Monopoles of the Dirac type and color confinement inQCD

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T. Suzuki, arXiv:1402.1294 (2014),
T. Suzuki et al, P.R. D80, 054504 (2009),
T. Suzuki, K.Ishiguro, V.Bornyakov, P.R. D97, 034501, 099905(erratum) (2018),
T. Suzuki, P.R. D97, 034509 (2018)
T. Suzuki, K. Ishiguro and A. Hiraguchi, Talks at APLAT 2020

1. Introduction

Color confinement problem not yet solved.

Almost half a century history !!!

1. 1963: Quark model (Gell-Mann and Zweig): fractionally charged quarks are searched, but not observed.
2. 1974-75: Idea of dual superconductor (electric $\leftrightarrow$ magnetic) as the color-confinement mechanism ('tHooft-Mandelstam): Something color magnetic must be condensed in QCD.
3. 1981: 'tHooft idea of monopole in QCD: A partial gauge-fixing $SU(3) \rightarrow U(1) \times U(1)$ and Abelian projection: Monopoles appear as a topological object. Numerical data supporting this idea are shown especially on the basis of maximally Abelian gauge. But this idea has a serious problem of gauge dependence.

No one knows what is a gauge-independent color magnetic quantity, a magnetic monopole in QCD without any additional or artificial assumption.

2. Abelian magnetic monopoles of the Dirac type in QCD

Note the Jacobi identities:

$$\epsilon_{\mu\nu\rho\sigma}[D_\nu, [D_\rho, D_\sigma]] = 0,$$

where $D_\mu \equiv \partial_\mu - igA_\mu$. Calculate explicitly:

$$\begin{aligned}
[D_\rho, D_\sigma] &= [\partial_\rho - igA_\rho, \partial_\sigma - igA_\sigma] \\
&= -ig(\partial_\rho A_\sigma - \partial_\sigma A_\rho - ig[A_\rho, A_\sigma]) + [\partial_\rho, \partial_\sigma] \\
&= -igG_{\rho\sigma} + [\partial_\rho, \partial_\sigma]
\end{aligned}$$

$[\partial_\rho, \partial_\sigma] = 0 \to D_\nu G^*_{\mu\nu} = 0$: Non-Abelian Bianchi identity (NABI)

$f_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu = (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)\sigma^a/2$

$\partial_\nu f^*_{\mu\nu} = 0 \to$ Abelian-like Bianchi identity:

$[\partial_\rho, \partial_\sigma] \neq 0 \rightarrow$ Jacobi identity + $[D_\nu, G_{\rho\sigma}] = D_\nu G_{\rho\sigma}$

$$
\begin{aligned}
\Longrightarrow D_\nu G^*_{\mu\nu} &= \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} D_\nu G_{\rho\sigma} \\
&= -\frac{i}{2g}\epsilon_{\mu\nu\rho\sigma}[D_\nu, [\partial_\rho, \partial_\sigma]] \\
&= \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}[\partial_\rho, \partial_\sigma]A_\nu = \partial_\nu f^*_{\mu\nu} \\
J_\mu = \frac{1}{2}J^a_\mu \sigma^a = D_\nu G^*_{\mu\nu} &= \partial_\nu f^*_{\mu\nu} = \frac{1}{2}k^a_\mu \sigma^a = k_\mu
\end{aligned}
$$

$k^a_\mu \neq 0 \rightarrow$ color magnetic Abelian-like monopole: $\partial_\mu k_\mu = 0$
$J^a_\mu \neq 0 \rightarrow$ Violation of NABI

Color magnetic monopoles= Violation of non-Abelian Bianchi identity (VNABI)

:Reference C. Bonati et al,, P.R.D81, 085022 (2010)

$$[\partial_\rho, \partial_\sigma]A_\nu \neq 0$$
$$\Downarrow$$

Line singularities existing in gauge fields $A_\mu(x)$ themselves!!! are the origin of Abelian monopoles in QCD. $N^2 - 1$ monopoles exist in $SU(N)$.

