Glueballs, localization and thermal monopoles in trace-deformed Yang-Mills theory

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Motivations

Yang-Mills theory at LOW-T:

- Confined.
- Strongly coupled $\Rightarrow$ **no** perturbative methods.
- Center symmetry is realized $\rightarrow \langle \mathrm{Tr} P \rangle = 0$.

Yang-Mills theory at HIGH-T:

- Deconfined.
- Weakly coupled $\Rightarrow$ perturbative/semiclassical methods.
- Center symmetry is spontaneously broken $\rightarrow \langle \mathrm{Tr} P \rangle \neq 0$.

Is it possible to use semiclassical methods to study the low-T regime?

Is it possible to preserve Eguchi-Kawai volume independence? (T. Eguchi and H. Kawai: PRL, **48**, (1982))

The Deformed Theory

Center symmetry is recovered even at high-T.

(M. Unsal and L. Yaffe: PRD **78**, (2008) 065035)

Lattice Study $\rightarrow$ (J.C. Myers and C. Ogilvie: PRD **77**, (2008) 125030).

$$\boxed{\mathrm{S}^{\mathrm{def}} = \mathrm{S}_{\mathrm{W}} + \underbrace{h \sum_{\vec{n}} \left| \mathrm{Tr} P(\vec{n}) \right|^2}_{\text{deformation}}}$$

where S_W is the Wilson action.

- Gauge configurations with $\langle \mathrm{Tr} P \rangle \neq 0$ are **suppressed**.
- The parameter h is chosen in order to restore center symmetry.
- The theory is on $\mathcal{R}^3 \times S^1$ + PBC.

The Aim of This Talk

1. Start deep in the deconfined phase.
2. Switch on the deformation.
3. Study the properties of the re-confined phase.
4. Study the re-confinement transition.

We want to investigate:

- Glueball spectrum in the deformed theory.
- Localization properties of Dirac operator and thermal monopoles across the re-confinement transition.

What we know:

- $T = 0$ Topological properties are recovered in the deformed theory at high-T.
  (C. Bonati, MC and M. D'Elia. PRD, **98** (2018))
  (C. Bonati, MC, M. D'Elia and F. Mazziotti. PRD, **101** (2020))

Glueball spectrum

A. Athenodorou, MC and M. D'Elia arXiv:2010.03618.

Glueball spectrum

YM theory has a non trivial spectrum made of bound states built from the gluon fields.

The states can be labelled using their Angular momentum (J), parity (P) and charge (C).

It is possible to extract the mass from lattice simulation using the relation

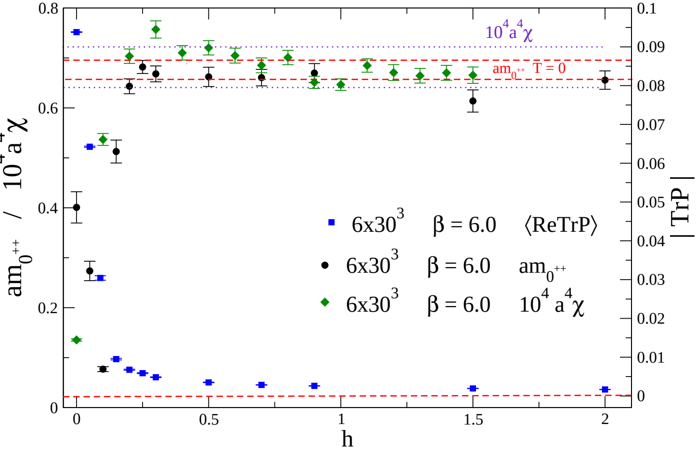
$$\langle O^*(x_0)O(x_0=0)\rangle = \sum_n |\langle n|\,\hat{O}\,|\Omega\rangle|^2 \mathrm{e}^{-E_n x_0}$$

where O is an operator built with link variables and $|\Omega\rangle$ is the ground state.

