

Role of inhomogeneities in the flattening of the quantum effective potential

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- Spontaneous symmetry breaking
- Constrained Monte Carlo simulations
- Summary



Spontaneous symmetry breaking

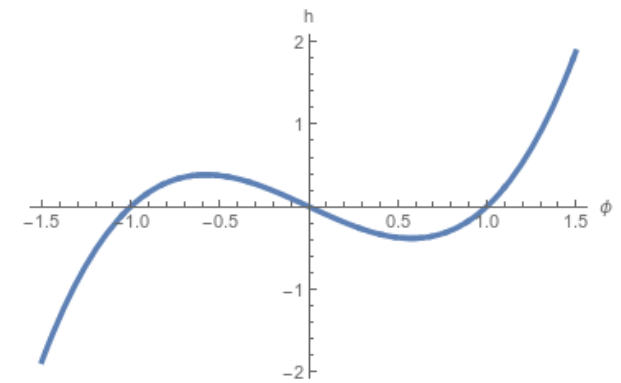
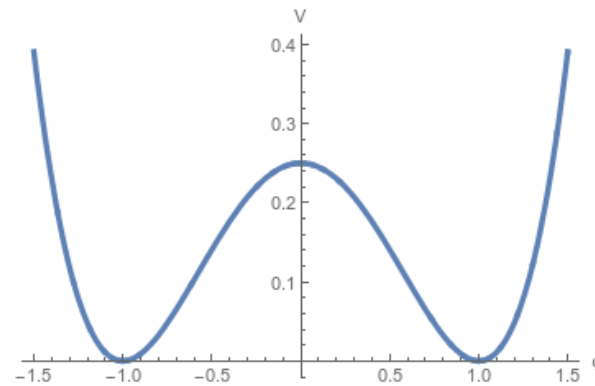
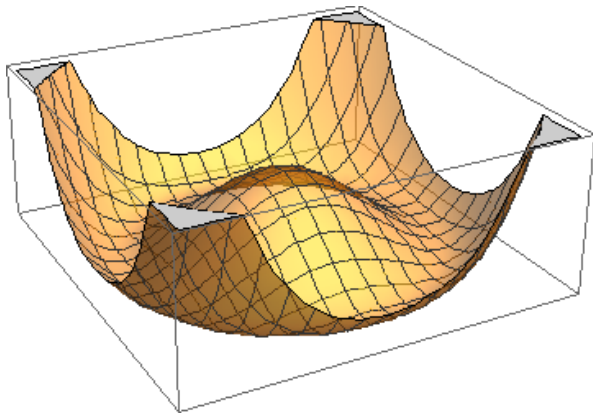
- More than one vacua exist connected by transformations by the broken symmetry.
- Spontaneous breaking is defined as a **double-limit**: 1) volume, 2) explicit breaking: $\bar{\phi}_{\min} = \lim_{h \rightarrow 0} \lim_{V \rightarrow \infty} \langle \phi \rangle_{V,h}$.
- Crossing from one direction of h to another through $h = 0$ is a **first order transition**.
- The **effective potential** between the different vacua is **flat**, but cannot be accessed by usual simulations.
- Discrete symmetry: **domains** of different vacua, if the order parameter is constrained inside the flat region.
- What happens if the broken symmetry is continuous? Expectation: regions of different vacua will change continuously.
- How exactly? What configurations dominate the effective potential?
- What happens to translational invariance?

