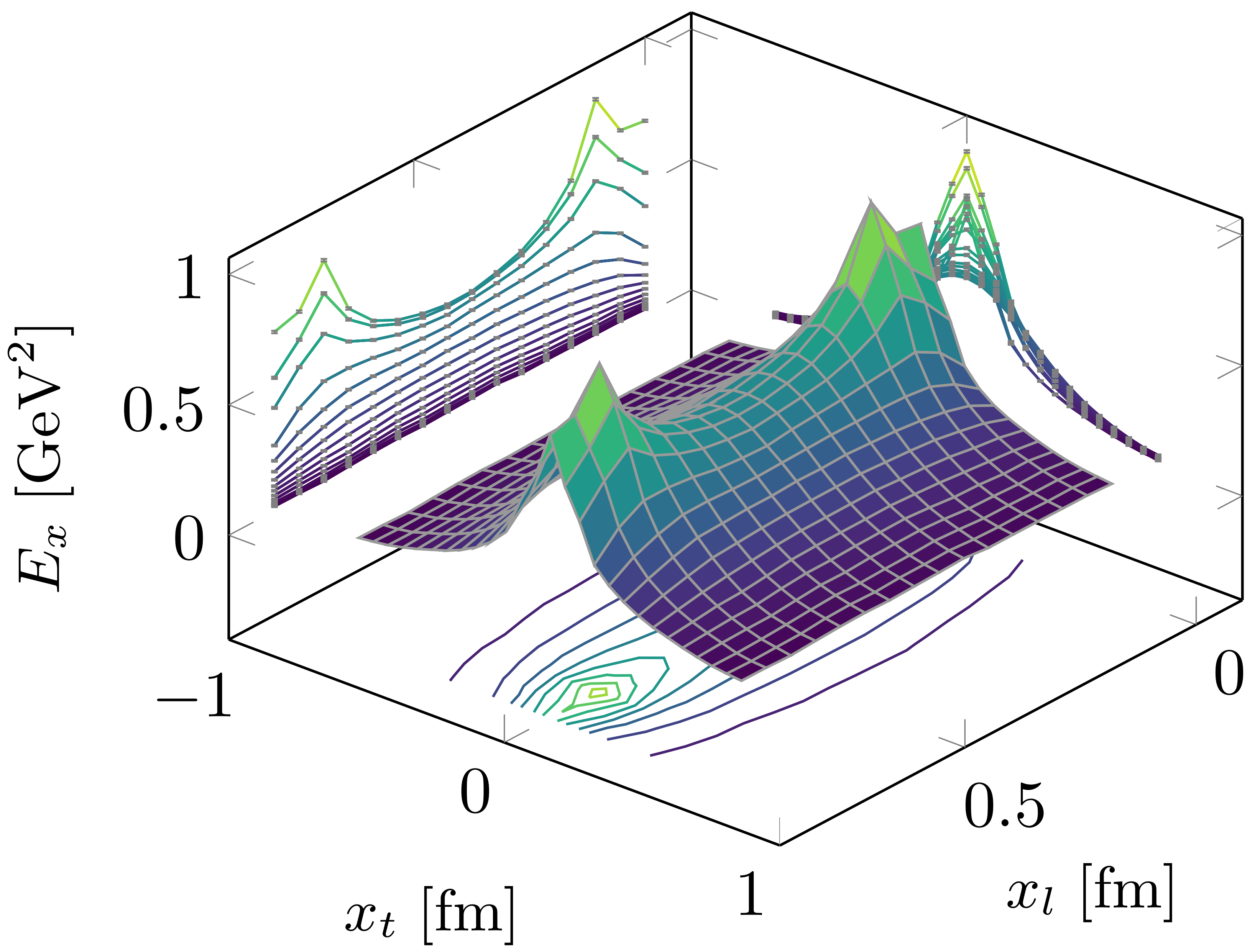
$B_z(x_t, x_l)$

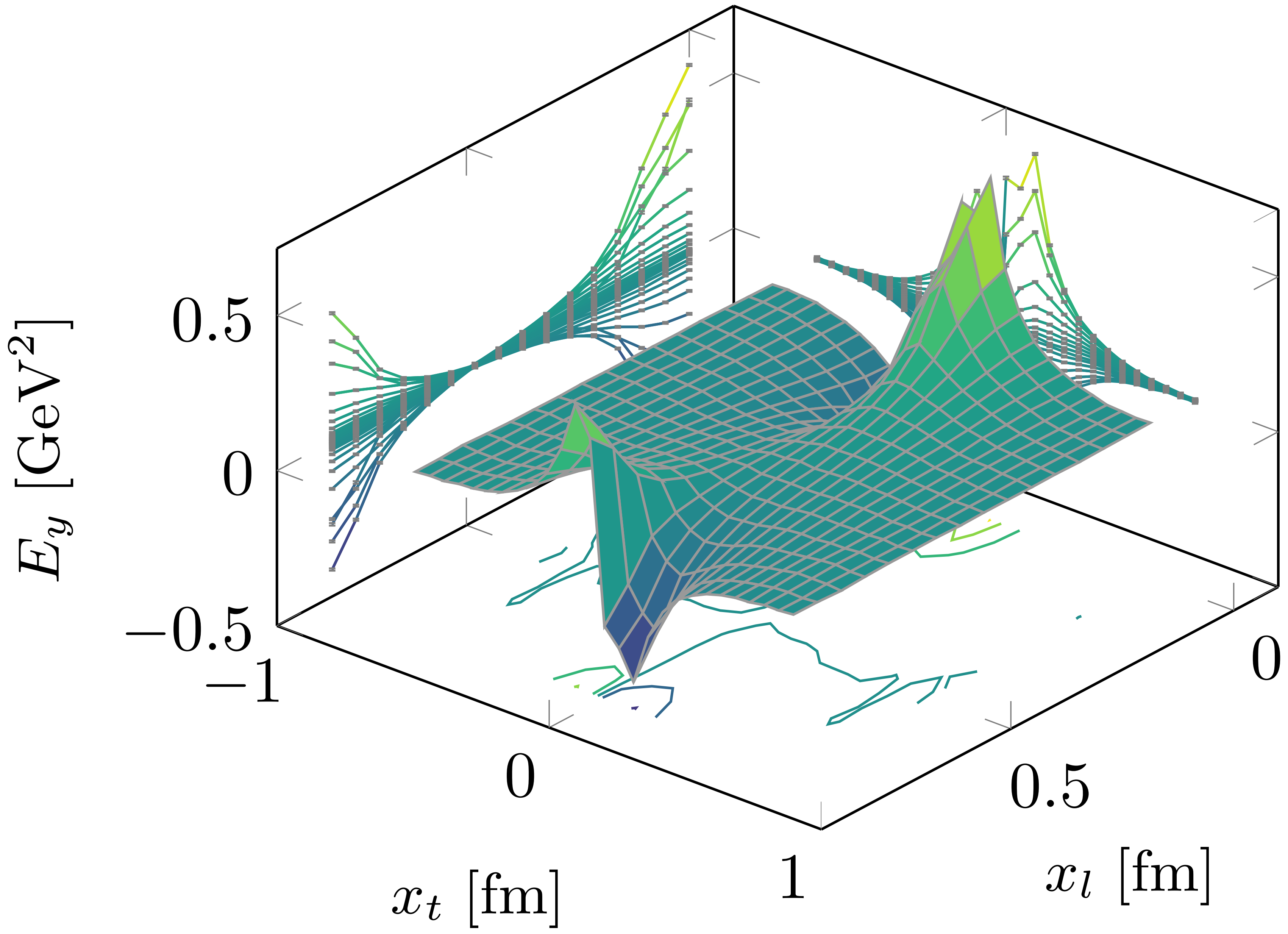


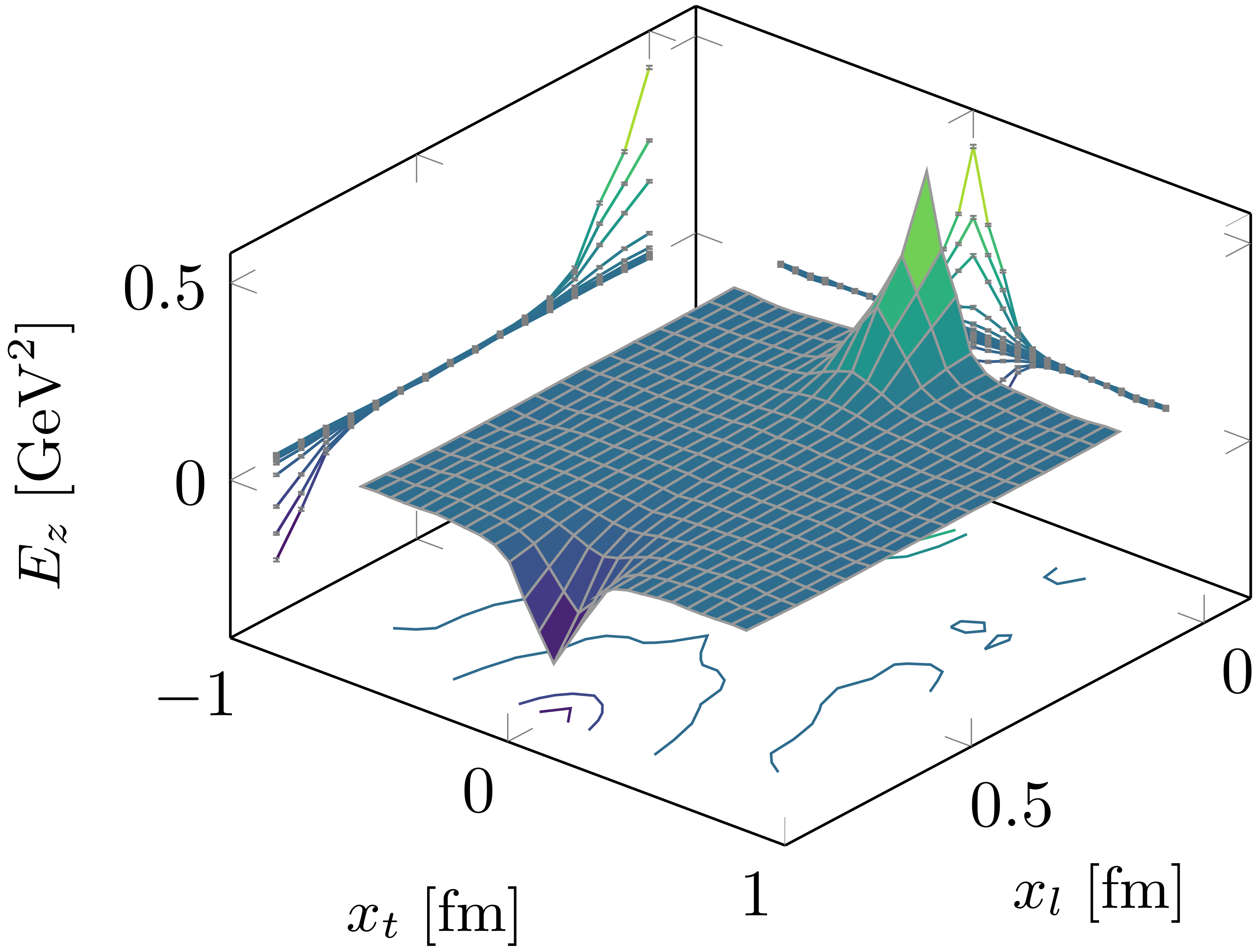
$\beta = 6.240 \quad d = 1.14$ fm

SU(3)

$\beta = 6.370 \quad d = 0.85$ fm







[M. Baker, P. Cea, V. Chelnolov, L.C., F. Cuteri, A. Papa, arXiv:1810.07133, arXiv:1912.04739]

The confining field of the QCD flux tube

- The chromomagnetic field is everywhere much smaller than the longitudinal chromoelectric field and is compatible with zero within statistical errors.
- The dominant component of the chromoelectric field is longitudinal.
- The transverse components of the chromoelectric field are also smaller than the longitudinal component but can be matched to the transverse components of an **effective Coulomb-like field** $\vec{E}^C(\vec{r})$ satisfying the conditions:

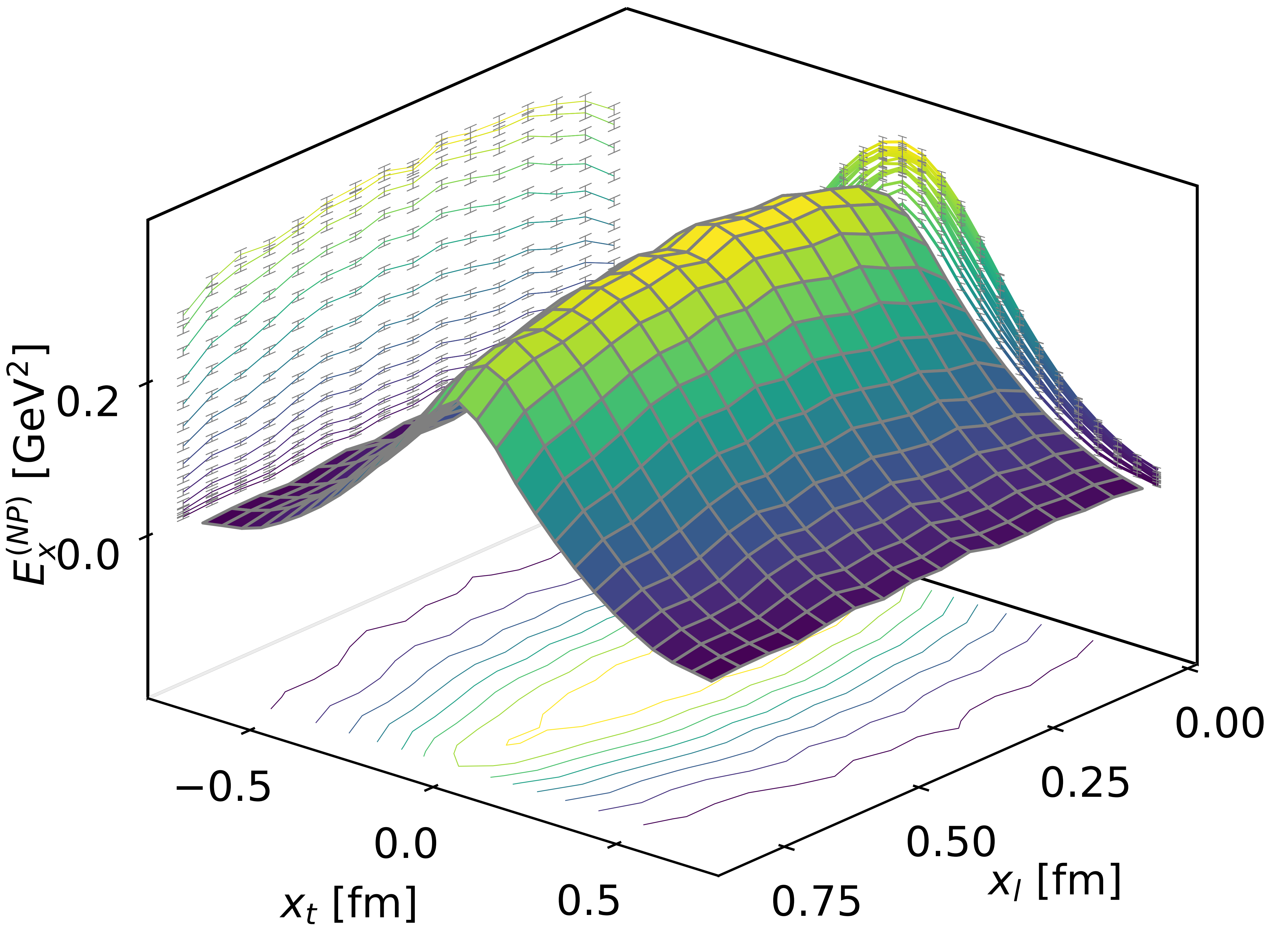
1. The transverse component E_y of the chromoelectric field is identified with the transverse component E_y^C of the perturbative field

$$E_y^C \equiv E_y$$

2. The perturbative field $\mathbf{E}^C$ is irrotational

$$\mathrm{curl}\, \mathbf{E}^C = 0$$

$$E_x^{\mathrm{NP}} = E_x - E_x^C$$



$$\beta = 6.370 \quad d = 0.85 \text{ fm}$$

[M. Baker, P. Cea, V. Chelnolov, L.C., F. Cuteri, A. Papa, arXiv:1810.07133, arXiv:1912.04739]