$x_0 \to \infty \quad \Rightarrow$ we can compute E_0 looking at the decay.

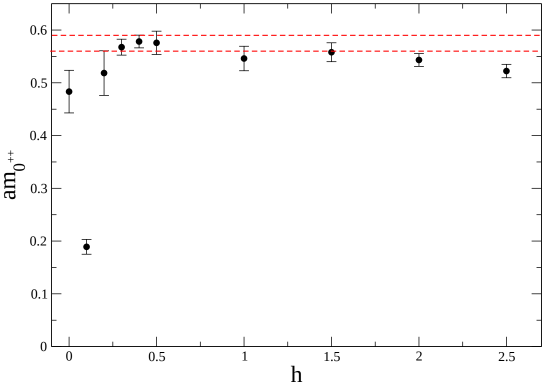
More detail $\to$ (B. Lucini, M. Teper, U. Wegner: JHEP 0464).

Deformed glueball: numerical results



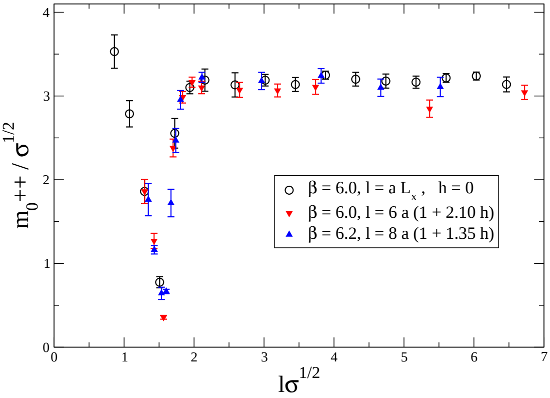
The $T = 0$ value of am_{0++} is reached growing with h.

Glueball for $N = 4$



- In $SU(4)$ we need two deformations.

Behaviour across the transition



- increasing h $\to$ increasing l.
- Possible Taylor approximation: $l_{\text{eff}}(l, h) = l(1 + Ah)$.

THE RE-CONFINEMENT PHASE TRANSITION

C. Bonati, MC, M. D'Elia, M. Giordano, F. Mazziotti: PRD, **103**, (2021)

The re-confinement phase transition

We want to study the transition from the deconfined phase to the re-confined one.

- We study the localization properties of the Dirac operator.
- We compute the thermal monopoles approaching re-confinement.

How these quantities change across the re-confinement?

Thermal monopoles

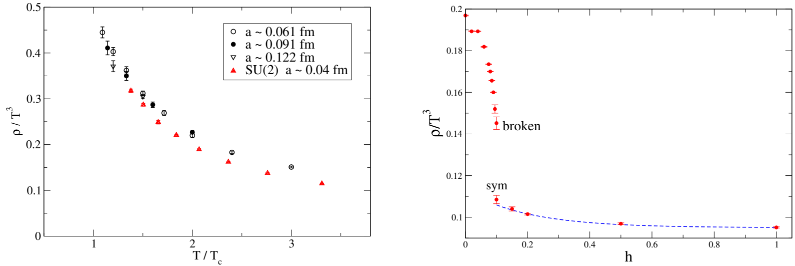
Abelian monopoles $\rightarrow U(1)^{N-1} \subset SU(N)$ gauge group.

We can identify the Abelian subgroup with a gauge fixing procedure. Once the gauge is fixed we can compute the Abelian current with the De Grand - Toussaint construction:

$$\boxed{m_\mu = \frac{1}{2\pi}\epsilon_{\mu\nu\rho\sigma}\hat{\partial}_\nu\bar{\theta}_{\rho\sigma} \qquad \theta_{\rho\sigma} = \bar{\theta}_{\rho\sigma} + 2\pi n_{\rho\sigma} \qquad n_{\rho\sigma} \in \mathbb{N}}$$

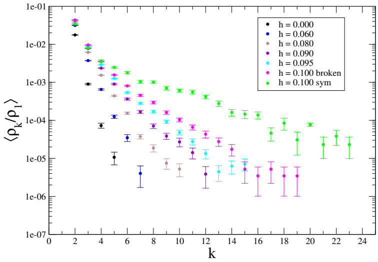
Abelian currents wrapping k times around temporal extension $\rightarrow$ set of k identical particles which permutate cyclically.

