Building quark-Diquark model from LQCD

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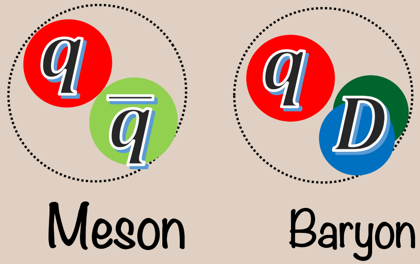
Introduction : Diquarks (1)

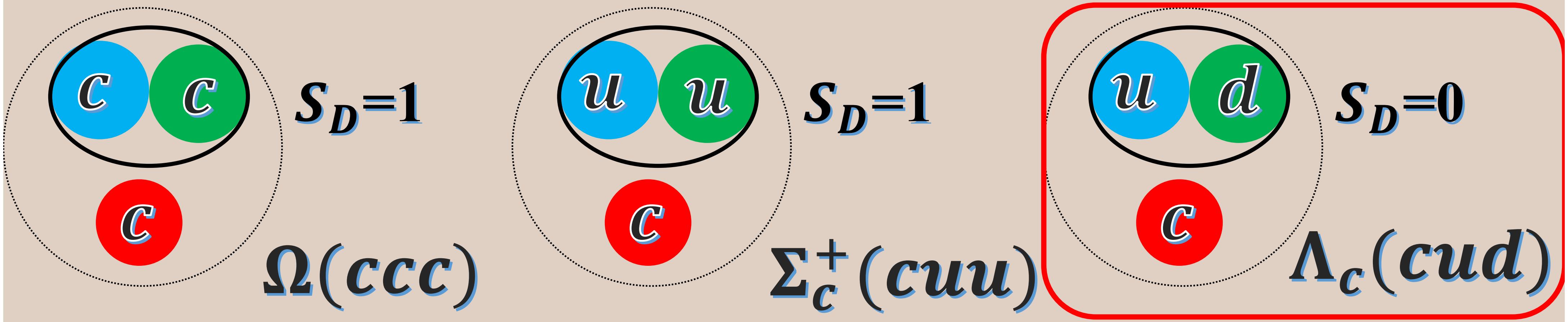
- Diquark's color = anti-symmetric = $\bar{3}$
  - Baryon = Diquark-quark Bound State

$$\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{6}$$



- Diquark inside a baryon
  - Λc baryon (udc) ⇨ scalar-diquark
  - Σc baryon (uuc,ddc,udc) ⇨ axialvector-diquark
  - Ωccc baryon (ccc) ⇨ axialvector-diquark

This work



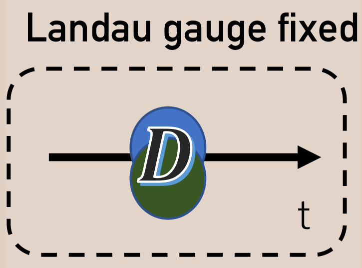
Introduction : Diquarks (2)

- No direct measurements ⇦ *color confinement*
  - Naive introduction of quark-diquark interaction & diquark mass

- Some Lattice QCD results
  - From Landau gauge fixed correlator
    - Diquark Mass : ~700 [MeV]
    - Quark mass : ~ 340 [MeV]

Chiral limit

M. Hess, et. al.PRD 58 (1998)



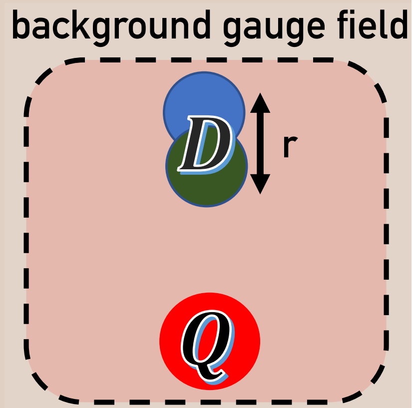
  - Diquark Size : ~ 1.1 [fm]
    - Presence of static quark (quench)

C. Alexandrou, et.al PRL 97, (2006)

  - Diquark Size : ~ 0.6 [fm]
    - Presence of static quark (full)

$m_\pi \simeq 156\ MeV$

A.Francis, et.al. arXiv(2021)



HAL QCD method and the Kawanai-Sasaki method

- HALQCD method : Hadron-Hadron pot. from LQCD
  - Potential reproduces the equal-time NBS wave func. from LQCD
    - And the scattering phase shift