The string tension and the width of the chromoelectric flux tube

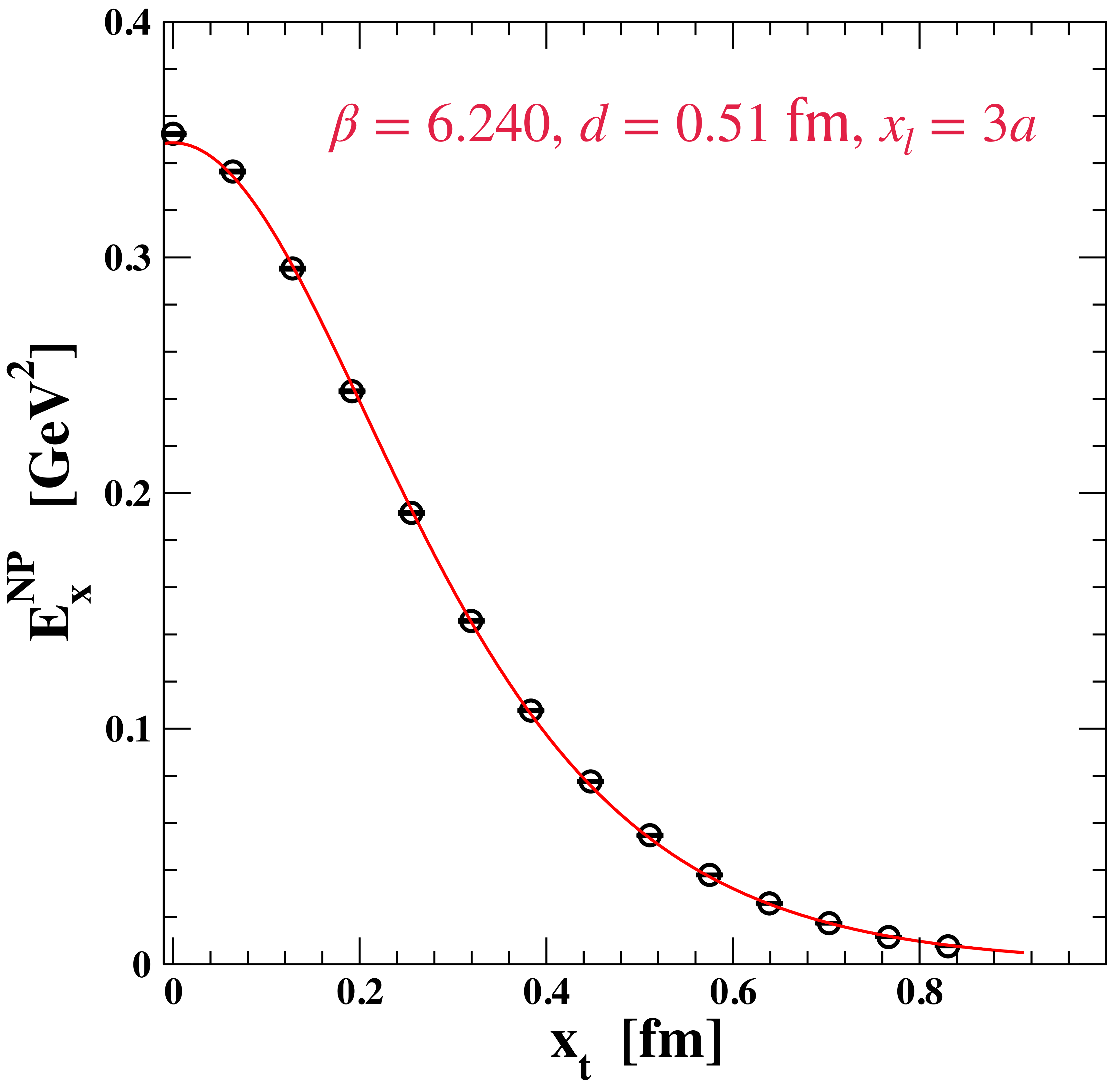
We can compute the (square root of the) **string tension** as:

$$\sqrt{\sigma} = \sqrt{\int d^2x_t \frac{(E_x^{\mathrm{NP}})^2(x_t)}{2}}$$

This determination can be done:

1) by a direct numerical integration
2) analytically, by fitting the numerical data for the transverse distribution of $E_x^{\mathrm{NP}}(x_t)$ to the Clem parameterization of the field surrounding a magnetic vortex in a superconductor:

$$E_x^{\mathrm{NP}}(x_t) = \frac{\phi}{2\pi}\frac{\mu^2}{\alpha}\frac{K_0[(\mu^2 x_t^2 + \alpha^2)^{1/2}]}{K_1[\alpha]}$$

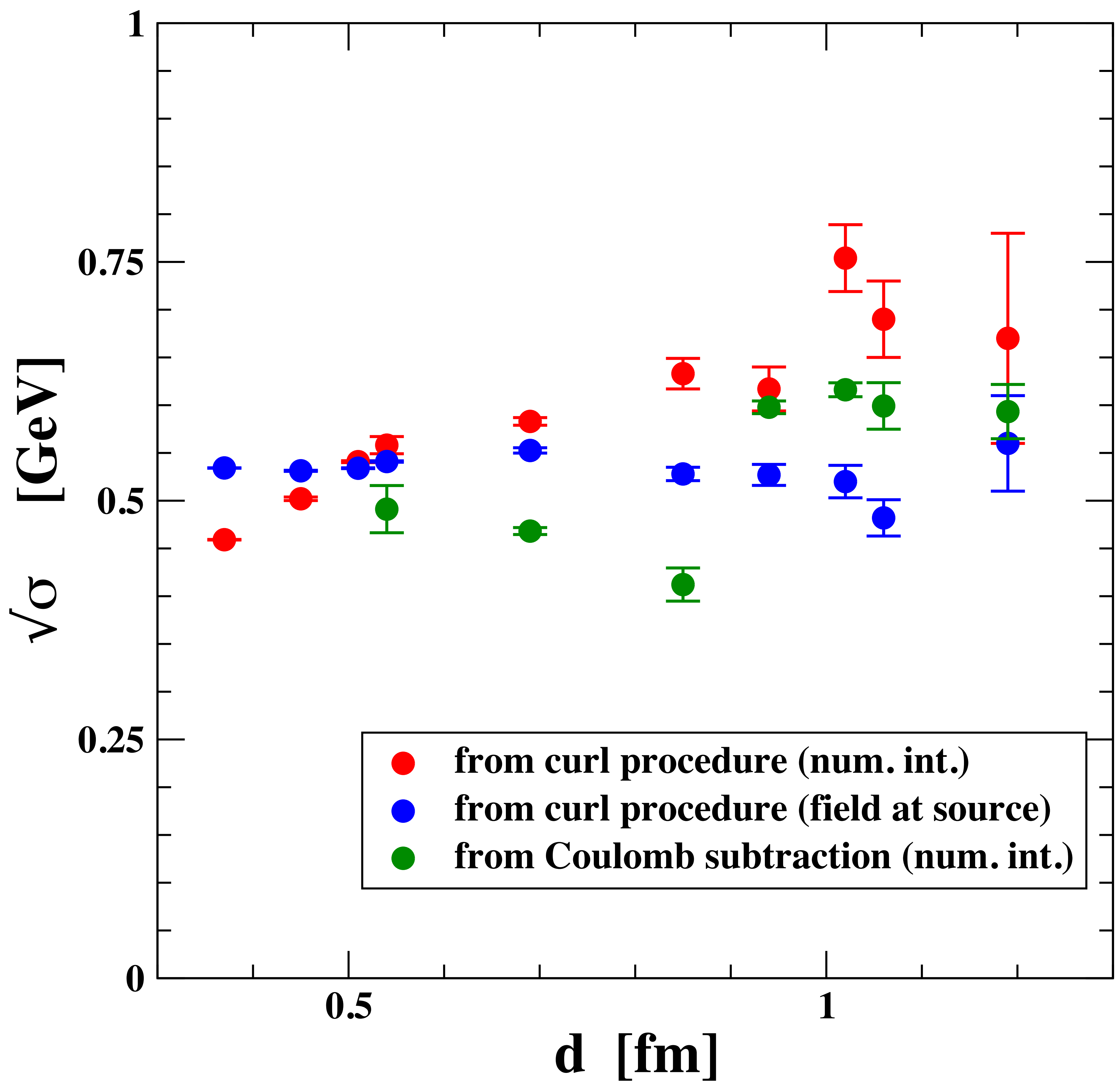


We can compute the **mean square root width of the flux tube**:

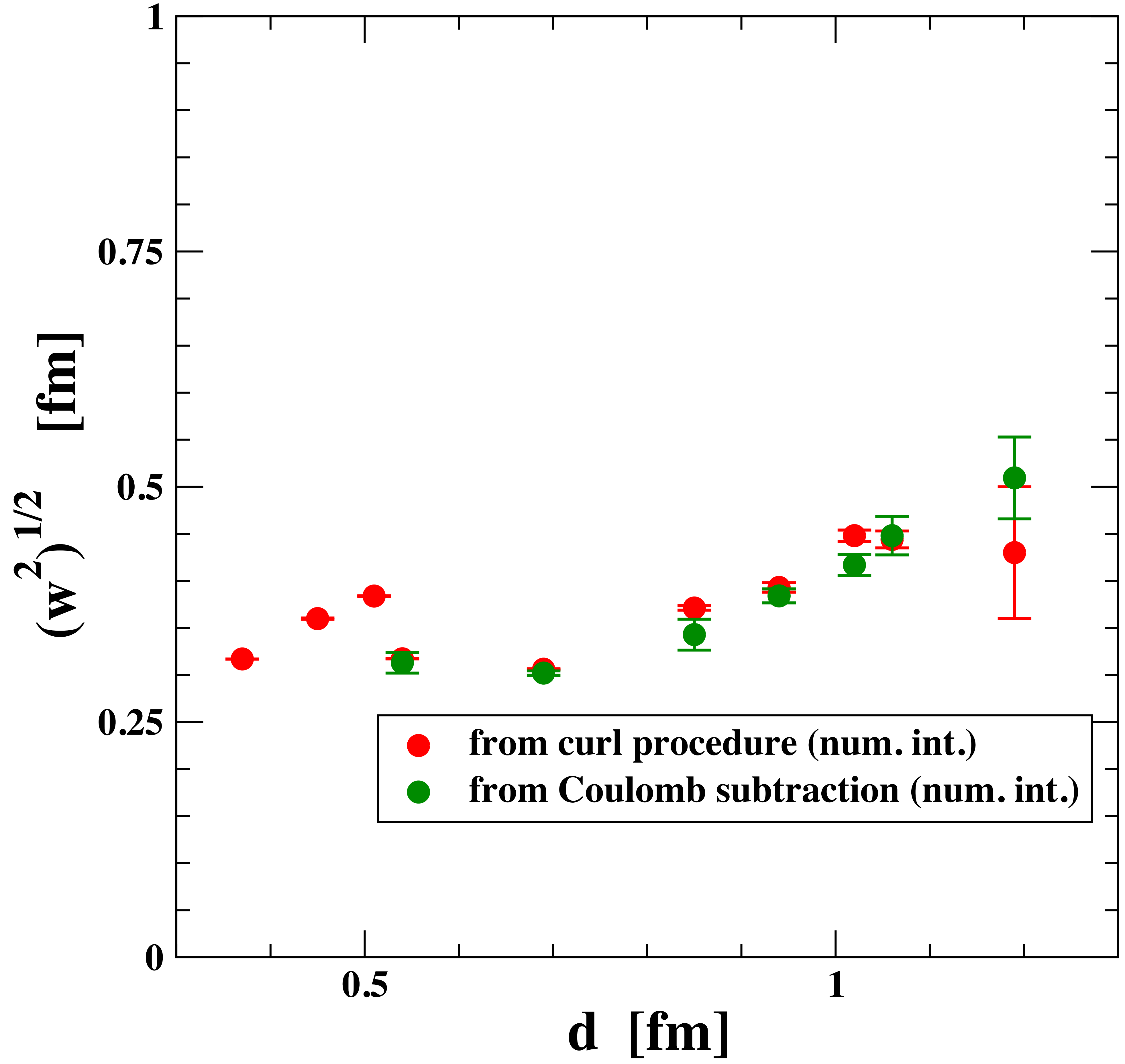
$$\sqrt{w^2} = \sqrt{\frac{\int d^2x_t\, x_t^2 E_x(x_t)}{\int d^2x_t\, E_x(x_t)}}$$