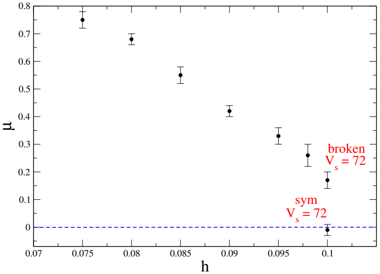
Thermal monopoles density.



- In both cases there is a sudden change at the transition.
- In the deformed theory ρ/T^3 decreases!
- Condensation $\rightarrow$ appearance of non-zero density of particles in the zero-momentum state.

Monopoles in the deformed theory





- We set $h = h_c$ and analysed separately configurations with $\langle \mathrm{Tr} P \rangle = 0$ and $\langle \mathrm{Tr} P \rangle \neq 0$ for the last point.
- The density of trajectories with larger k increases as $h \to h_c$.

Localisation properties of the Dirac spectrum

C. Bonati, MC, M. D'Elia, M. Giordano, F. Mazziotti: PRD, **103**, (2021)

Localization of the Dirac spectrum

The low part of the Dirac spectrum is localised in the spatial volume, while the high part is delocalised.

The separation from localized to delocalized $\rightarrow$ Mobility edge.

In the confining phase all the modes are delocalised.

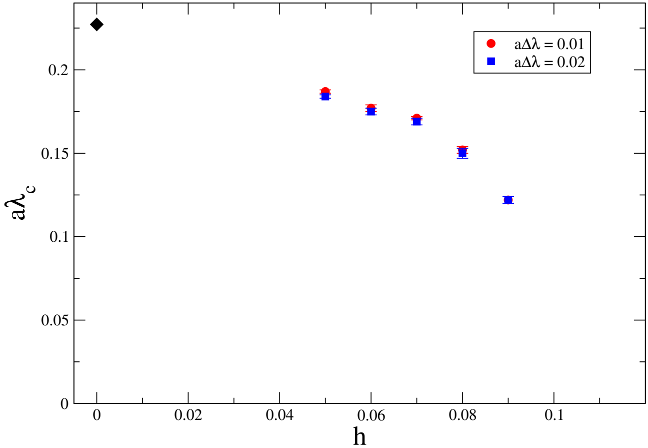
More details:

(T.G. Kovacs, R. A. Vig: PRD **97** (2018)).

(T.G. Kovacs, R. A. Vig: PRD **101** (2020)).

QCD $\rightarrow$ (T.G. Kovacs, F. Pittler: PRD **86** (2012)).

Mobility Edge



Sharp gap at $h = h_c$

Summary part 1

Why trace deformation?

- A tool to understand how the confining properties are related to the realization of center symmetry.
- Exploit volume independence.

Lattice results:

- The first Glueball mass in the reconfined phase is the same of the one at $T = 0$.

Summary part 2

Lattice results:

- The transition from the deconfined phase to the reconfined phase is a first order.
- There is a condensation of thermal monopoles approaching the reconfined phase.
- The spectrum of the Dirac operator becomes delocalised also in the lower part when we approach the reconfined phase.

Future perspectives

- What happens when we compactify more than one dimension?
- Try to compute higher glueball states and compare them with the $T = 0$ ones.
- Decrease the length of the compactified dimension in physical units in order to check semi-classical predictions. Numerically difficult!

Future perspectives

- What happens when we compactify more than one dimension?
- Try to compute higher glueball states and compare them with the $T = 0$ ones.
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THANK YOU!