※Nambu-Bethe-Salpeter

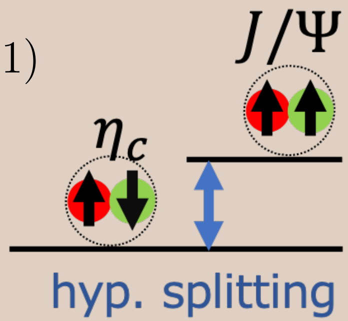
- Application to the quark-antiquark system
  - Naive choice of quark mass

Y. Ikeda and H. Iida, Progress of Theoretical Physics **128**, 941 (2012)

- Kawanai-Sasaki method
  - Quark mass determined from spin-spin interaction and hyp. splitting
  - Not applicable to quark-(scalar)diquark system (No spin-spin interaction)

T. Kawanai and S. Sasaki, Phys. Rev. Lett. **107**, 091601 (2011)

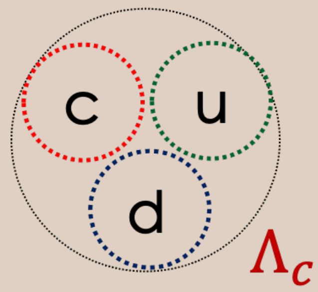
➢ We propose an alternative
  - Extend HALQCD method to quark-diquark system