SU(3)

The string tension



The width of the chromoelectric flux tube

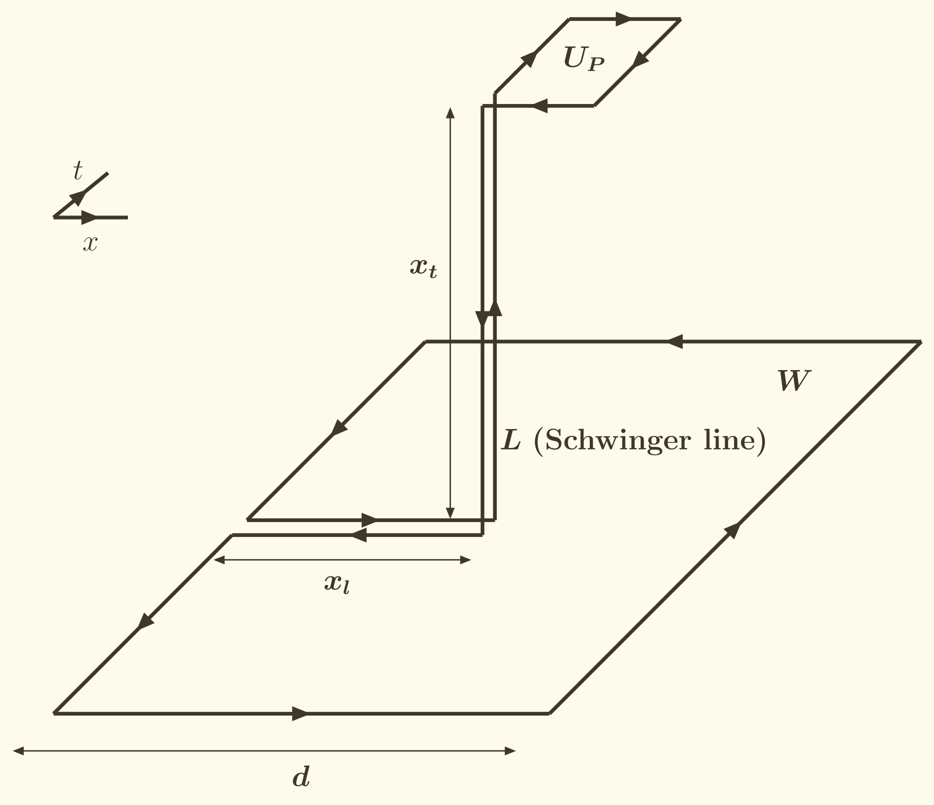


QCD with (2+1) HISQ flavors

preliminary results

QCD (2+1) HISQ flavors - Lattice setup

We want to compute the **flux tube operator** (as defined for the SU(3) pure gage case):



$$\rho^{\text{conn}}_{W,\,\mu\nu} = \frac{\langle \mathrm{tr}\,(W L U_P L^\dagger)\rangle}{\langle \mathrm{tr}(W)\rangle} - \frac{1}{N}\frac{\langle \mathrm{tr}(U_P)\mathrm{tr}(W)\rangle}{\langle \mathrm{tr}(W)\rangle}$$

- **Highly Improved Staggered Quark** action with **tree level improved Symanzik gauge action** (**HISQ/tree**) with 2+1 flavors
- **Line of constant physics (LCP)** determined (*) by fixing the strange quark mass to its physical value m_s at each value of the gauge coupling β. The light-quark mass has been fixed at $m_l = m_s/20$ ($m_\pi = 160$ MeV) (*) Bazavov et al (HotQCD Collaboration), PRD 85, 054503 (2012), arXiv:1111.1710
- **MILC code** for producing configurations and for making the measurement of the flux tube operator
- **Scale setting** —> "r_1 scale" as in Bazavov et al. arXiv:1111.1710)
- **Smoothing of the gauge configurations**: **1 HYP** on temporal links + **n HYP3d** on space links
- **Configurations** at several values of lattice spacing and lattice size:

  - 24^4, $\beta = 6.445$, $a(\beta) = 0.145$ fm $\longrightarrow$ 3330 configurations
  - 32^4, $\beta = 7.158$, $a(\beta) = 0.074$ fm $\longrightarrow$ 2029 configurations
  - 48^4, $\beta = 6.885$, $a(\beta) = 0.095$ fm $\longrightarrow$ 779 configurations

Smoothing of the gauge configurations

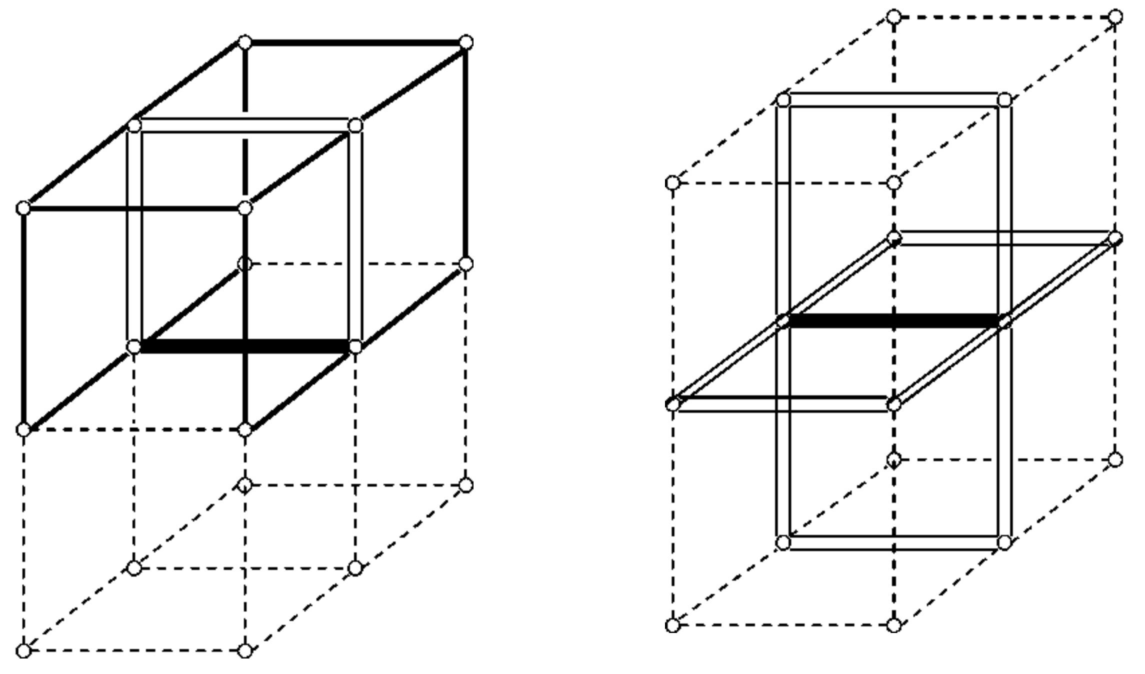
The connected correlator ρ_W exhibits large fluctuations at the scale of the lattice spacing, which are responsible for a bad signal-to-noise ratio.
To extract the physical information carried by fluctuations at the physical scale (and, therefore, at large distances in lattice units) we smoothed out configurations by a smearing procedure. [HYP smearing, A. Hasenfratz, F. Knechtli, PRD D 64 034504 (2001)]

1 HYP smearing on temporal links + n HYP3d smearing on spatial links

$$V_{n,\mu} = \mathrm{Proj}_{SU(3)}[(1-\alpha_1)U_{n,\mu} + \frac{\alpha_1}{6}\sum_{\pm\nu\neq\mu}\widetilde{V}_{n,\nu;\mu}\widetilde{V}_{n+\hat{\nu},\mu;\nu}\widetilde{V}^{\dagger}_{n+\hat{\mu},\nu;\mu}]$$

$$\widetilde{V}_{n,\mu;\nu} = \mathrm{Proj}_{SU(3)}[(1-\alpha_2)U_{n,\mu} + \frac{\alpha_2}{4}\sum_{\pm\rho\neq\nu,\mu}\overline{V}_{n,\rho;\nu\,\mu}\overline{V}_{n+\hat{\rho},\mu;\rho\,\nu}\overline{V}^{\dagger}_{n+\hat{\mu},\rho;\nu\,\mu}]$$

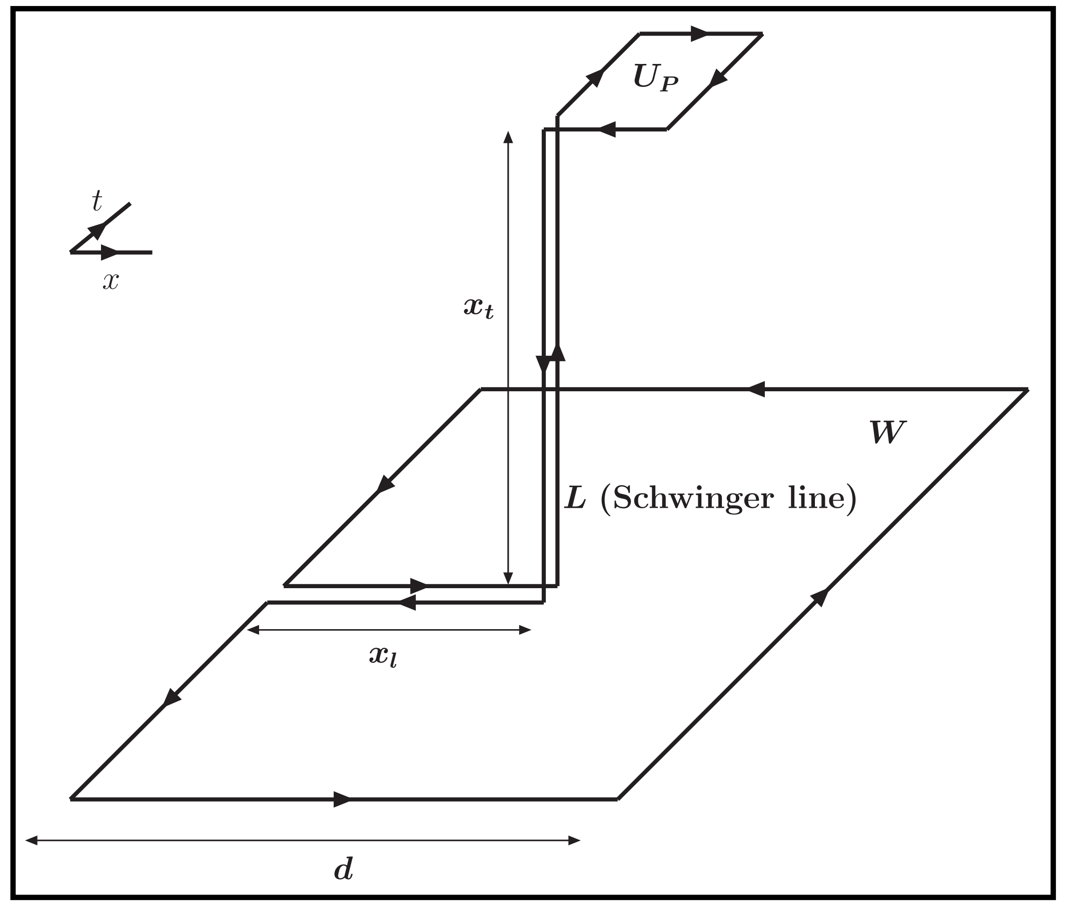
$$\overline{V}_{n,\mu;\nu\,\rho} = \mathrm{Proj}_{SU(3)}[(1-\alpha_3)U_{n,\mu} + \frac{\alpha_3}{2}\sum_{\pm\eta\neq\rho,\nu,\mu}U_{n,\eta}U_{n+\hat{\eta},\mu}U^{\dagger}_{n+\hat{\mu},\eta}]$$



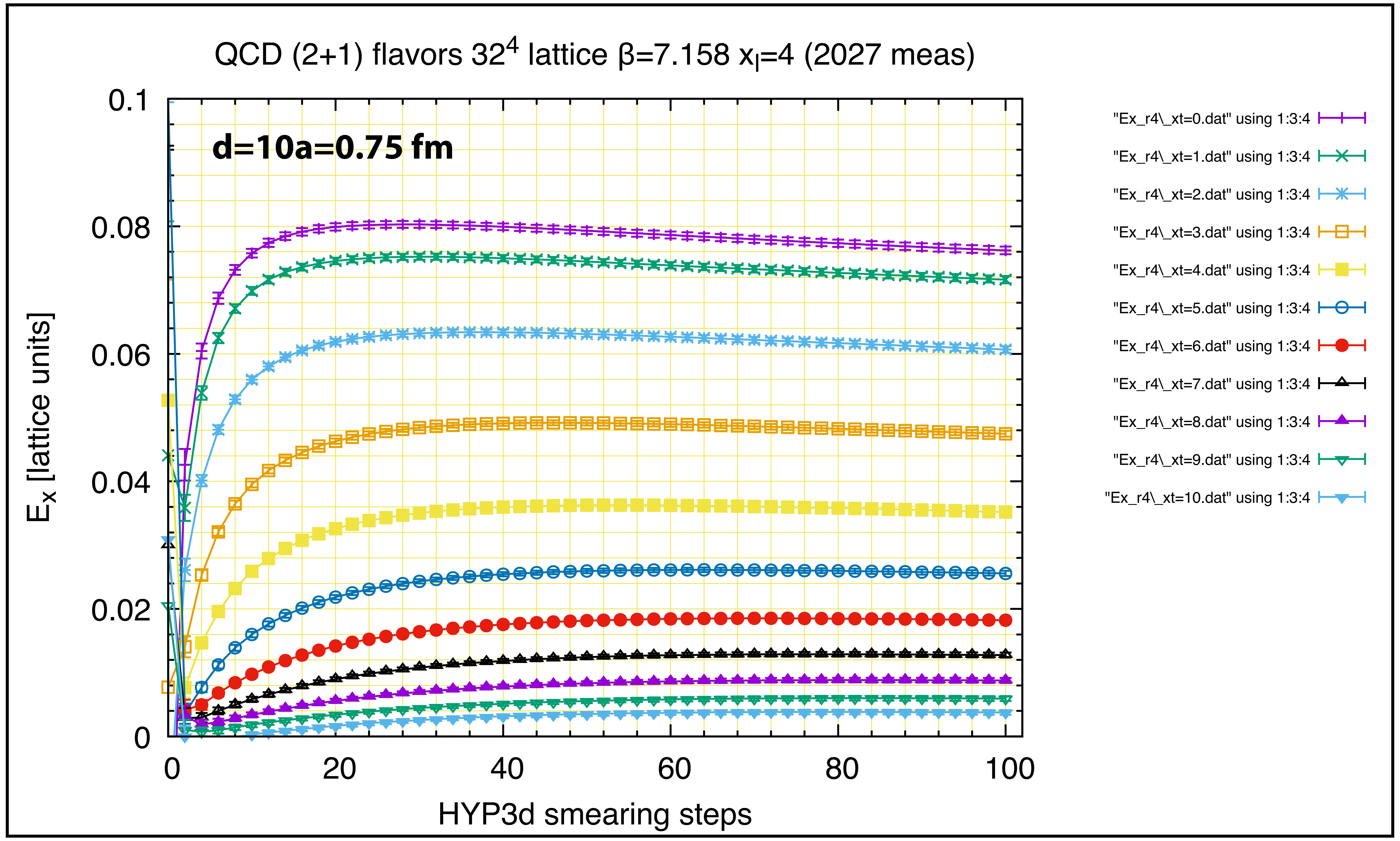
The flux tube operator ρ_W undergoes a non-trivial renormalization, which depends on x_t (the length of the Schwinger line) as discussed in a recent work [N. Battelli. C. Bonati, arXiv:1903.10463], but the renormalization procedure outlined in arXiv:1903.10463 is prohibitively demanding from the computational point of view for Wilson loops and Schwinger lines with linear dimension of the order of 1 fm, where the interesting physics is expected to take place.