Back-up slides

Motivation part 2: volume independence

It has been shown that YM theory is volume independent in the large N limit as long as center symmetry is not broken. (T. Eguchi and H. Kawai: PRL, **48**, (1982)).

It could be possible to study infinite volume YM performing simulations on smaller lattices, even on a single site one!

Large N limit + Center symmetry

$\Rightarrow$ Volume independence.

Question: Is it possible to preserve center symmetry also for small compactification radii without changing the physical properties?

Possible Answer: The double trace deformation.

The double trace deformation: $SU(3)$ case

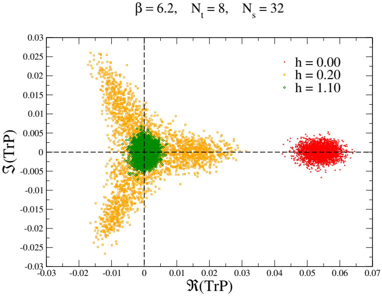
$$\boxed{S^{\mathrm{def}} = S_W + \underbrace{h \sum_{\vec{n}} \left| \mathrm{Tr} P(\vec{n}) \right|^2}_{\text{deformation}}}$$

The deformation is used to avoid the only possible phase transition in $SU(3)$ YM:

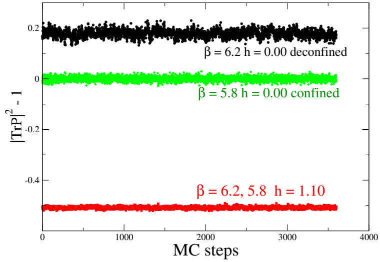
$$\boxed{\mathbb{Z}_3 \;\rightarrow\; \mathbb{1}}$$

In the updating procedure the configurations are drawn with a statistical weight $e^{-S^{\mathrm{def}}}$, thus **configurations with $\mathrm{Tr} P \neq 0$ are suppressed.**

Restoration of center symmetry



- Center Symmetry is recovered increasing h.
- The local value of TrP? Adjoint Polyakov loop.



- $P^{\mathrm{adj}} = |\mathrm{Tr}P|^2 - 1$.
- A negative value implies that TrP is close to zero locally.

The double trace deformation: $SU(4)$ case

$$\boxed{S^{\mathrm{def}} = S_W + \underbrace{h_1 \sum_{\vec{n}} \left|\mathrm{Tr}P(\vec{n})\right|^2 + h_2 \sum_{\vec{n}} \left|\mathrm{Tr}P(\vec{n})^2\right|^2}_{\text{deformations}}}$$

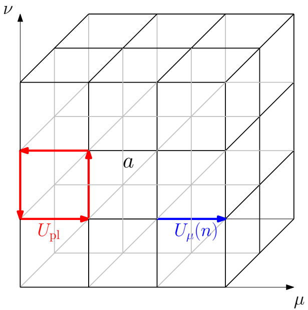
$SU(4)$ YM has two possible symmetry breaking patterns:

$$\boxed{\mathbb{Z}_4 \quad \rightarrow \quad \mathbb{1}} \qquad \boxed{\mathbb{Z}_4 \quad \rightarrow \quad \mathbb{Z}_2}$$

We need to deal with two different order parameter:

$$\boxed{\langle \mathrm{Tr} P \rangle} \qquad \boxed{\langle \mathrm{Tr} P^2 \rangle}$$

Lattice formulation



$$\boxed{U_\mu(n) = e^{igA_\mu(n)}}$$

$$\boxed{U_{\mathrm{pl}}(n) = e^{iga^2 F_{\mu\nu}(n)}}$$

$$\boxed{S_W = \sum_{\mathrm{pl}} \beta \left[1 - \mathrm{ReTr}(U_{\mathrm{pl}})\right]}$$

Finite temperature on the lattice can be obtained using different lattice extensions:

$$\boxed{L_x = L_y = L_z > L_t}$$

Center symmetry (on the lattice)