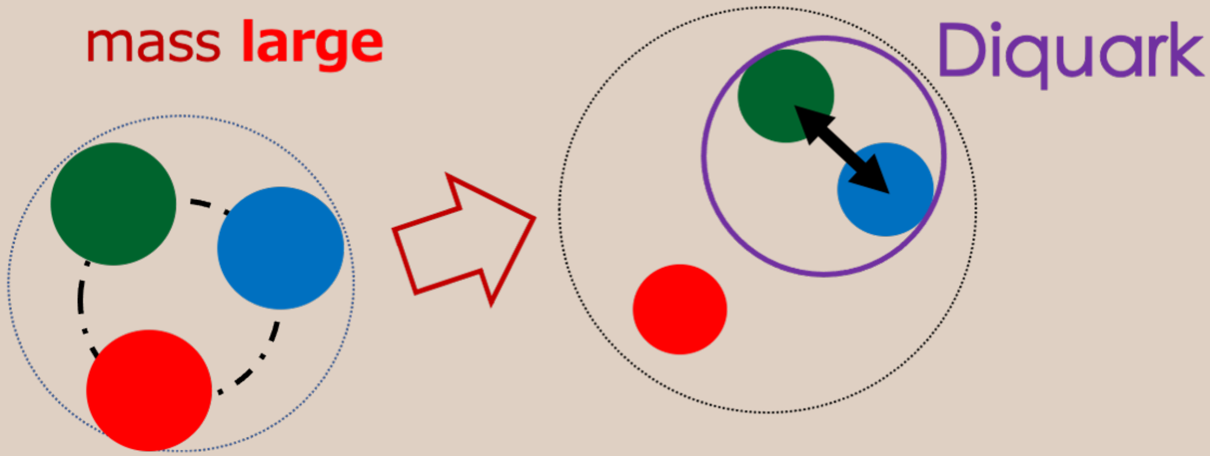
Aim of our current works

- Investigate the interaction and mass of diquarks

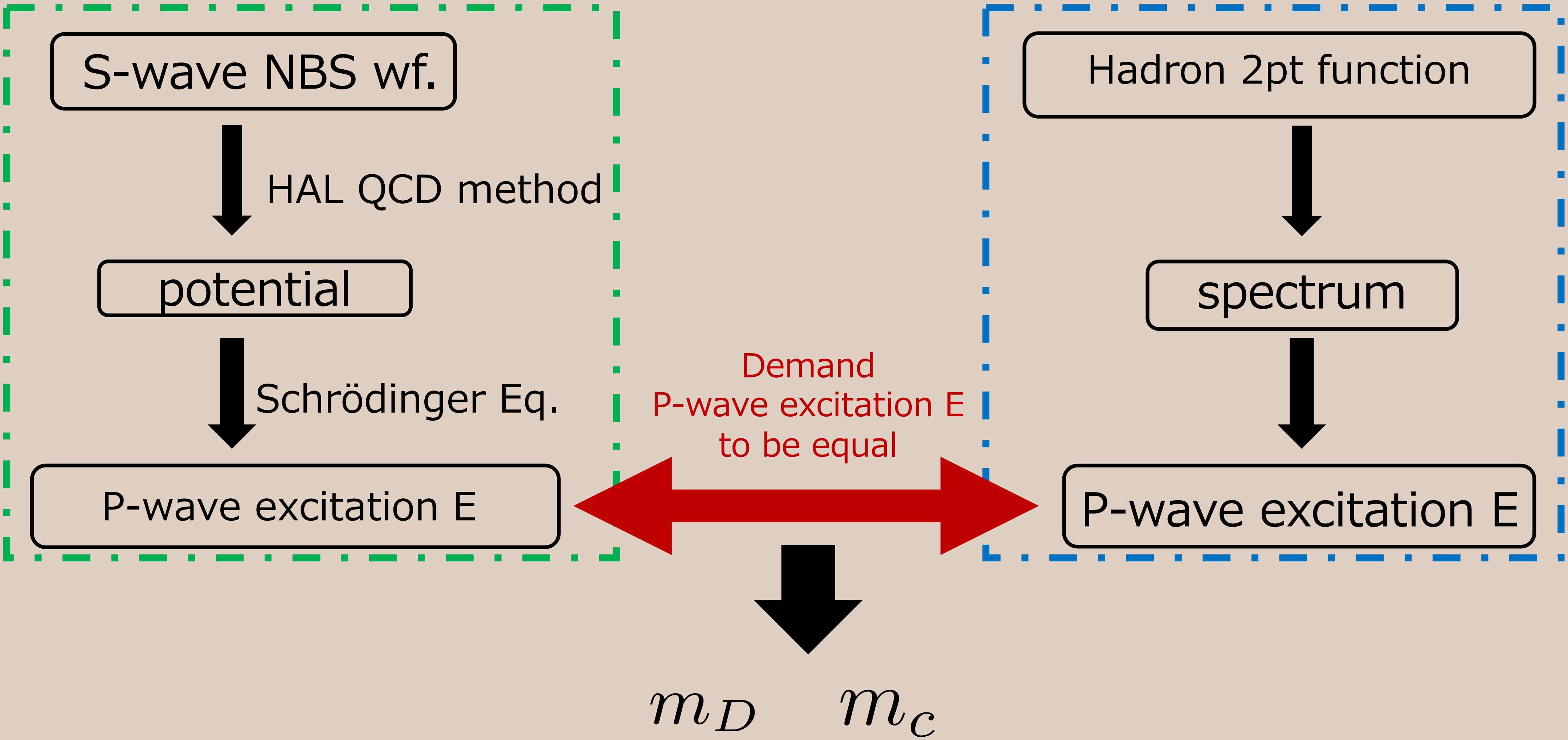
  - Consider charmed baryon Λc(udc)
    - ➢heavy quark acts like a spectator -> Diquark enhanced



  - Consider iso-scalar scalar diquark (0^+)



strategy


S-wave NBS wf.
HAL QCD method
potential
Schrödinger Eq.
P-wave excitation E
Hadron 2pt function
spectrum
P-wave excitation E
Demand
P-wave excitation E
to be equal
m_D m_c

Quark–diquark NBS wave function for $\Lambda_c(1^+/2)$

- The equal-time NBS wave function

$$\psi(\boldsymbol{x},\boldsymbol{y}) \equiv \langle 0|\, D(\boldsymbol{x}) c(\boldsymbol{y}) \,|\Lambda_c\,(1^+/2)\rangle$$

$$\overline{\mathcal{J}}_{cud,\alpha}\,(1/2) = \{[uC\gamma_5 d]\, c_{3,\alpha'}\}^\dagger\, \gamma_{\alpha'\alpha}$$

Λc (1/2) operator

  - In Lattice QCD, extracted from 4pt. func.

$$D^c(\boldsymbol{x}) \equiv \epsilon^{abc} u_a(\boldsymbol{x}) C\gamma_5 d_b(\boldsymbol{x})$$

Diquark operator

$$C_{\alpha\beta}(\boldsymbol{x},\boldsymbol{y},t) \equiv \langle 0|\, D^i(\boldsymbol{x},t) c_{i,\alpha}(\boldsymbol{y},t) \overline{\mathcal{J}}_{cud,\beta}\,(1/2)\,|0\rangle$$