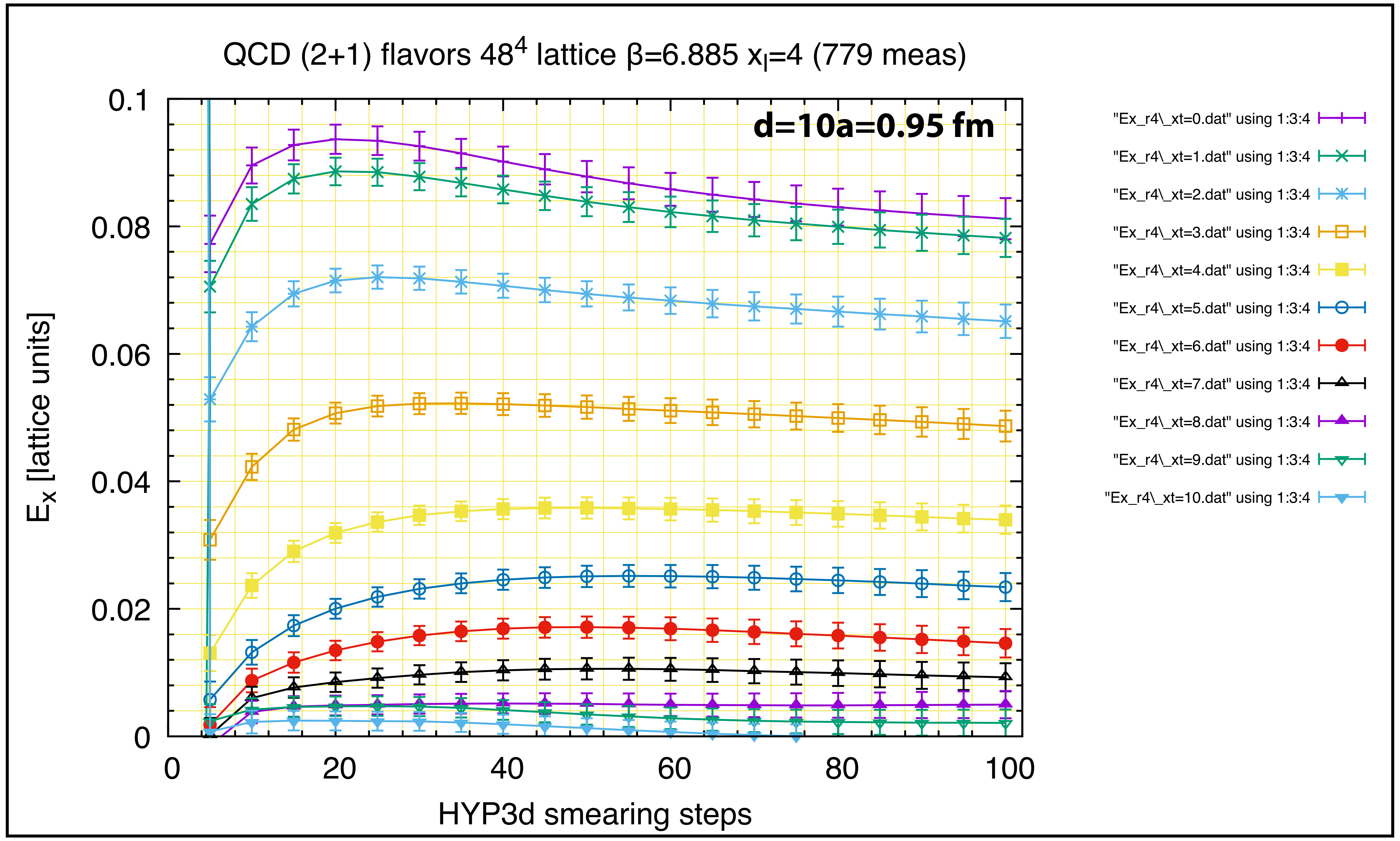
We adopt here the approach to perform smearing on the Monte Carlo ensemble configurations before taking measurements.

Actually smearing behaves as an effective renormalization, effectively pushing the system towards the continuum, where renormalization effects become negligible. *(The a posteriori validation of the smearing procedure is provided by the observation of continuum scaling: fields obtained in the same physical setup, but at different values of the coupling, are in perfect agreement in the range of parameters used in the present work)*



Smoothing of the gauge configurations





$$\mathbf{HYP1t} \rightarrow (\alpha_1 = 1.0,\ \alpha_2 = 1.0,\ \alpha_3 = 0.5)$$

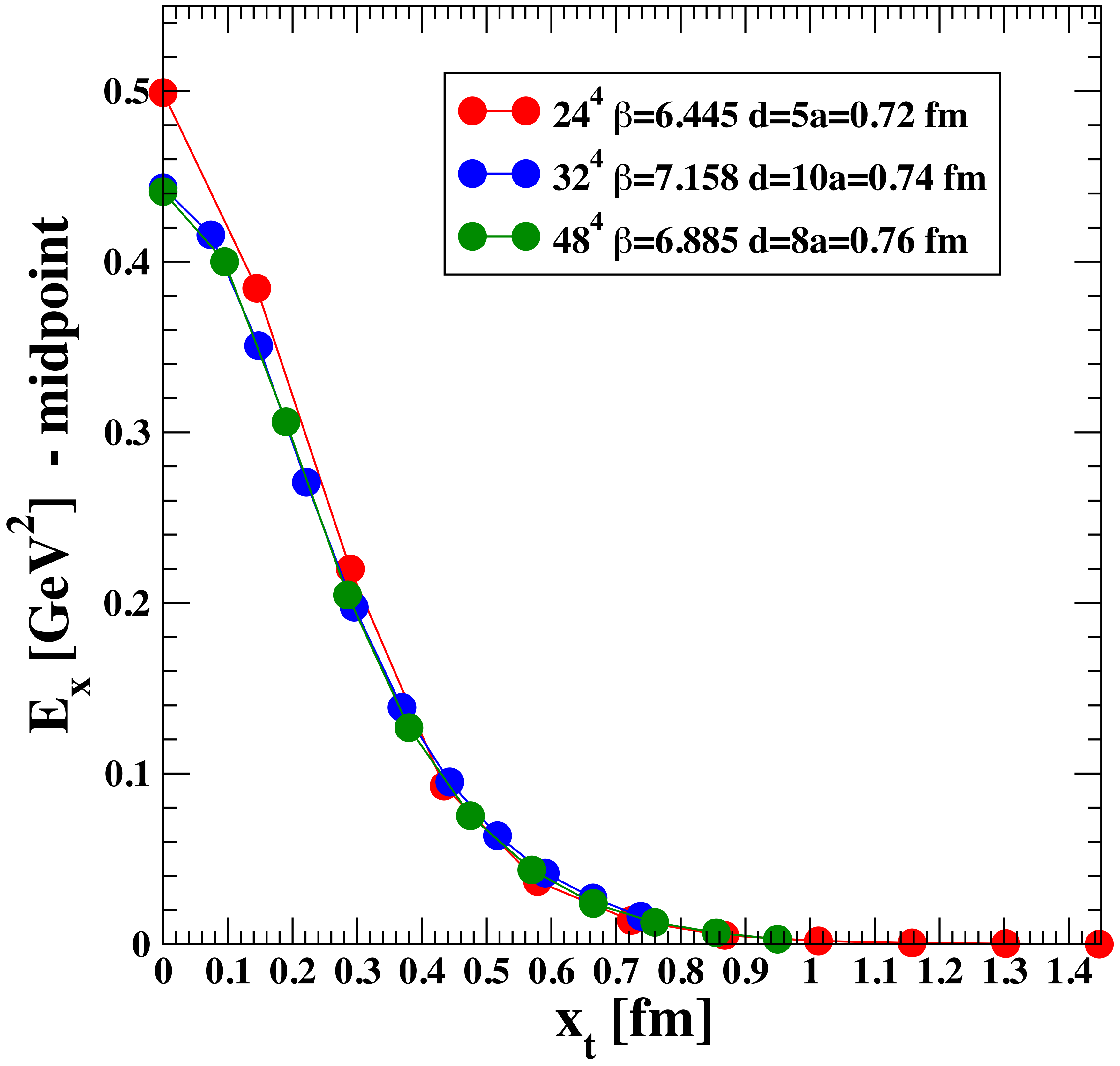
$$\mathbf{HYP3d} \rightarrow (\alpha_1^{\mathbf{HYP3d}} = 1.0,\ \alpha_3^{\mathbf{HYP3d}} = 0.5)$$

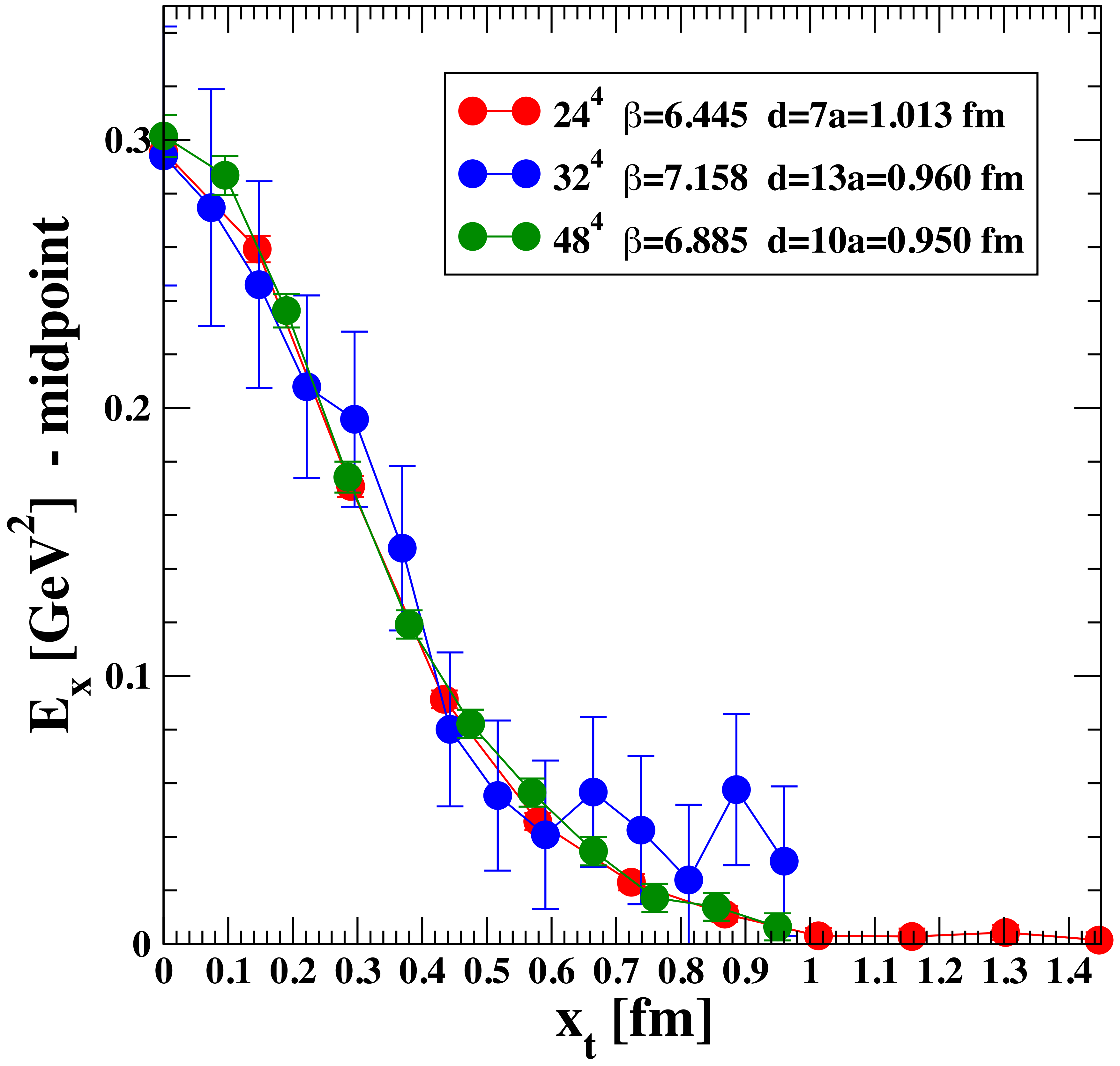
Scaling

$$a(\beta) = r_1 F(\beta) = (0.3106 \times F(\beta))\ \text{fm} \qquad F(\beta) = \frac{c_0 f(\beta) + c_2 (f(\beta))^3 (10/\beta)}{1 + d_2 (f(\beta))^2 (10/\beta)} \qquad c_0 = 44.06\,, c_2 = 272102\,, d_2 = 4281$$

$$f(\beta) = (b_0 \frac{10}{\beta})^{-b_1/(2b_0)^2} \exp(-\frac{\beta}{20 b_0}) \qquad N = 3, n_f = 3 \longrightarrow b_0 = \frac{9}{16\pi^2}\ b_2 = \frac{1}{4\pi^4}$$