Multiply all the temporal link variables at a given time slice U_t by an element of the center of the gauge group C.

$$\boxed{U_t(\vec{n}, n_t) \longrightarrow zU_t(\vec{n}, n_t) \qquad z \in C \subset SU(3)}$$

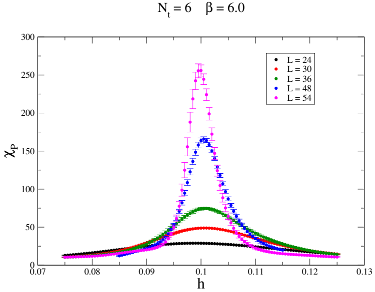
The center of a given group is the subset of elements which commute with all the other elements of the group. The center of $SU(3)$ is Z_3, i.e. the cube root of the identity. In general, the center of $SU(N)$ is $\mathbb{Z}_n$.

$$\boxed{z \in \left(\mathbb{1}_{3\times 3},\ e^{\frac{2\pi i}{3}}\mathbb{1}_{3\times 3},\ e^{\frac{4\pi i}{3}}\mathbb{1}_{3\times 3}\right)}$$

Polyakov loop susceptibility

We compute on the lattice:

$$\boxed{\chi_P = V_s\left(\langle|P|^2\rangle - \langle|P|\rangle^2\right)}$$



We estimate the peak of χ_P and then we do a fit using $\chi_P^{\mathrm{max}} = aV^{\mathrm{d}}$.

If the transition is first order $\Rightarrow$ $d = 1$.

Using $V = 36, 48, 54$ we obtain
$d = 0.98(3)$ $h_c = 0.995(5)$.

Participation Ratio (PR)

Localisation/delocalisation $\rightarrow$ compute the IPR on the lattice

$$\boxed{V_s * PR = \left(\sum_x |\psi_i^\dagger(x)\psi_i(x)| \right)^{-1}}$$

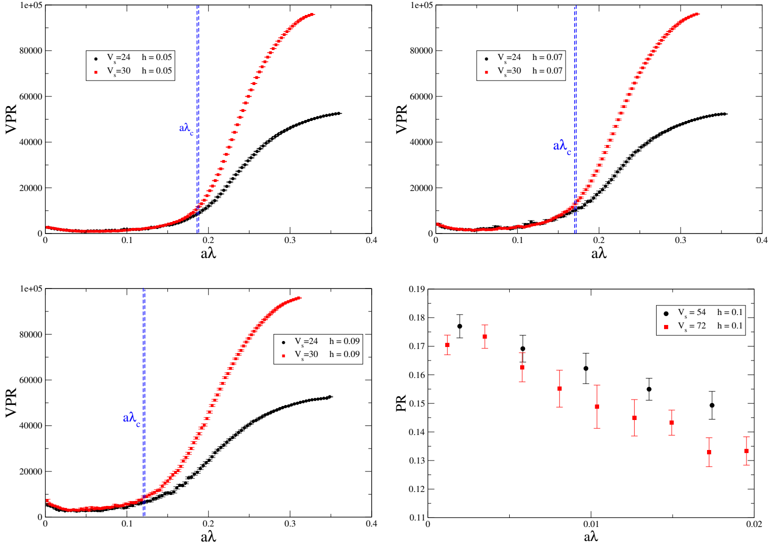
where $\psi_i(x)$ is the i-th eigenvector of the Dirac operator.

Localisation
$V_s * PR$ constant on different volumes.

Delocalisation
$V_s * PR$ grows with growing volumes.

We compute the spectrum of the staggered Dirac operator with stout smearing with stout parameter $\rho = 0.15$.