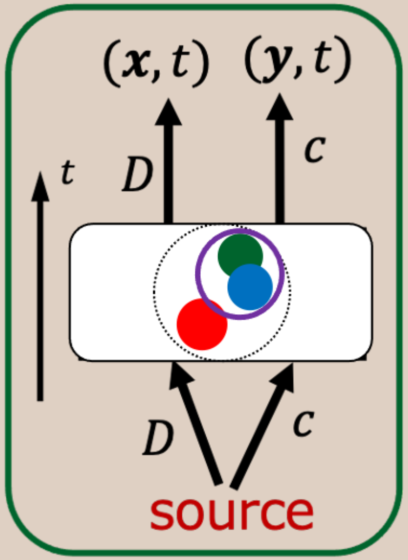
  - Projection to parity + state

$$C(\boldsymbol{x},\boldsymbol{y},t) \equiv \left[\frac{1+\gamma_0}{2}\right]_{\alpha,\beta} C_{\alpha\beta}(\boldsymbol{x},\boldsymbol{y},t)$$

  - In large time-region

$$C(\boldsymbol{x},\boldsymbol{y},t) = \psi(\boldsymbol{x},\boldsymbol{y}) \exp(-M_{\Lambda_c} t)$$

$\Lambda_c(1^+/2)$ mass



- Projection to C.o.M. and s-wave (A^+_1)

Schrödinger equation

- Demand NBS wave func. to satisfy Schrödinger equation

$$-\frac{\nabla^2\phi(r)}{2\mu} + \int d^3r' U(r,r')\phi(r') = E\phi(r)$$

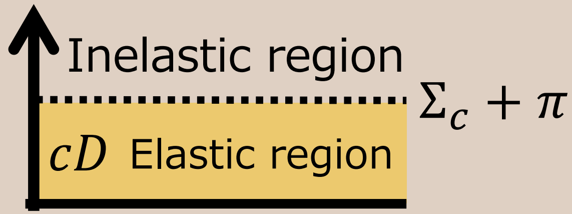
Non-local potential

$E \equiv M_{\Lambda_c} - m_c - m_D$ Binding E

$\mu = m_D m_c/(m_D + m_c)$ Reduced mass

  - Derivative expansion of non-local potential (to the lowest)

$$-\frac{\nabla^2\phi(r)}{2\mu} + U(r)\phi(r) = E\phi(r)$$



$$U(r,r') = U(r)\delta(r-r') \cancel{+ U^{(1)}(r)\nabla\delta(r-r') + U^{(2)}(r)\nabla^2\delta(r-r') + \cdots}.$$

LS force, etc.

  - Define "pre-potential" from the NBS wave function

$$-\frac{\nabla^2\phi(r)}{\phi(r)} = 2\mu\{U(r) - E\}$$

Pre-potential

- Evaluate the spectrum from pre-potentials
  - Demand μ to reproduce the p-wave excitation energy from 2pt func.

Lattice QCD setup

- PACS-CS 2+1 flavor dynamical configuration (#conf =399)
  - Iwasaki gauge action Ukita et.al. (PACS-CS collaboration) PRD 79 (2009)
  - Improved Wilson quark action
  - Lattice size & lattice spacing $M_\pi \simeq 700$ [MeV]

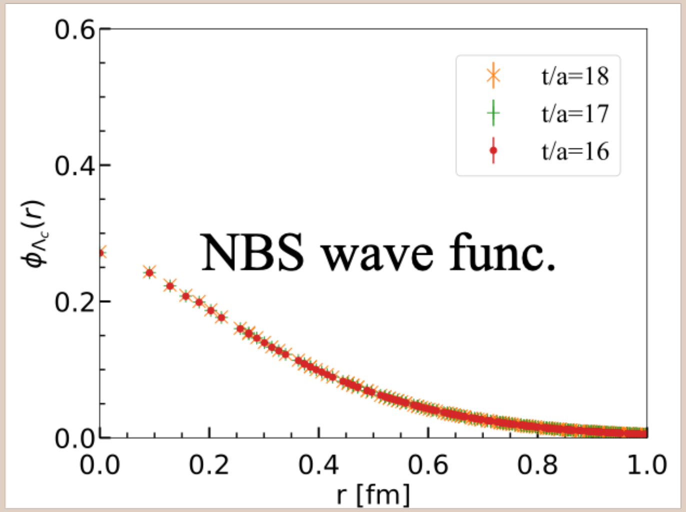
$$V \times T = 32^3 \times 64 \ , a \cong 0.0907 \text{ [fm]} \quad \Rightarrow \quad La \simeq 3(fm)$$