[Bazavov et al (HotQCD Collaboration), PRD 85, 054503 (2012), arXiv:1111.1710]



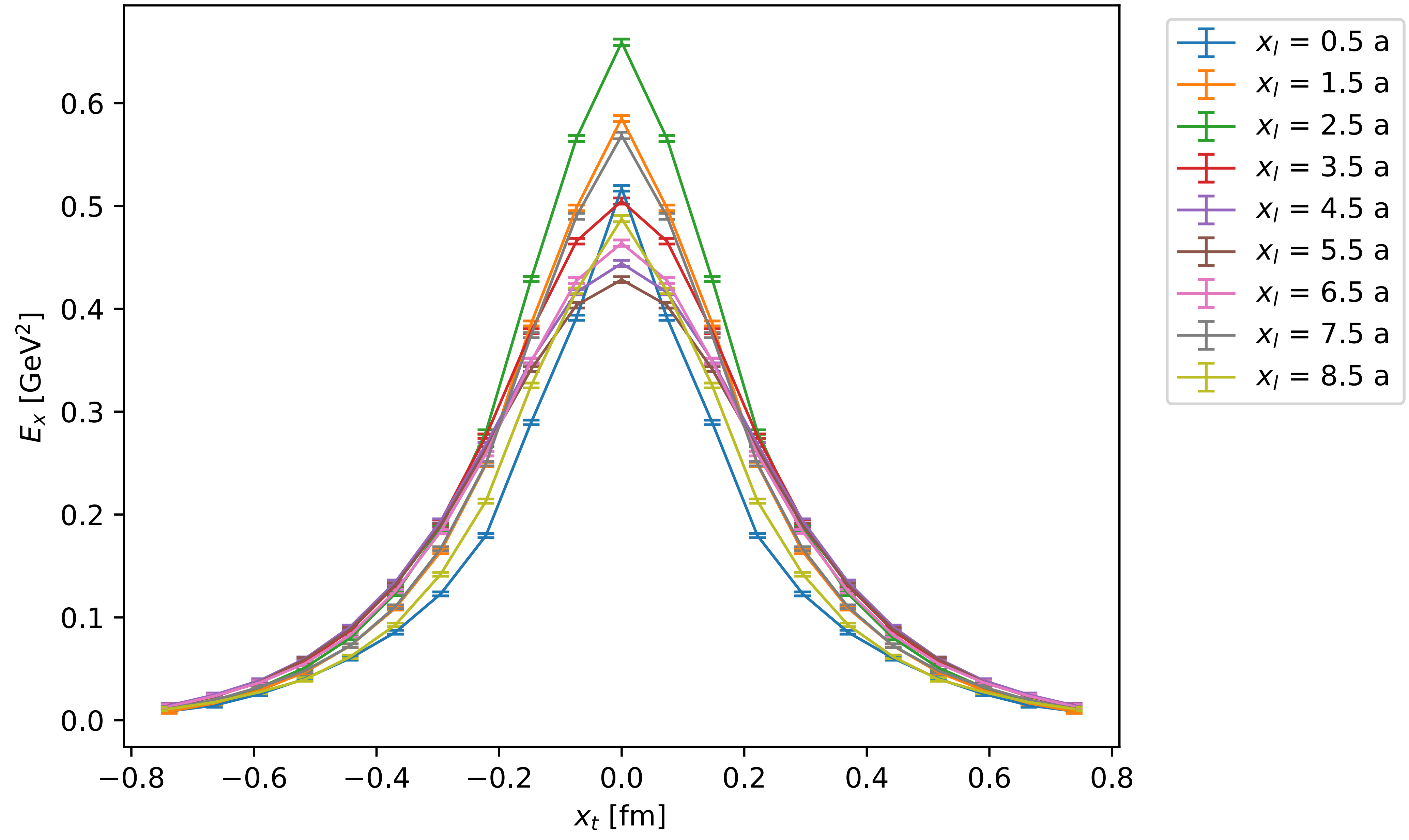


The non-perturbative E_x field in the QCD flux tube

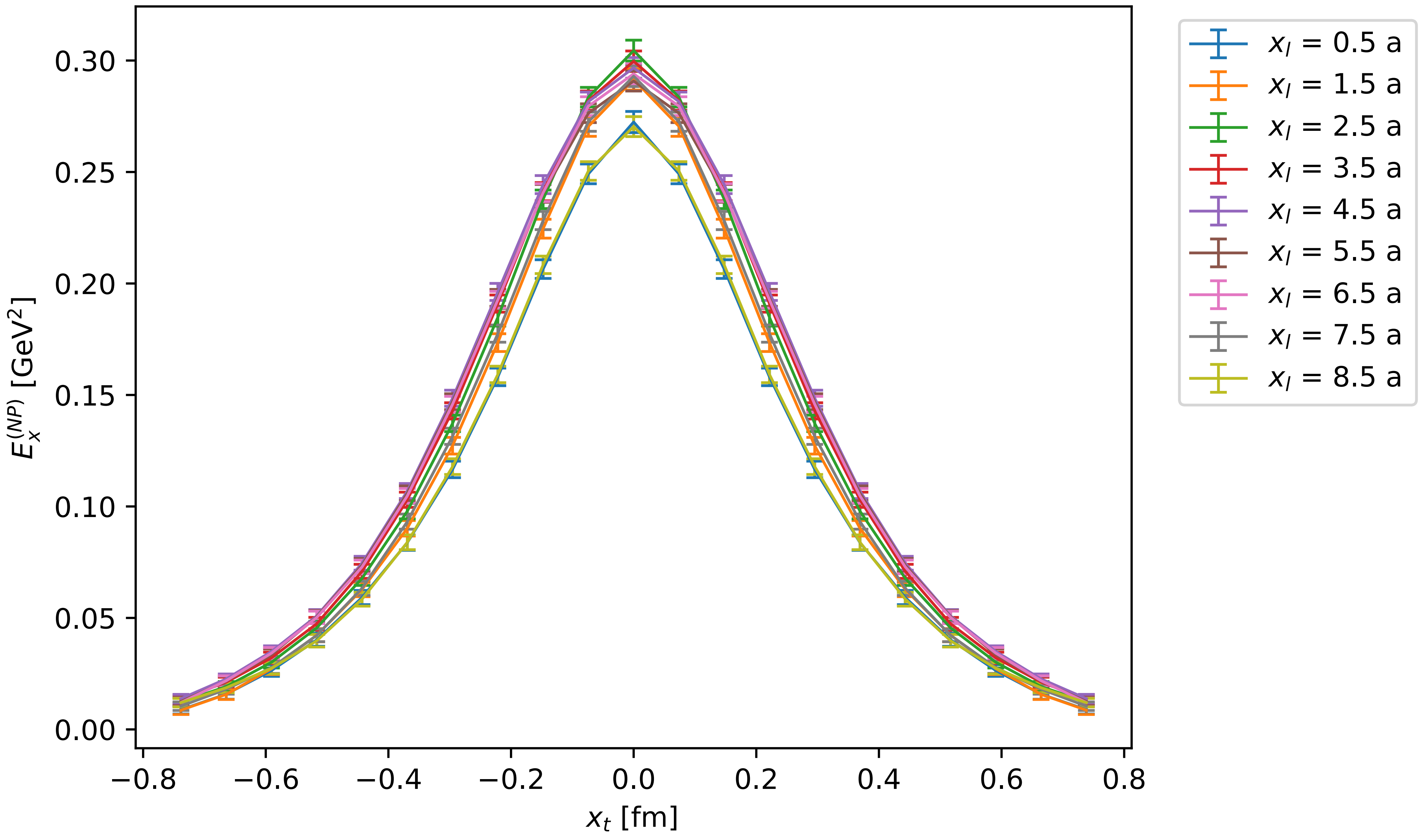
$\beta = 7.158,\ d = 10a = 0.74\,\mathrm{fm}$

QCD (2+1) HISQ flavors ($m_\pi = 160$ MeV)

E_x



E_x^{NP}

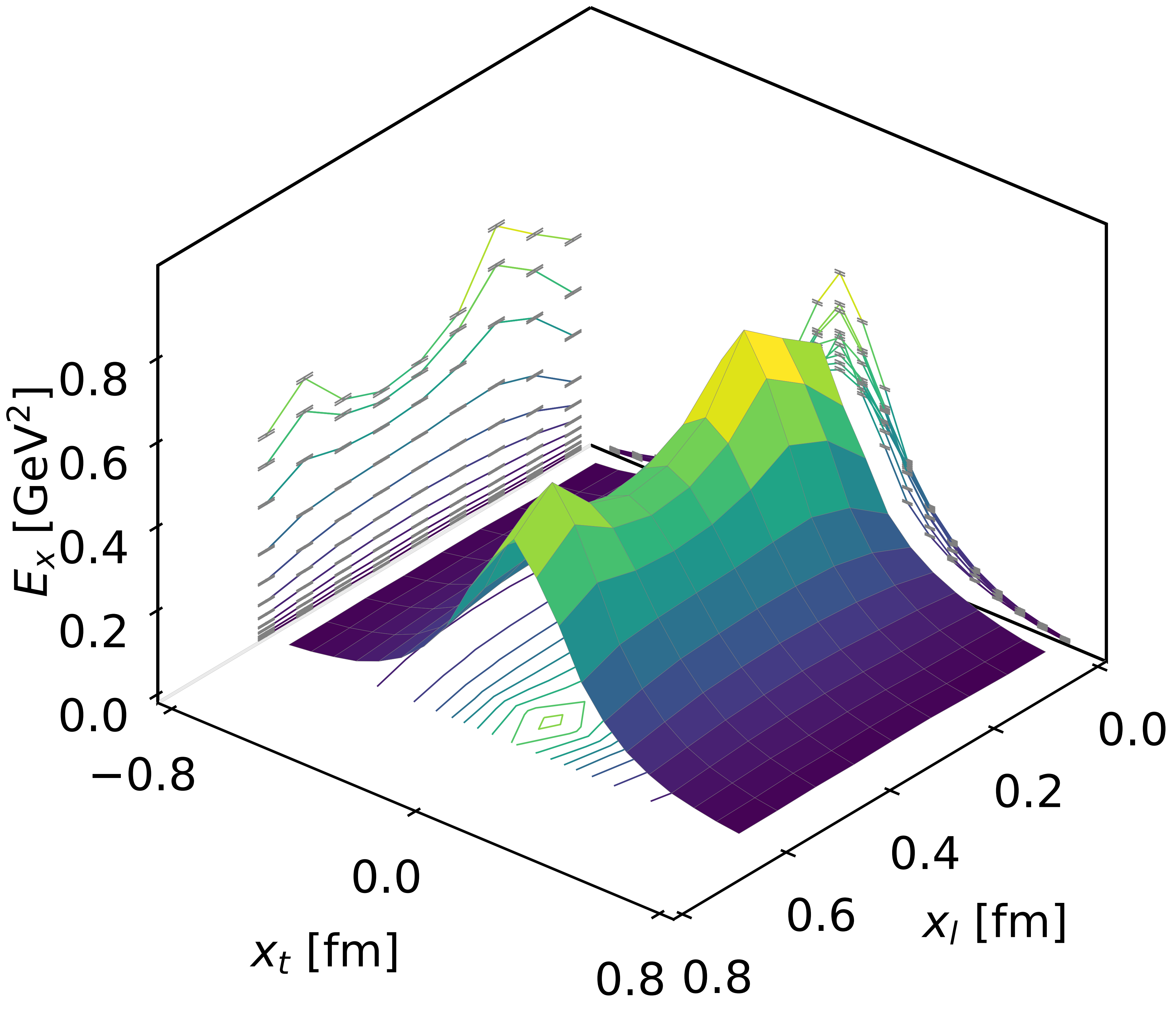


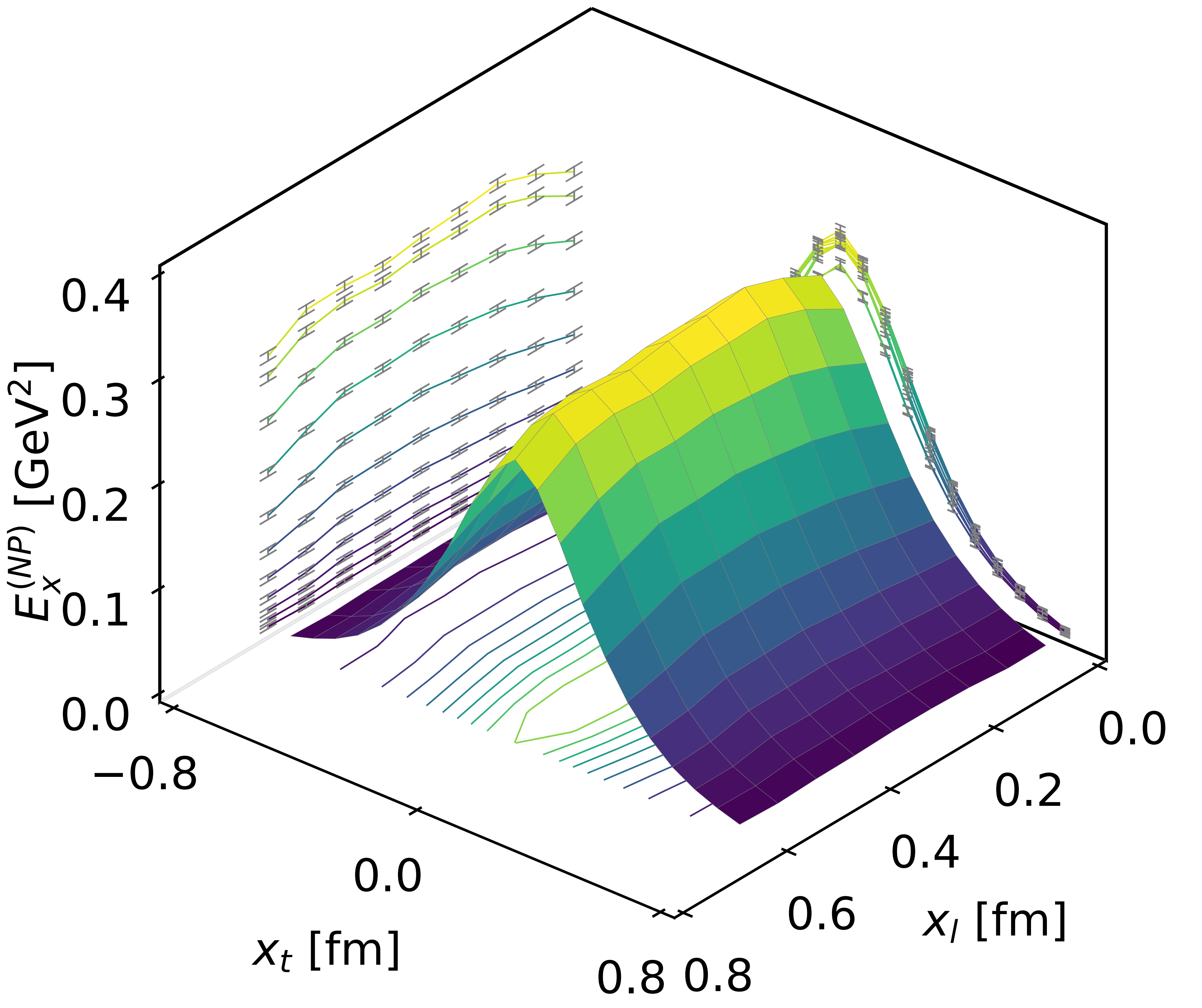
$$E_x^{\mathrm{NP}} = E_x - E_x^C$$

The non-perturbative E_x field in the QCD flux tube

$\beta = 7.158,\ d = 10a = 0.74\ \mathrm{fm}$

QCD (2+1) HISQ flavors ($m_\pi = 160$ MeV)