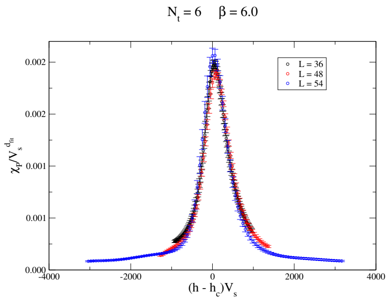
PR in the deformed theory



Finite Size Scaling (FSS)

We perform a FSS using the function

$$\boxed{y(x) = \frac{\chi_P(x)}{V_s^{d_{\rm fit}}} \quad \text{where} \quad x = (h - h_c)V_s^{d_{\rm fit}}}$$



The functions computed on different volumes collapse on the same curve.

The deconfinement/re-confinement phase transition is a first order, as the usual YM phase transition.

Spontaneous breaking of center symmetry

Deconfinement $\Rightarrow$ Center symmetry is broken

The Polyakov loop P is the order parameter of the deconfinement phase transition.

$$\boxed{P = \prod_{t=1}^{N_t} U_4(\vec{n}, t)} \qquad \boxed{\langle \mathrm{TrP} \rangle \quad \rightarrow \quad z \langle \mathrm{TrP} \rangle}$$

Low-T (Confinement)	High-T (Deconfinement)
$\langle \mathrm{Tr} P \rangle = 0$	$\langle \mathrm{Tr} P \rangle \neq 0$

Discretisation of The Topological Charge

- In our simulations we will use the discretisation of the topological charge with definite parity

$$q_L(x) = -\frac{1}{2^9\pi^2} \sum_{\mu\nu\rho\sigma=\pm 1}^{\pm 4} \varepsilon_{\mu\nu\rho\sigma} \mathrm{tr}\left[\Pi_{\mu\nu}\Pi_{\rho\sigma}\right]$$

- In the continuum limit $q_L(x)$ must be corrected by a renormalization factor Z introduced by the lattice discretisation

$$q_L(x) \;\rightarrow\; a^4 Z q(x) + O(a^6)$$

- We remove UV fluctuation using the Cooling procedure.

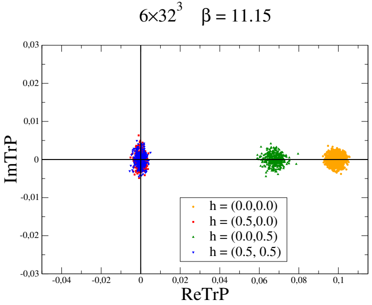
Lattice Spacing and the Deformation on $SU(3)$

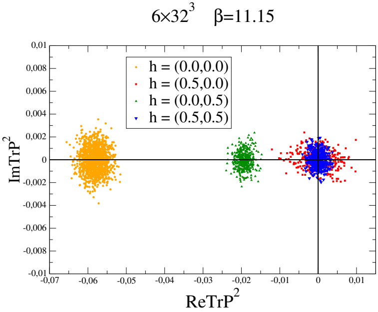
β	h	t_0/a^2
5.96	0.0	2.7854(62)
5.96	1.0	2.8087(69)
5.96	2.0	2.8063(74)

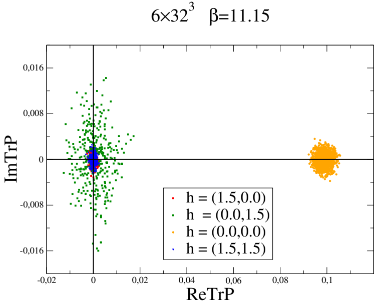
β	h	t_0/a^2
6.17	0.0	5.489(14)
6.17	1.0	5.530(16)
6.17	2.0	5.498(16)

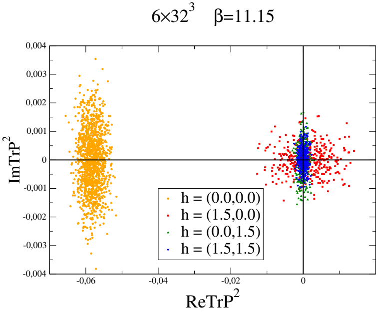
- To test the independence of the lattice spacing on h we determined the scale t_0 defined by gradeient flow. See
  (M. Luscher: JHEP **1403**, 092 (2014)).
- $\beta = 5.96 \quad \rightarrow \quad 24^4$ lattices.
  $\beta = 6.17 \quad \rightarrow \quad 32^4$ lattices.
- Data coincides with those at $h = 0$ up to less than 1%.