3. Lattice studies of the new QCD magnetic monopoles

Consider one-colored monopole $k^1(s,\mu)$ among three $k^a(s,\mu)$ ($a = 1 \sim 3$ in $SU(2)$) or eight ($a = 1 \sim 8$ in $SU(3)$) monopoles defined following DeGrand-Toussait.

Lattice monopole is not gauge-invariant. But Elitzer's theorem says that gauge-invariant contents, if exist, can be extracted by Monte-Carlo average of gauge-variant quantities.
S. Elitzur, P.R. D12 (1975) 3978.

Lattice monopole after Abelian projection

Maximize $R = \sum_{s,\mu} Re\,\mathrm{Tr}\, e^{i\theta_1(s,\mu)\lambda_1} U^\dagger(s,\mu)$

$$\Downarrow$$

$$\begin{aligned}
\theta_1(s,\mu) &= \tan^{-1}\frac{Im(U_{12}(s,\mu)+U_{21}(s,\mu))}{Re(U_{11}(s,\mu)+U_{22}(s,\mu))} \\
\theta_1(s,\mu\nu) &= \partial_\mu\theta_1(s,\nu) - \partial_\nu\theta_1(s,\mu) \\
&= \bar{\theta}_1(s,\mu\nu) + 2\pi n_1(s,\mu\nu) \quad (|\bar{\theta}_1(s,\mu\nu)| < \pi) \\
k_\mu^1(s) &= -(1/2)\epsilon_{\mu\alpha\beta\gamma}\partial_\alpha\bar{\theta}_1(s+\hat{\mu},\beta\gamma) \\
&= (1/2)\epsilon_{\mu\alpha\beta\gamma}\partial_\alpha n_1(s+\hat{\mu},\beta\gamma)
\end{aligned}$$

Pefect abelian and monopole dominance w.r.t. the string tension

Evaluate

$$V(R) = -\frac{1}{aN_t} \ln\langle P(0)P^*(R)\rangle \ .$$

$$P_{\rm A} = \exp[i\sum_{k=0}^{N_t-1} \theta_1(s+k\hat{4},4)] = P_{\rm ph} \cdot P_{\rm mon} \ ,$$

$$P_{\rm ph} = \exp\{-i\sum_{k=0}^{N_t-1}\sum_{s'} D(s+k\hat{4}-s')\partial'_\nu \bar{\Theta}_1(s',\nu 4)\} \ ,$$

$$P_{\rm mon} = \exp\{-2\pi i\sum_{k=0}^{N_t-1}\sum_{s'} D(s+k\hat{4}-s')\partial'_\nu n_1(s',\nu 4)\}$$

Perfect Abelian dominance can be proved using the Lüscher's multilevel method.

$\beta = 5.6$	$16^3 \times 16$			$\beta = 5.8$	$12^3 \times 12$	
	σ	α	C	σ	α	C
$V_{\rm NA}$	0.239(2)	-0.39(4)	0.79(2)	0.101(3)	-0.28(1)	0.82(1)
$V_{\rm A}$	0.25(2)	-0.3(1)	2.6(1)	0.102(9)	-0.27(2)	2.60(3)

To evaluate $< P_{\rm mon} P^*_{\rm mon} >$, we take average over $4000 \sim 7000$ (in $SU(2)$) and more than 60000 (in $SU(3)$ thermalized vacua and their random gauge-transformed vacua 4000 times for each one.