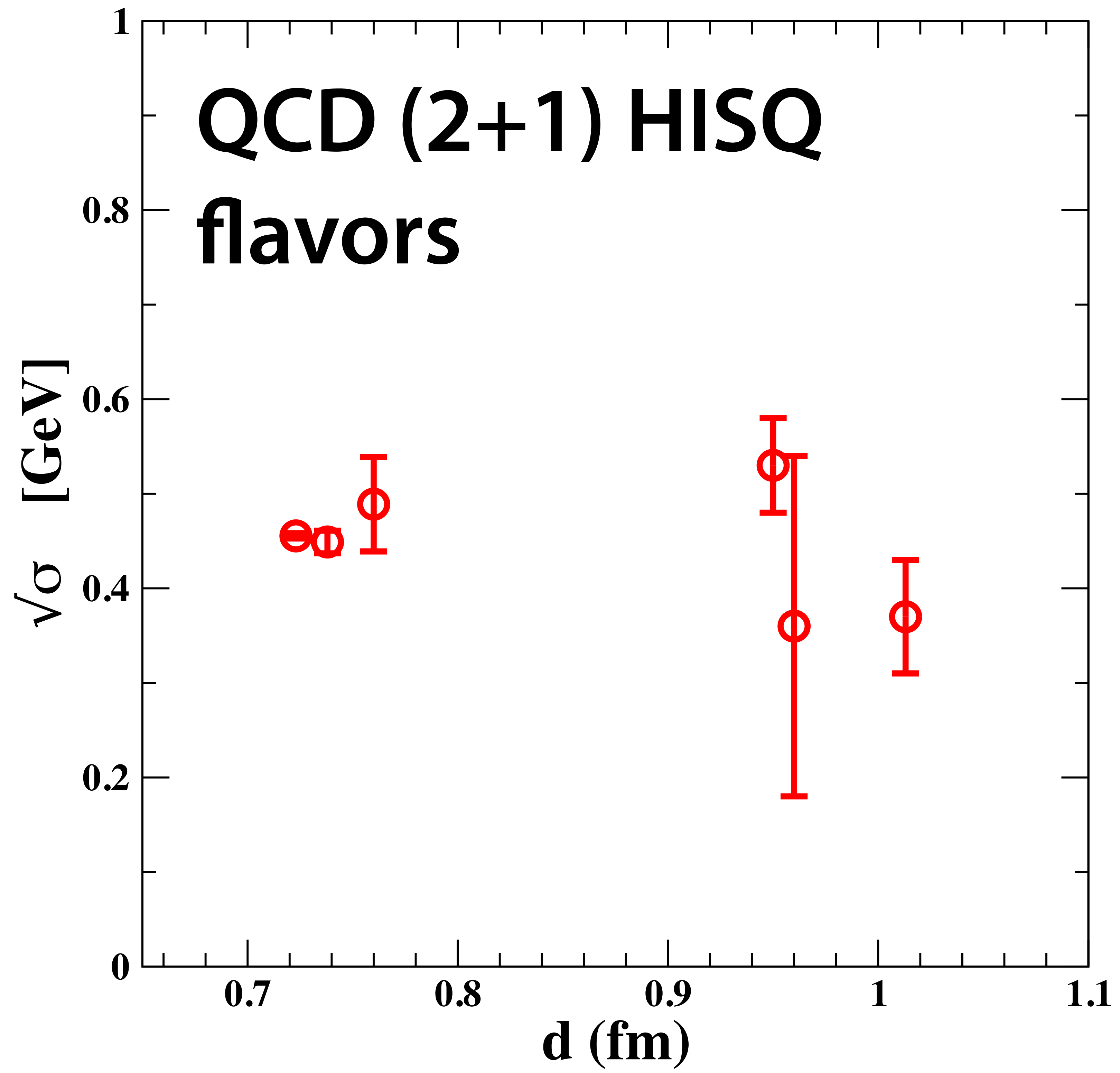


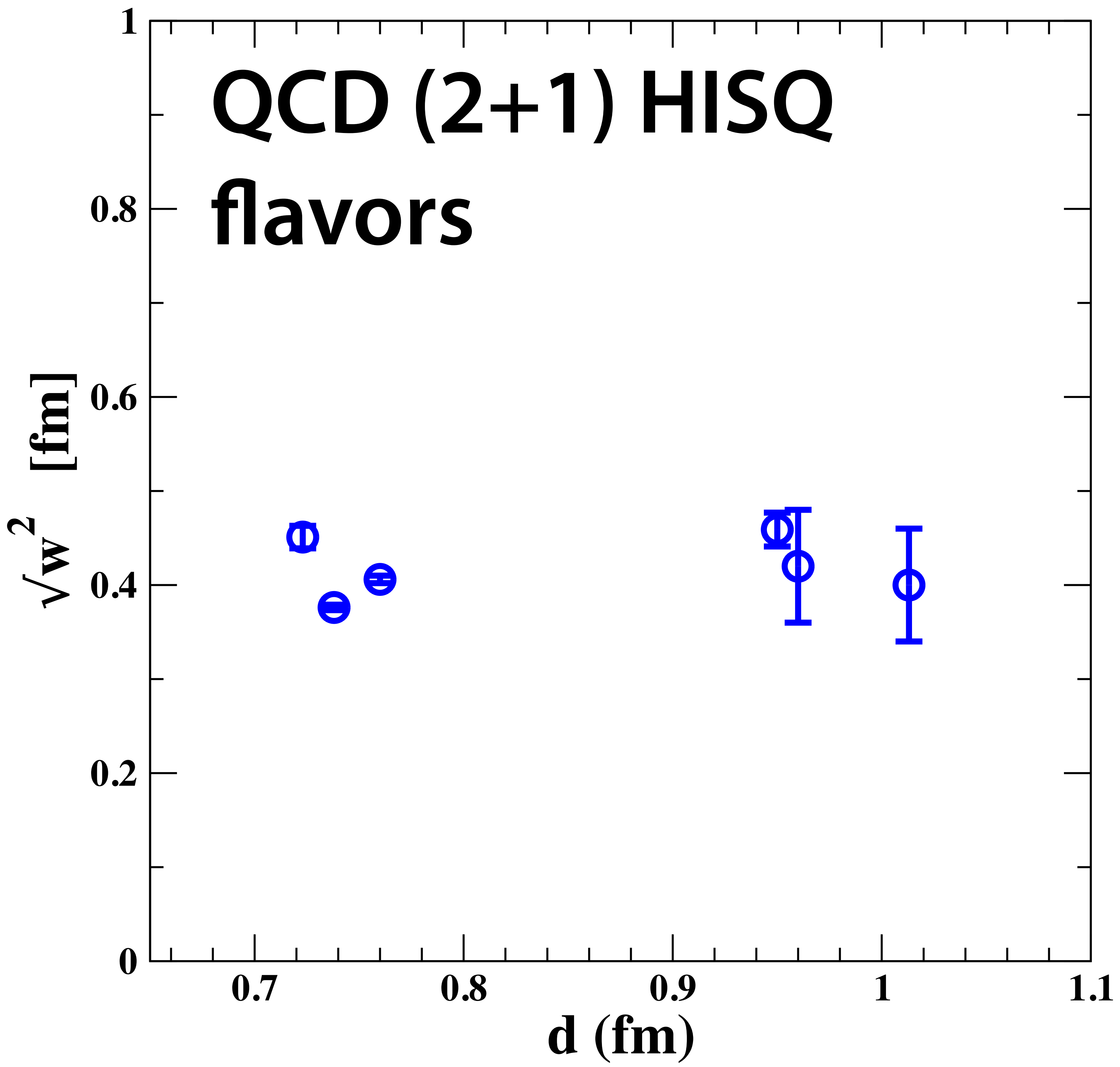
$\beta = 7.158,\ d = 10a = 0.74\ \mathrm{fm}$

Preliminary results for: string tension and width of the flux tube

$$\sqrt{\sigma}(\mathrm{NP}) = \sqrt{\int d^2x_t \frac{(E_x^{NP})^2(x_t)}{2}}$$

$$\sqrt{w^2}(\mathrm{NP}) = \sqrt{\frac{\int d^2x_t\, x_t^2 E_x^{NP}(x_t)}{\int d^2x_t\, E_x^{NP}(x_t)}}$$





In progress: systematic study for several distances between the quark sources

- 24^4, $\beta = 6.445$, $a(\beta) = 0.145$ fm $\longrightarrow$

distance between quark sources	
in lattice units	in physical units (fm)
5	0.723
7	1.013
8	1.158
10	1.447

- 32^4, $\beta = 7.158$, $a(\beta) = 0.074$ fm $\longrightarrow$

distance between quark sources	
in lattice units	in physical units (fm)
10	0.738
13	0.960
14	1.034

- 48^4, $\beta = 6.885$, $a(\beta) = 0.095$ fm $\longrightarrow$

distance between quark sources	
in lattice units	in physical units (fm)
6	0.570
8	0.760
10	0.950
11	1.045
12	1.140

BACKUP SLIDES

The confining field of the QCD flux tube

- The chromomagnetic field is everywhere much smaller than the longitudinal chromoelectric field and is compatible with zero within statistical errors
- The dominant component of the chromoelectric field is longitudinal
- The transverse components of the chromoelectric field are also smaller than the longitudinal component but can be matched to the transverse components of an **effective Coulomb-like field**

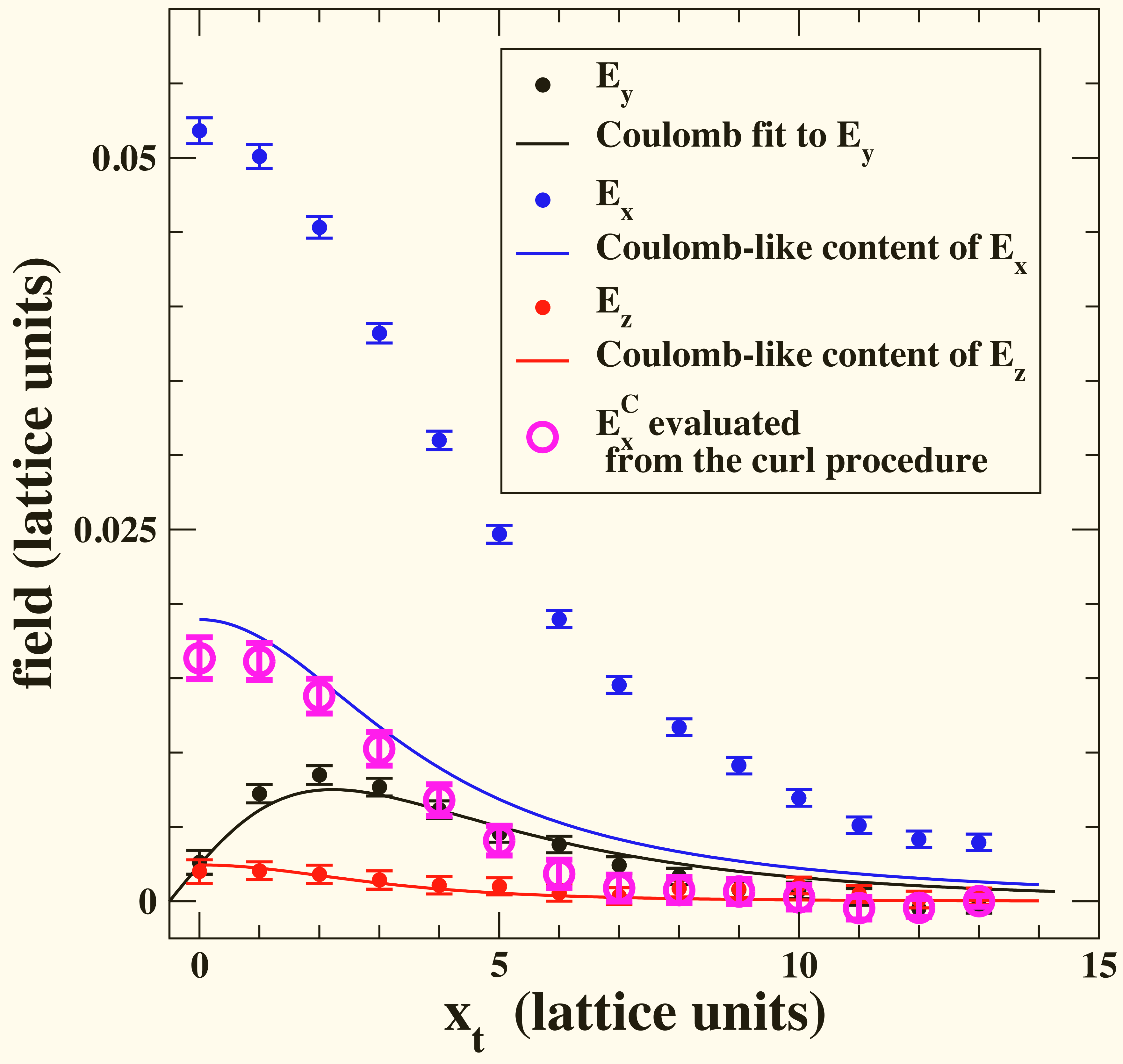
$$\vec{E}^{C}(\vec{r}) = Q\left(\frac{\vec{r}_1}{\max(r_1, R_0)^3} - \frac{\vec{r}_2}{\max(r_2, R_0)^3}\right)$$

$$\vec{r}_1 \equiv \vec{r} - \vec{r}_Q \;, \quad \vec{r}_2 \equiv \vec{r} - \vec{r}_{-Q}$$