Scatter Plots $SU(4)$ $\beta = 11.15$

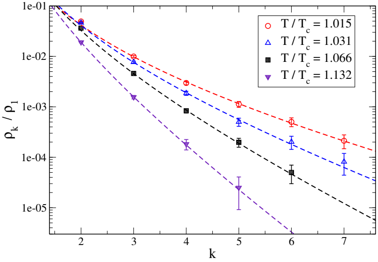


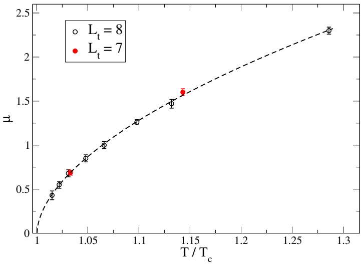






Thermal monopoles in $SU(3)$





- (C. Bonati, M. D'Elia: Nucl. Phys. B, **877**, (2013)).
- The density of the trajectories wrapping multiple times grows approaching (from above) the deconfinement phase transition.
- There is a monopoles condensation with a vanishing chemical potential μ.

Torelon mass

Torelon $\rightarrow$ flux tube of length l that wraps once around the compactified direction.

The ground state energy $m_T(l)$ can be computed using the Nambu-Goto formula:

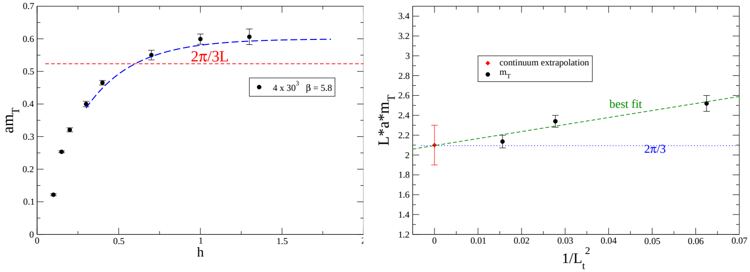
$$m_T(h=0, l > l_c) = \sigma l \left(1 - \frac{2\pi}{3\sigma l^2} \right)^{1/2},$$

where σ is the string tension.

Deformed Theory: Torelons $\rightarrow$ Kaluza-Klein modes:

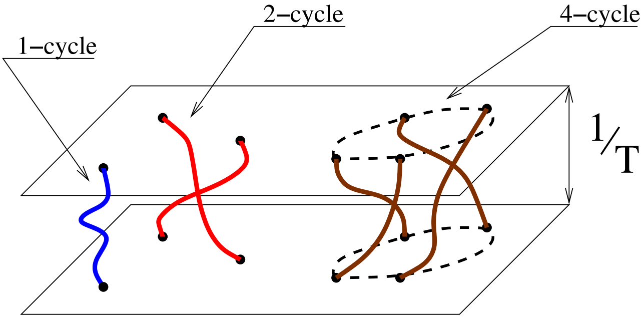
$$m_T(l) = \frac{2\pi}{Nl} = \frac{2\pi}{3l},$$

Deformed torelons: numerical results



- We fit the datas with $am_T(h) = am_T(\infty) + be^{-ch}$.
- The semi-classical approximation works for $N\Lambda l \ll 1$.
- The continuum limit is $am_T L = 2.1(2) \quad \to$ good agreement with semi-classical approximation!

Monopoles trajectories



- (A. D'Alessandro, M. D'Elia, E. V. Shuryak: PRD **81** (2010)).
- Wrapping of monopoles trajectories → Euclidean path integral representation of an ensemble of identical monopoles at thermal equilibrium.