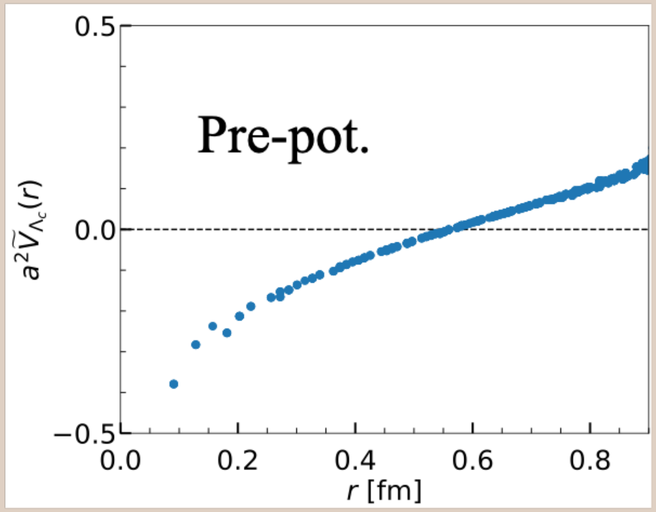
- Relativistic heavy quark action for charm quark

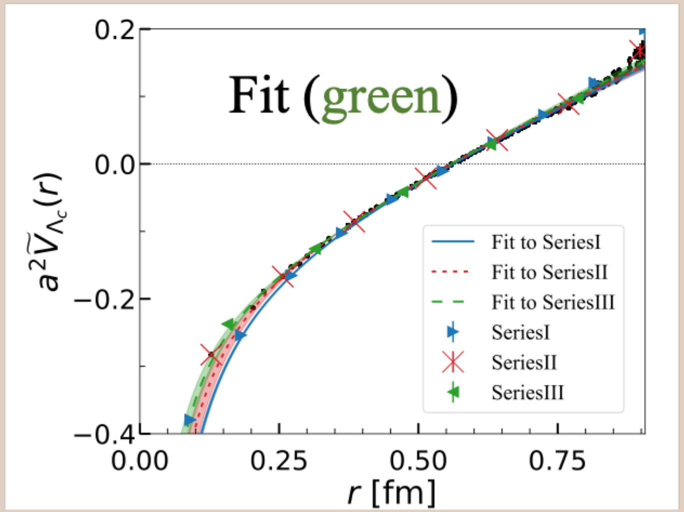
Namekawa et.al. (PACS-CS collaboration) PRD 87 (2013)

- Wall source for all quarks
- Fixed to Coulomb gauge

Numerical results of NBS wave functions and pre-potentials







- Fit the pre-potential at $0.3 \leq r \leq 1.0$ fm
  - Fit function ; Cornell function

$$F_{fit}(r) = -\frac{A}{r} + Br + C$$

G. S. Bali, Phys. Rept. **343**, 1 (2001)

- log term is present in charmonium
  - finite quark mass effect

Y. Koma and M. Koma, Few-Body Systems **54** (2013).

$$F_{fit}(r) = -\frac{A}{r} + Br + C\log(r) + D$$

Eigen value problem

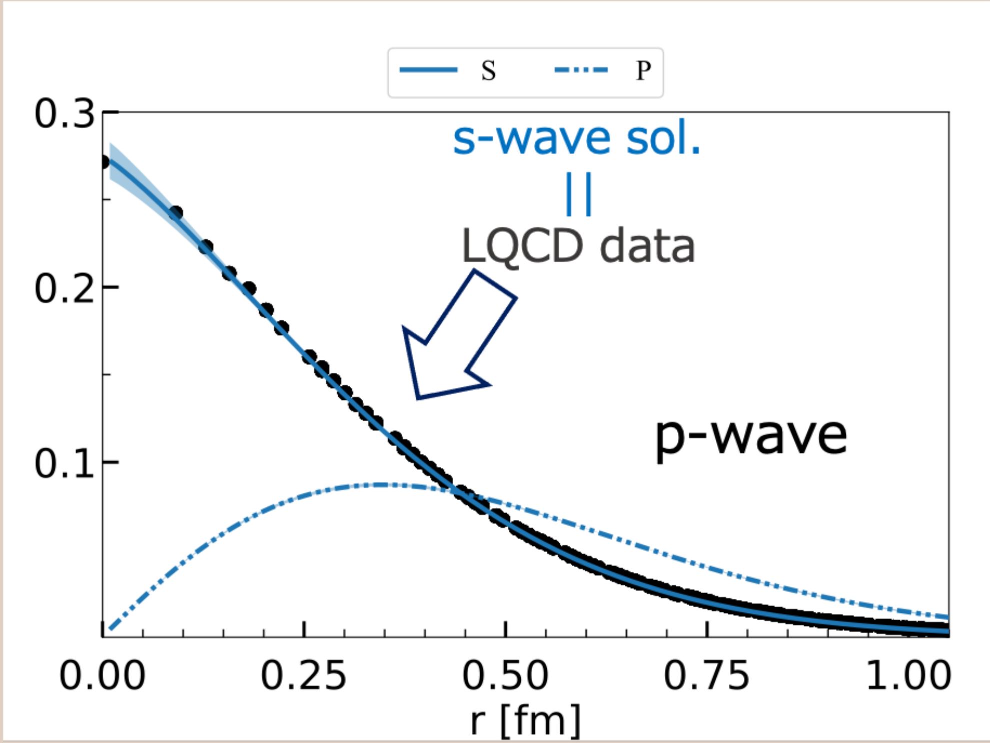
- Solve eigenvalue problem for the pre-potential