To the extent that we can fit the transverse components of the field E to those of E_C with an appropriate choice of Q :

the confining field of the QCD flux tube

$$\boxed{\vec{E}^{\rm NP} \equiv \vec{E} - \vec{E}^{\rm C}}$$



[M. Baker, P. Cea, V. Chelnolov, L.C., F. Cuteri, A. Papa, arXiv:1810.07133, arXiv:1912.04739]

The confining field of the QCD flux tube: the *curl* procedure

- To extract the confining part of the chromoelectric field in the data it is preferable to have a procedure which avoids the use of an explicit fitting function, and which can work close to the quark sources (*).
- In order to separate the field into 'perturbative' and 'non-perturbative' components.
  1) We identify the transverse component E_y of the field with the transverse component E_y^C of the perturbative field: $E_y^C \equiv E_y$
  2) We impose the condition that the perturbative field is **irrotational**: curl $E^C = 0$

The irrotational condition on a discrete lattice (on a plaquette):

$$E_x^{\mathrm{C}}(x,y) + E_y^{\mathrm{C}}(x+1,y) - E_x^{\mathrm{C}}(x,y+1) - E_y^{\mathrm{C}}(x,y) = 0$$

Solve this equation for E_x^C

$$E_x^{\mathrm{C}}(x,y) = \sum_{y'=y}^{y_{\max}} \left(E_y(x,y') - E_y(x+1,y')\right) + E_x^{\mathrm{C}}(x, y_{\max}+1)$$

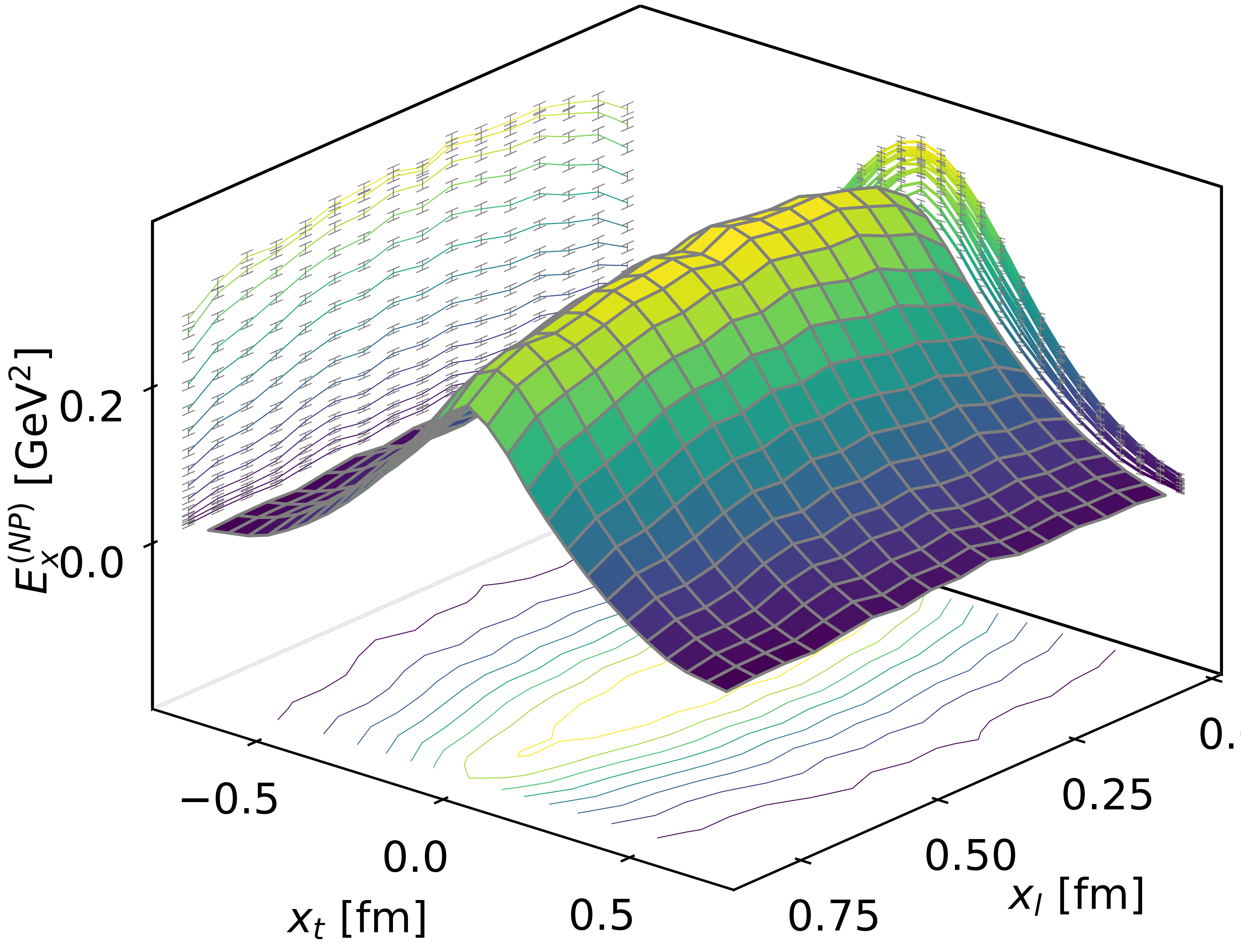
We further assume: $E_x^{\mathrm{C}}(x, y_{\max}+1) = 0$

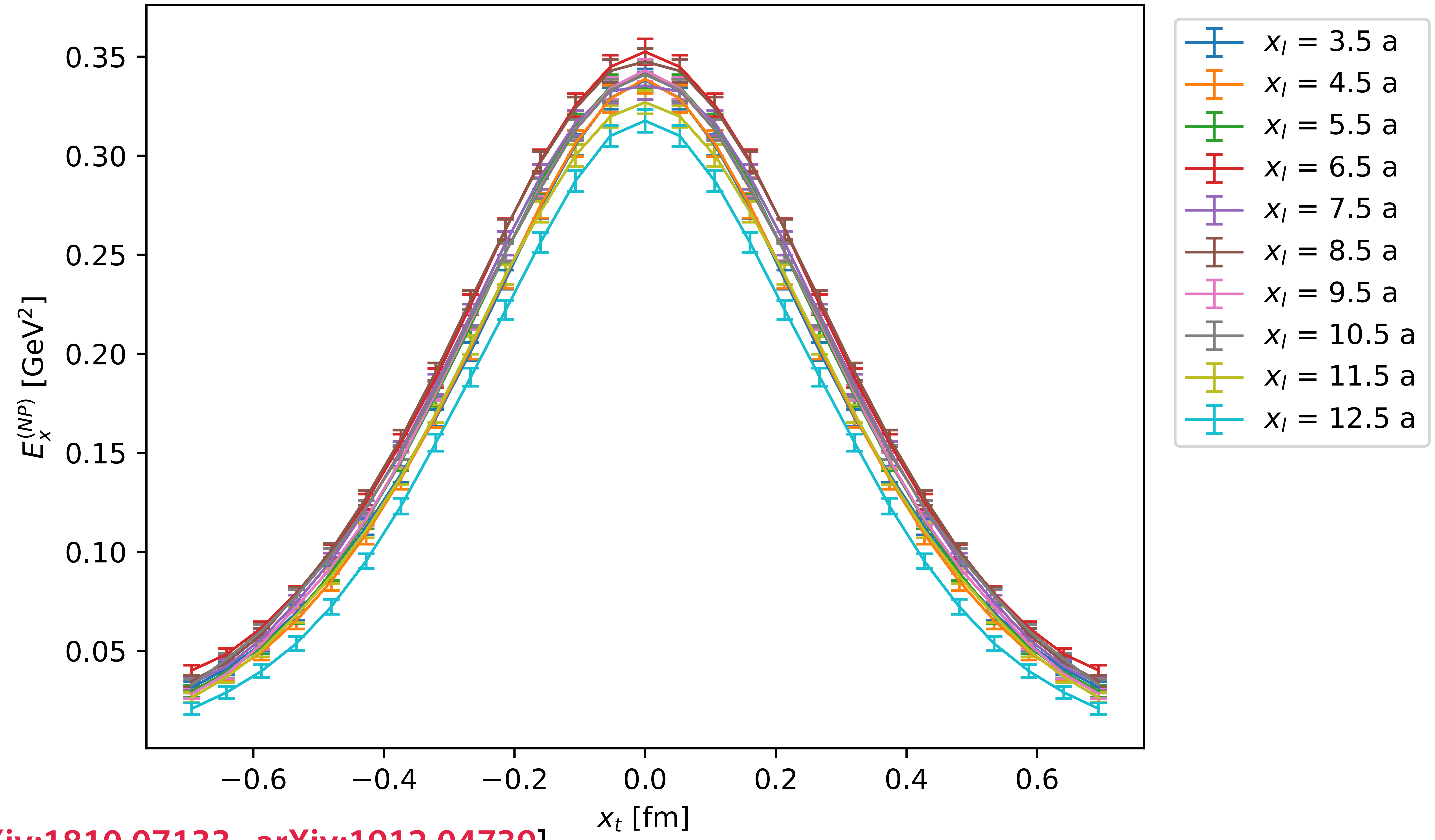
(*) Indeed close to the quark sources (two lattice spacings from the sources) for effects due to lattice discretisation and/or non-spherical form of the effective charges the Coulomb fit procedure does not give a good description of the transverse components of the chromoelectric field.

The non-perturbative component of the E_x field:

After the estimation of the perturbative longitudinal field E_x^C one can subtract it from the total field E_x, obtaining the non-perturbative component:

$$\beta = 6.370 \quad d = 0.85 \text{ fm}$$





[M. Baker, P. Cea, V. Chelnolov, L.C., F. Cuteri, A. Papa, arXiv:1810.07133, arXiv:1912.04739]

SU(3) Wilson action

Scale: $a(\beta) = r_0 \times \exp\left[c_0 + c_1(\beta-6) + c_2(\beta-6)^2 + c_3(\beta-6)^3\right]$