Monopoles gas

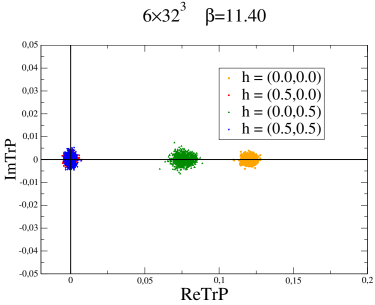
We can try to describe monopoles trajectories as a gas of identical particles. The density of a k-cycle is:

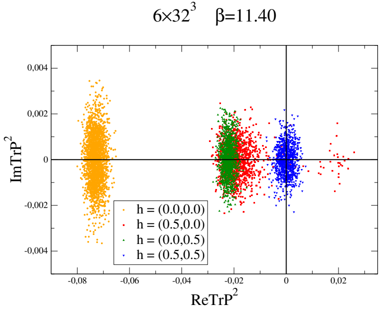
$$\boxed{\rho_k = \frac{\langle n_k \rangle}{V} = \frac{e^{-\hat{\mu} T}}{\lambda^3 k^{\frac{5}{2}}} \quad \hat{\mu} = -\frac{\mu}{T} \geqslant 0}$$

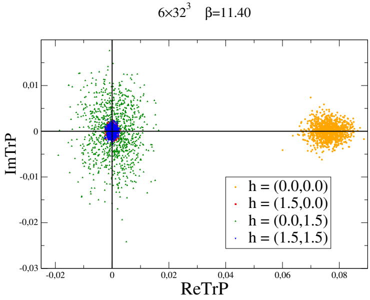
where λ is the De Broglie wavelength.

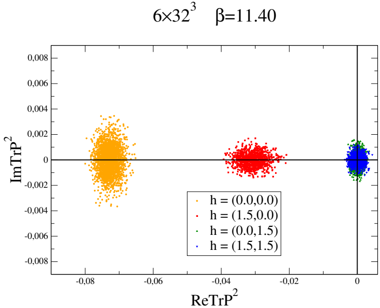
We can fit the density of k-cycles and see if there is a critical density for which $\hat{\mu}$ vanishes.

Scatter Plots $SU(4)$ $\beta = 11.40$

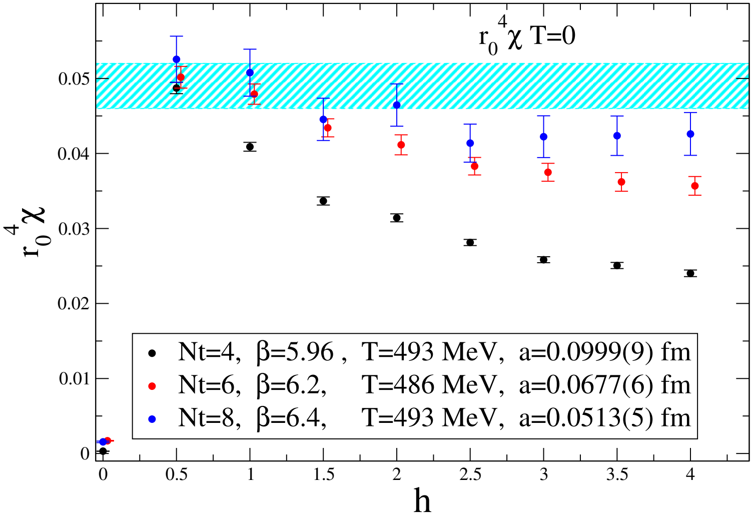








Beware of N_t



$r_0^4\chi$ T=0
$r_0^4\chi$
h
0
0.5
1
1.5
2
2.5
3
3.5
4
0.01
0.02
0.03
0.04
0.05
Nt=4, β=5.96 , T=493 MeV, a=0.0999(9) fm
Nt=6, β=6.2, T=486 MeV, a=0.0677(6) fm
Nt=8, β=6.4, T=493 MeV, a=0.0513(5) fm