$$\phi_0(r) = \phi(r), E_0 = E$$

$$-\frac{1}{r^2}\frac{d}{dr}(r^2\frac{d}{dr})\phi_l(r) + \underbrace{2\mu\left[U(r) - E_l\right]}_{\text{pre-potential}}\phi_l(r) + \frac{l(l+1)}{r^2}\phi_l(r) = 0$$

  - p-wave "pre-excitation energy"

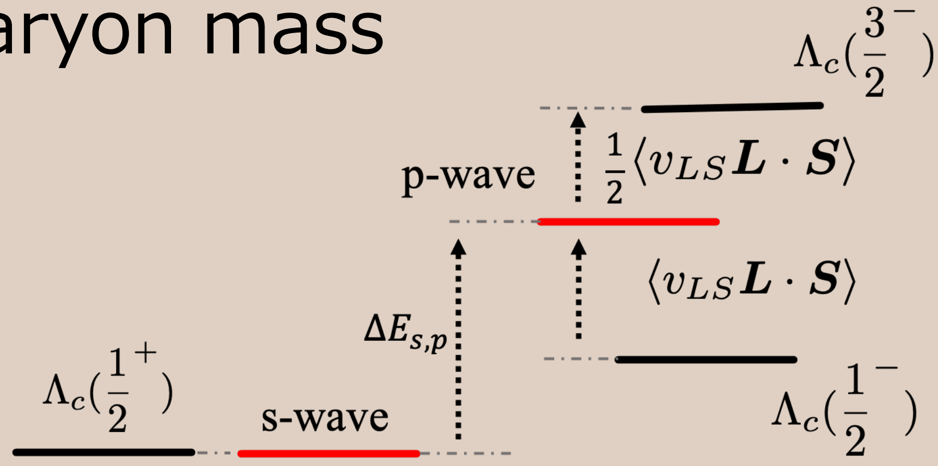
$$\boxed{2\mu\left(E_p - E_s\right) = 0.661(7)\ \mathrm{GeV}^2}$$



p-wave excitation energy from a two-point function

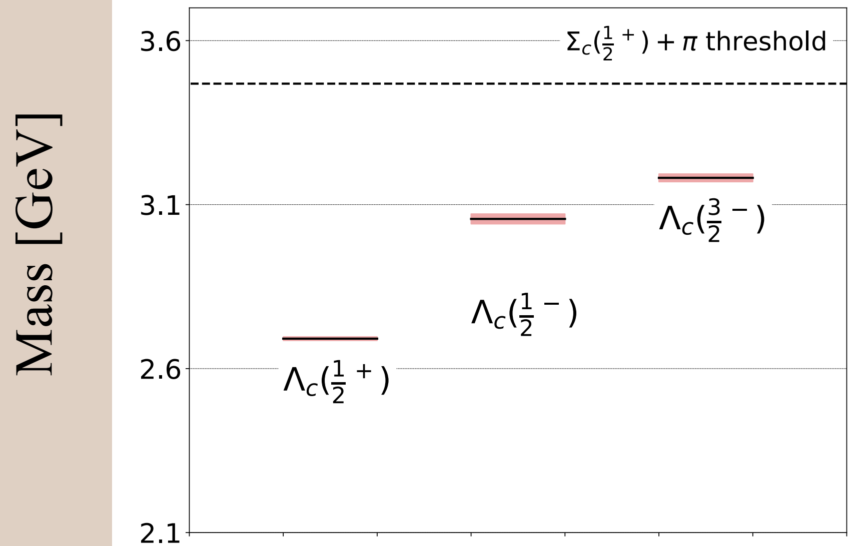
- p-wave excitation energy from baryon mass
  - p-wave splits due to LS force
    - (s=1/2) ⊗ (l=1) ⇨ J=1/2, 3/2
  - Restore p-wave Energy from J=1/2,3/2