$r_0 = 0.5 \text{ fm}$

$c_0 = -1.6804\,,\, c_1 = -1.7331$

$c_2 = 0.7849\,,\, c_3 = -0.4428$

S. Necco, R. Sommer, arXiv:hep-lat/0108008

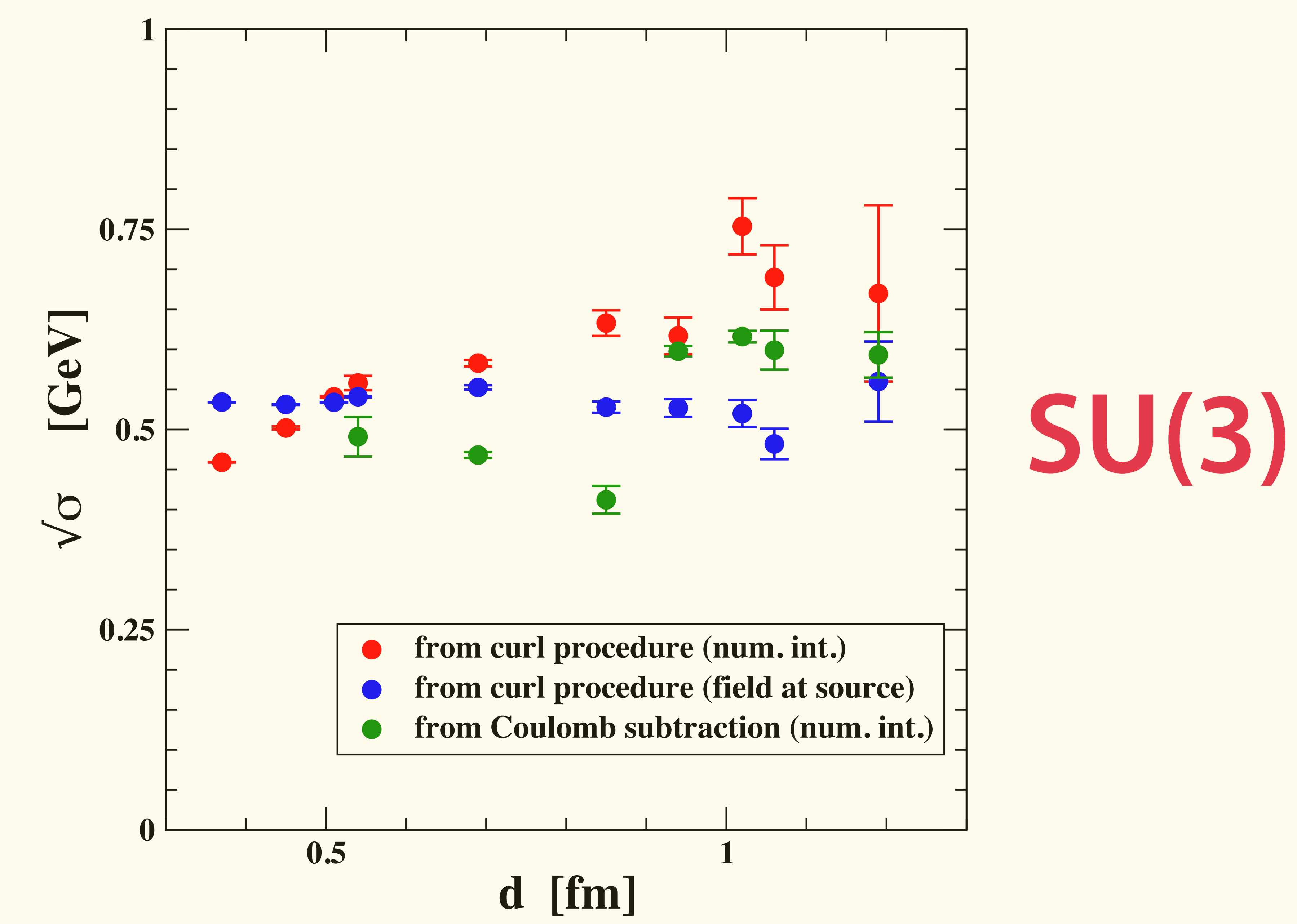
Smearing: 1 HYP t + n APE 3d

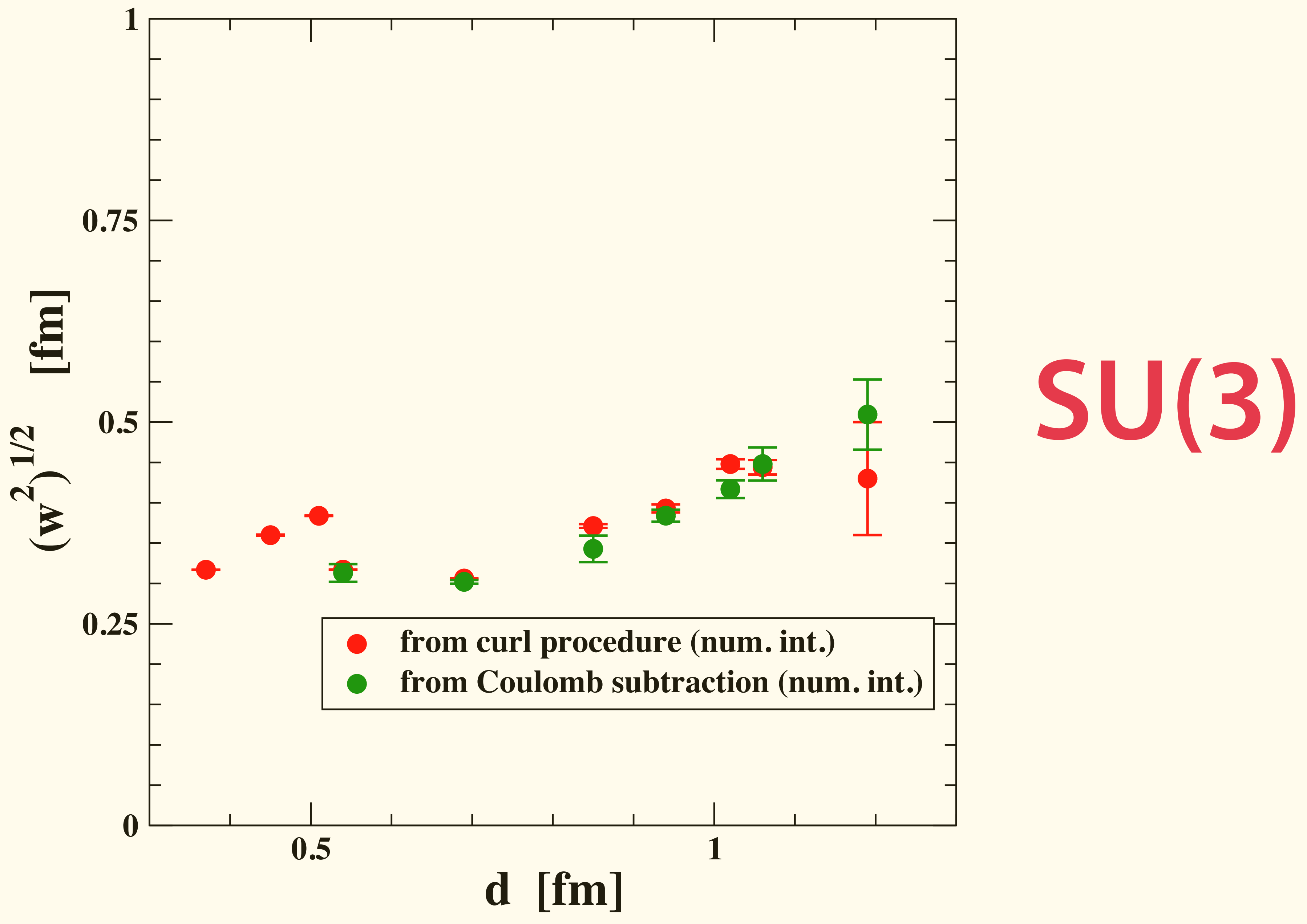
The string tension of the chromoelectric flux tube

β	d (fm)	$\sqrt{\sigma_{\rm int}}$ (GeV)	$\sqrt{\sigma_{\rm Clem}}$ (GeV)	$\sqrt{\sigma_0}$ (GeV)	$\sqrt{\sigma_{\rm Coulomb}}$ (GeV)
6.47466	0.37	0.4591(3)	0.4659(3)	0.53426(22)	–
6.333	0.45	0.5020(19)	0.5045(20)	0.5313(6)	–
6.240	0.51	0.5409(10)	0.5430(10)	0.5340(4)	–
6.500	0.54	0.5582(9)	0.5687(10)	0.5410(7)	0.491 (25)
6.539	0.69	0.583(4)	0.596(5)	0.5526(28)	0.468 (4)
6.370	0.85	0.633(16)	0.640(17)	0.528(7)	0.412 (17)
6.299	0.94	0.617(23)	0.620(24)	0.527(11)	0.598 (7)
6.240	1.02	0.75(4)	0.77(4)	0.520(17)	0.616 (7)
6.218	1.06	0.69(4)	0.62(3)	0.482(19)	0.599 (24)
6.136	1.19	0.67(11)	0.67(12)	0.56(5)	0.593 (28)

The width of the chromoelectric flux tube

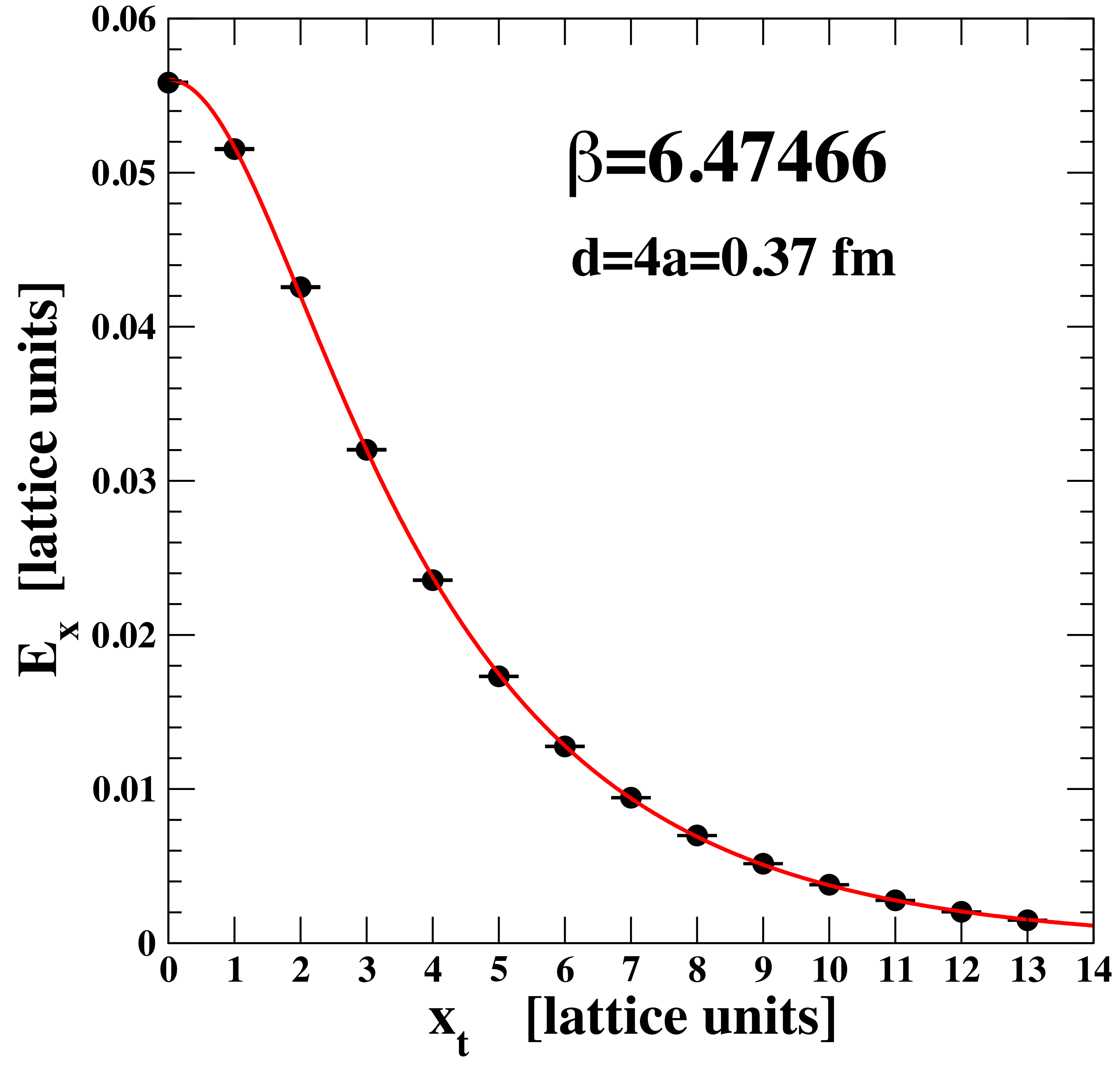
β	d (fm)	$\sqrt{w^2_{\rm int}}$ (fm)	$\sqrt{w^2_{\rm Clem}}$ (fm)	$\sqrt{w^2_{\rm Coulomb}}$ (fm)
6.47466	0.37	0.31696(6)	0.4795(6)	–
6.333	0.45	0.3598(7)	0.477(3)	–
6.240	0.51	0.3838(3)	0.4543(9)	–
6.500	0.54	0.31716(15)	0.4727(18)	0.313(11)
6.539	0.69	0.3061(5)	0.457(6)	0.3020(23)
6.370	0.85	0.3712(24)	0.497(21)	0.343(16)
6.299	0.94	0.393(5)	0.483(29)	0.384(7)
6.240	1.02	0.448(6)	0.63(5)	0.417(11)
6.218	1.06	0.444(9)	0.56(5)	0.448(21)
6.136	1.19	0.43(7)	0.46(6)	0.51(4)





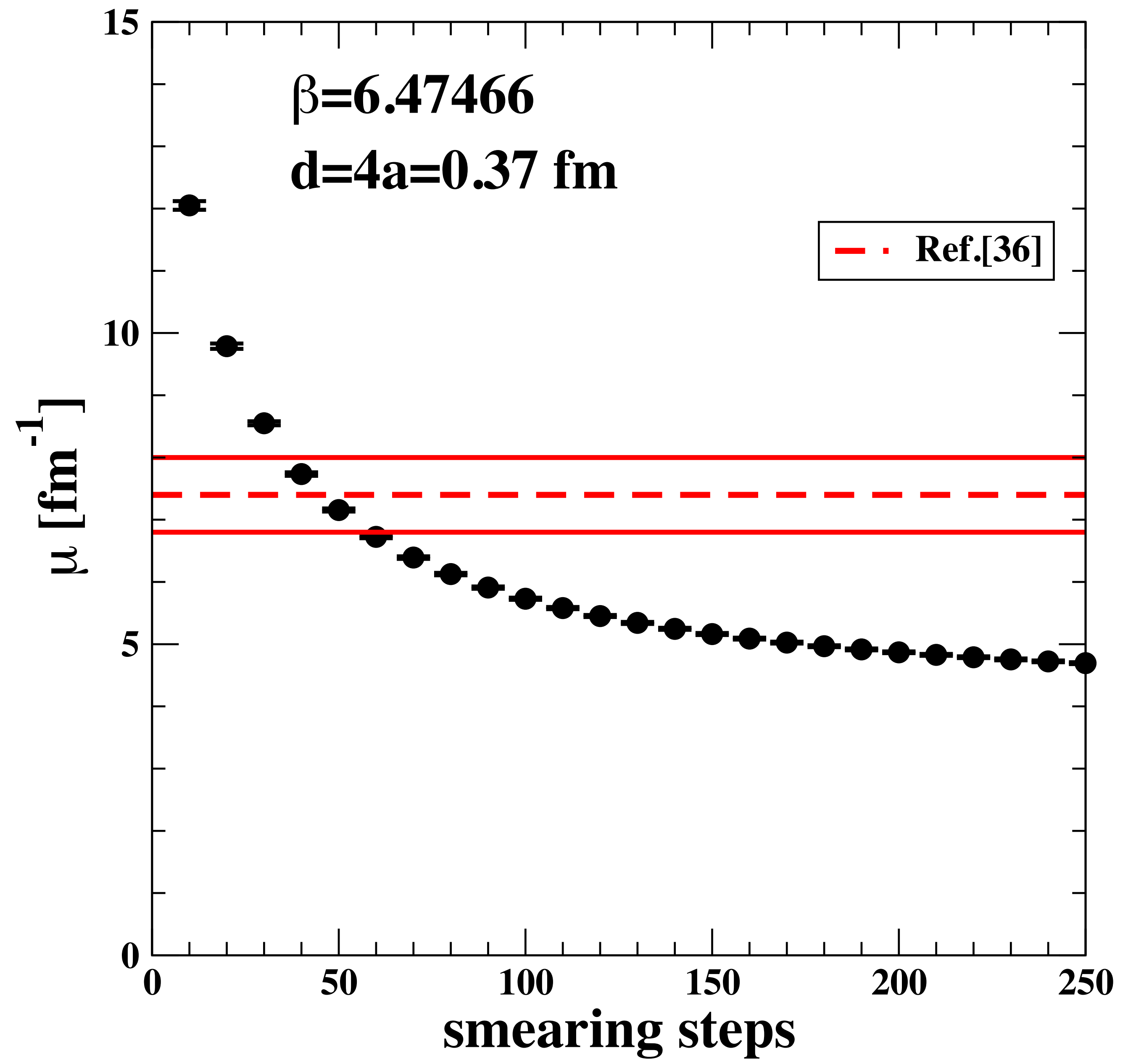
Smearing and renormalization

[comparison with N. Battelli, C. Bonati, arXiv:1903:10463]



$$E_x(x_t) = \frac{\phi}{2\pi}\frac{\mu^2}{\alpha}\frac{K_0[(\mu^2 x_t^2 + \alpha^2)^{1/2}]}{K_1[\alpha]}$$

$$\kappa = \frac{\sqrt{2}}{\alpha}\left[1 - K_0^2(\alpha)/K_1^2(\alpha)\right]^{1/2}$$



The smearing behaves as one effective renormalization, driving the parameters towards the values extracted from the renormalised field.