$SU(2)$	nconf=5000	ngf=1000	$\beta = 2.53$		
$36^3 \times 6$	σa^2	c	μa	FR(R/a)	$\chi^2/N_{\rm df}$
$V_{\rm NA}$	0.072(3)	0.48(9)	0.53(3)	4.6 - 12.1	1.03
$V_{\rm A}$	0.073(2)	0.47(6)	1.10(2)	4.3 - 11.2	1.03
$V_{\rm mon}$	0.073(3)	0.46(7)	1.43(3)	4.0 - 11.8	1.01
$V_{\rm ph}$	$-1.0(1) \times 10^{-4}$	0.0132(1)	0.4770(2)	6.4 - 11.5	1.03
$SU(3)$	nconf=60000	ngf=4000	$\beta = 5.6$		
$24^3 \times 4$	σa^2	c	μa	FR(R/a)	$\chi^2/N_{\rm df}$
$V_{\rm NA}$	0.193(4)	0.422(3)	1.146(20)	1-7	0.992
$V_{\rm A}$	0.184(15)	0.458(97)	2.912(80)	1-8	1.10
$V_{\rm mon}$	0.188(16)	0.453(99)	2.906(82)	1-8	0.967
$V_{\rm ph}$	$-0.0014(2)$	0.073(5)	1.521(3)	1 - 11	0.997

Perfect Abelian and monopole dominance are obtained.

4. Existence of the continuum limit

Does the continuum limit of $k^a(s,\mu)$ exist?

Study the monopole density in the continuum limit in pure SU2 QCD.

The lattice vacuum is contaminated with large amount of lattice artifact monopoles. To reduce lattice artifacts, various techniques smoothing the vacuum are introduced.

1. Tadpole improved action:
48^4 at $\beta = 3.0 \sim 3.9$ and 24^4 at $\beta = 3.0 \sim 3.7$

2. Introduction of various smooth gauge-fixings
1) Maximal center gauge(MCG): Maximization of $R = \sum_{s,\mu}(\mathrm{Tr}U(s,\mu))^2$
SU(2) $\rightarrow$ Z(2)

2) Direct Laplacian center gauge (DLCG)

3) Maximal Abelian Wilson loop gauge (AWL): Maximization of
$R = \sum_{s,\mu\neq\nu}\sum_a(cos(\theta^a_{\mu\nu}(s))$

4) Maximal Abelian and $U(1)$ Landau gauge (MAU1):

3. The blockspin transformation of monopoles

the blockspin transformation of monopoles

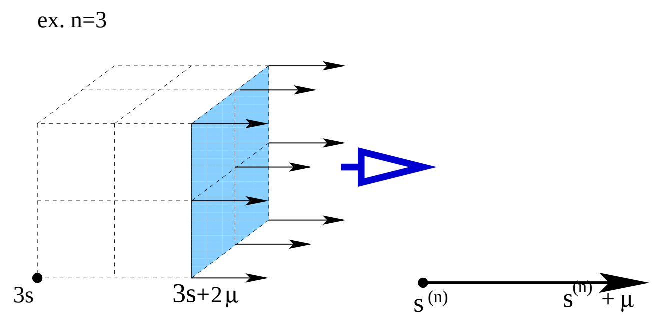


Figure 1: Blockspin definition of monopoles:
T.L. Ivanenko et al., Phys. Lett. **B252**, (1990) 631

Monopole is defined on a a^3 cube and the n-blocked monopole is defined on a cube with a lattice spacing $b = na$