$$\frac{1}{3}(2M_{\Lambda_c(3^-/2)}+M_{\Lambda_c(1^-/2)})$$



- Baryon spectrum
  - From the 2pt. correlation function

$$\boxed{\Delta E_{s,p} = 0.457(7)\ \text{GeV}}$$



Diquark mass

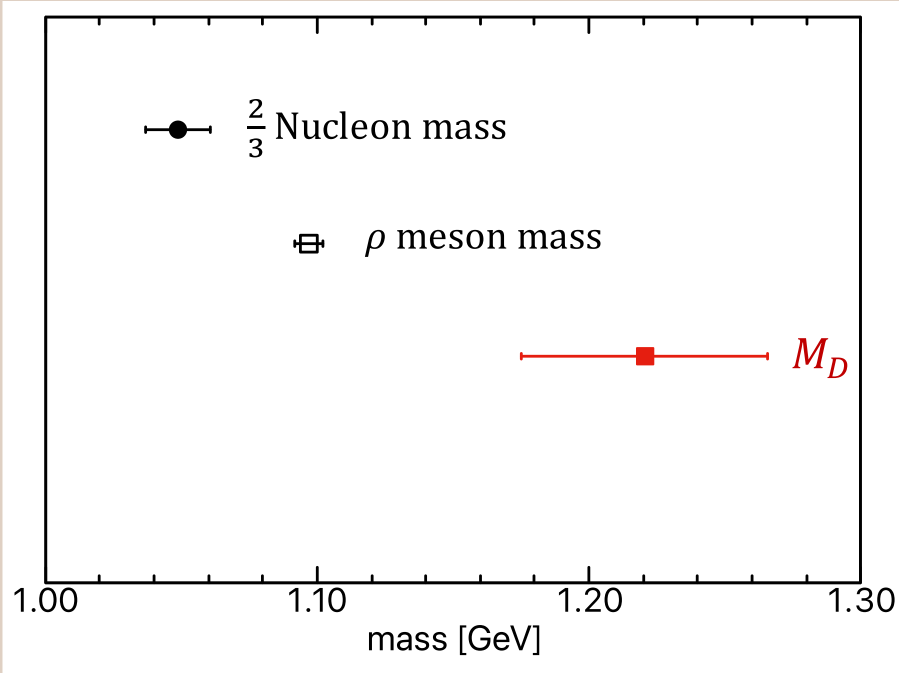
- Diquark mass

$$m_D = \frac{\mu m_c}{m_c - \mu}$$

$$M_D = 1.220(45)\ [\mathrm{GeV}]$$

- Quark mass (same method to $\bar{c}c$)

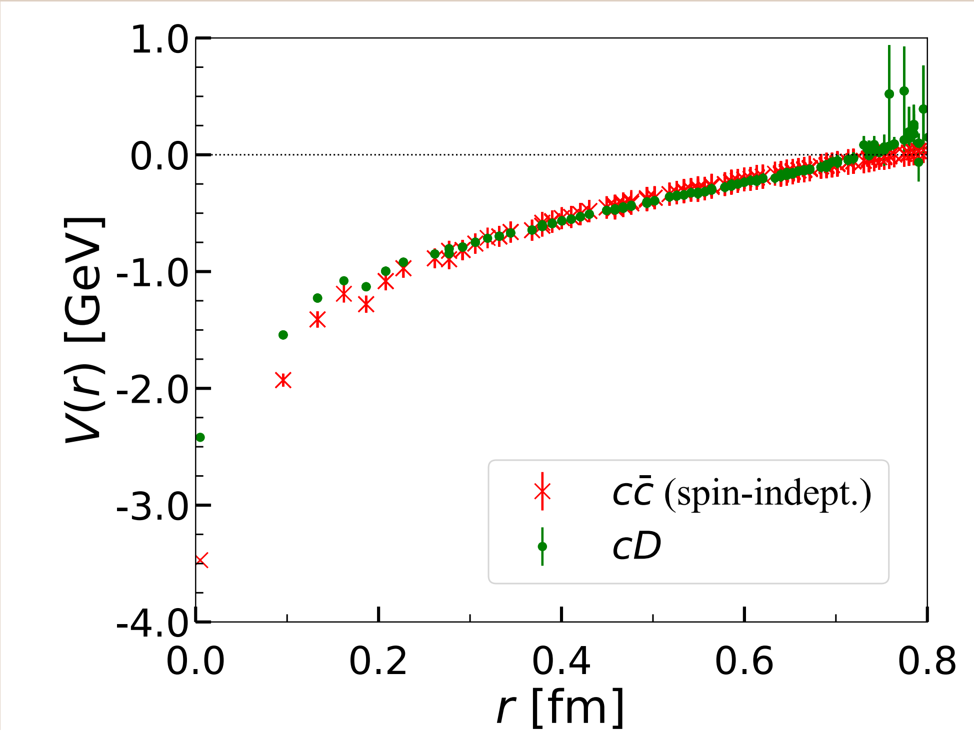
$$m_c = 1.771(48)\ [\mathrm{GeV}]$$

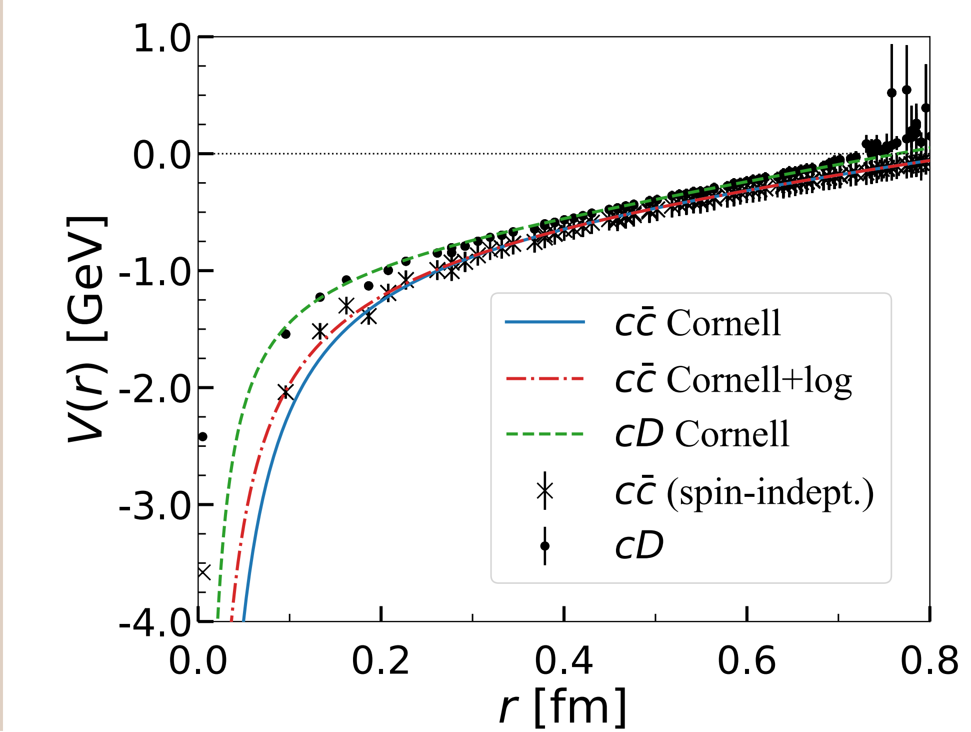


Potential

- cD potential and the $c\bar{c}$ potential
  - Coulomb coefficient
    - cD : $c\bar{c} \approx 1 : 3$
  - String tension
    - Roughly the same

	A [GeV·fm]	σ [GeV/fm]	B [GeV]	const [GeV]
$c\bar{c}$ (Cornell+log)	0.0972(30)	0.673(32)	0.284(8)	-0.554(18)
$c\bar{c}$ (Cornell)	0.173(13)	0.924(52)	-	-0.0355(468)
c-D (Cornell)	0.0665 (107)	1.31 (8)	-	-0.610(62)





Summary

- We proposed a new method to calculate the qD potential and the diquark mass from LQCD
  - P-wave excitation energy was evaluated from
    - ➢ HALQCD method
    - ➢ 2pt function
  - Diquark mass was determined by demanding these two results to agree

- In this study
  - Diquark Mass : Consistent with a naive expectation of quark models
    - Slightly heavier than ρ meson mass, 2/3 nucleon mass (roughly the same)

  - Quark-diquark potential : qualitatively similar to quark-antiquark potential
    - Consistent with Coulomb+linear behavior
      - The string tension is roughly the same
      - The Coulomb coefficient is ~1/3

- Future Works
  - Finer lattice calculation to determine the short range behavior
  - Chiral extrapolation
  - Consider higher excited states
  - Improving gauge fixing → Naive Lattice Coulomb gauge fixing may affect rotational symmetry

Thank you for your attention