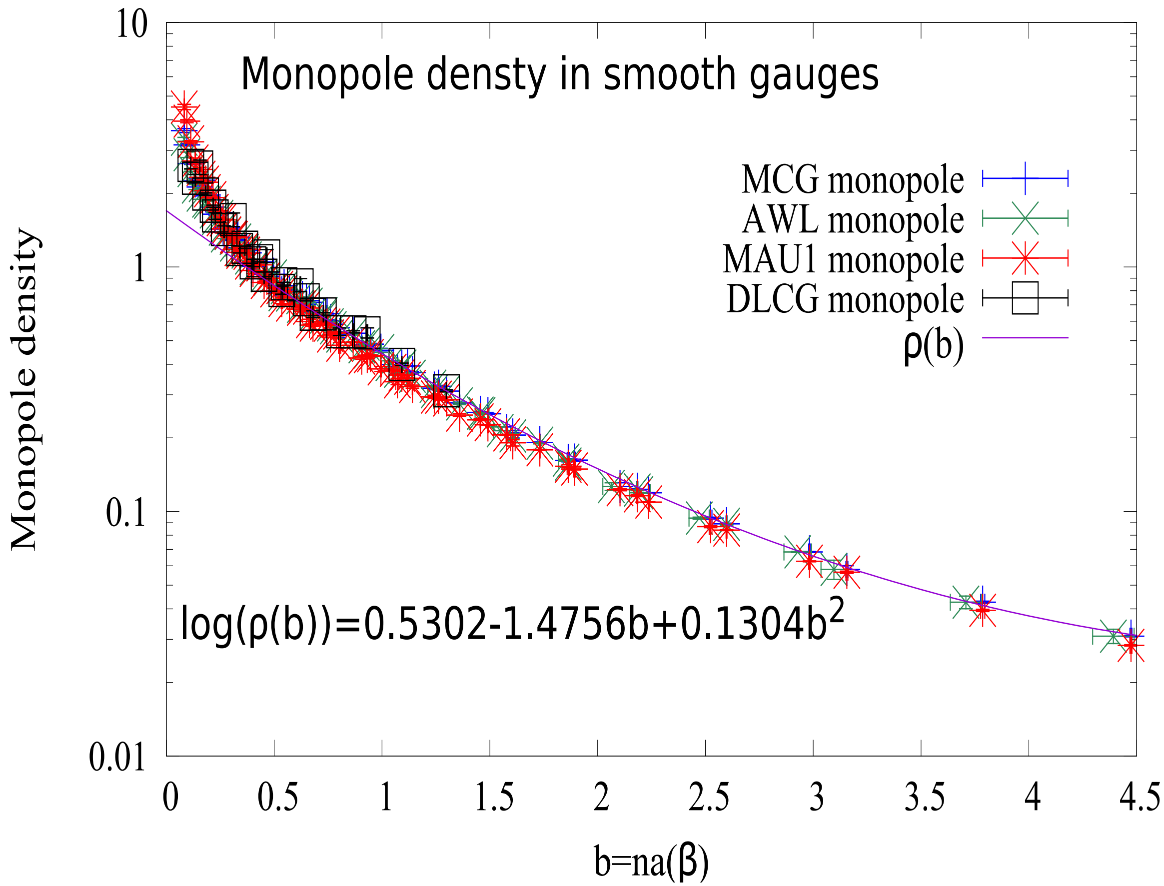
$$\boldsymbol{k}_{\mu}^{(n)}(s_n) = \sum_{i,j,l=0}^{n-1} k_{\mu}(ns_n + (n-1)\hat{\mu} + i\hat{\nu} + j\hat{\rho} + l\hat{\sigma})$$

$n = 1, 2, 3, 4, 6, 8, 12$ blockings adopted on 48^4 lattice.

Evaluate a gauge-invariant density of the n-blocked monopole:

$$\boldsymbol{\rho}(a(\beta), n) = \frac{\sum_{\mu, s_n} \sqrt{\sum_a (k_\mu^{(n)a}(s_n))^2}}{4\sqrt{3} V_n b^3}$$

Figure 2: Comparison of the VNABI (Abelian-like monopoles) densities versus $b = na(\beta)$ in MCG, AWL, DLCG and MAU1 cases.



Summary

1. Clear scaling behaviors are observed up to the 12-step blockspin transformations for $\beta = 3.0 \sim 3.9$. The density $\rho(a(\beta), n)$ is a function of $b = na(\beta)$ alone, i.e. $\rho(b)$. $n \to \infty$ means $a(\beta) \to 0$ for fixed $b = na$. Existence of the continuum limit!

2. When the vacuum becomes smooth enough shown here in MCG, DLCG, AWL, MAU1, the same $\rho(b)$ is obtained. Gauge independence!
   This is naturally expected in the continuum limit.

3. The similar scaling and gauge-independence are observed also with respect to the effective monopole actions under the block-spin transformation.
   $S(k) = \sum_i F_i S_i(k) \to F_i(b = na(\beta))$