Spontaneous symmetry breaking

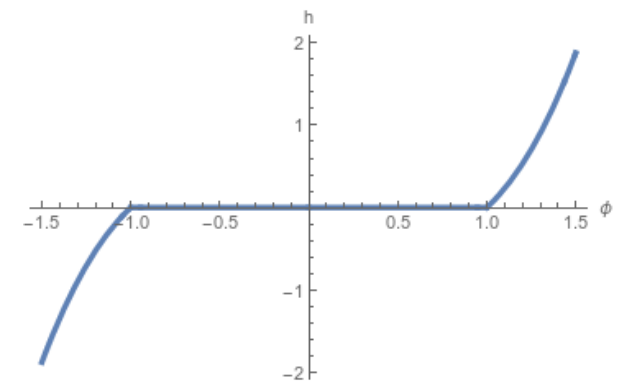
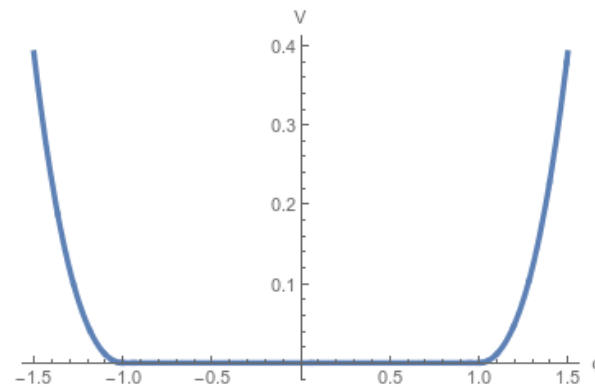
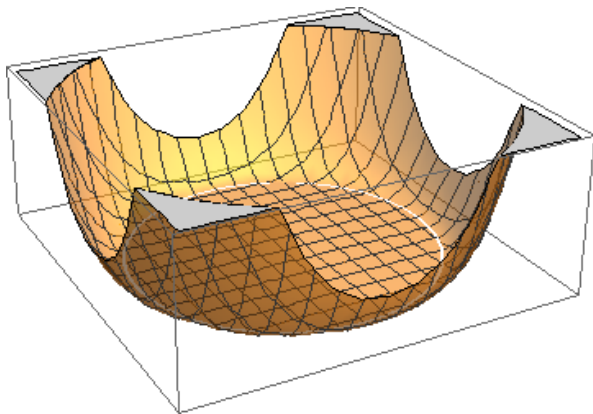
- Consider the 3D $O(2)$ symmetric scalar model

$$S = \int_{d^3x} \frac{(\partial_\mu \vec{\varphi})^2}{2} + \frac{m^2 \vec{\varphi}^2}{2} + \frac{g}{4!} (\vec{\varphi}^2)^2.$$

- Classically:



- QFT:



Accessing the flat region, constraint potential

Define the **constraint** effective potential

$$\exp(-V\Omega(\bar{\phi})) = \int \mathcal{D}\varphi \exp(-S[\varphi]) \delta\left(\int \varphi - V\bar{\phi}\right).$$

- In the **infinite volume** limit (and only there) agrees with the standard effective potential (Legendre-transform).

O’Raifeartaigh et al., NPB 271 (1986)

- Markov chain Monte Carlo techniques can be constructed which **satisfy** the constraint.

Fodor et al., PoS LATTICE2007 056 (2007)

- Analogous to changing from **canonical** (fixed h) to **microcanonical** (fixed $\bar{\phi}$) ensemble.

- h can be recovered as

$$h = \frac{d\Omega(\bar{\phi})}{d\bar{\phi}} = m^2\bar{\phi} + \frac{g}{6V} \left\langle \int_x \varphi^3(x) \right\rangle_{\bar{\phi}}.$$

Constrained simulations

- We carry out constrained MC simulations on 3D lattices of size L^3 with **periodic** boundary conditions.
- In analogy to using $\vec{h} = (h, 0)$ we constrain both field directions:

$$V^{-1} \int \varphi_1 = \bar{\phi} \quad \text{and} \quad V^{-1} \int \varphi_2 = 0.$$

- We only study volume dependence towards the **infinite volume** limit, not the lattice spacing dependence.
- We expect spin-wave like configurations, which break translational symmetry.

Constrained simulations

Looking at **typical configurations**:

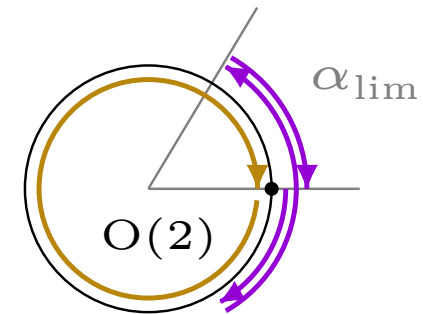
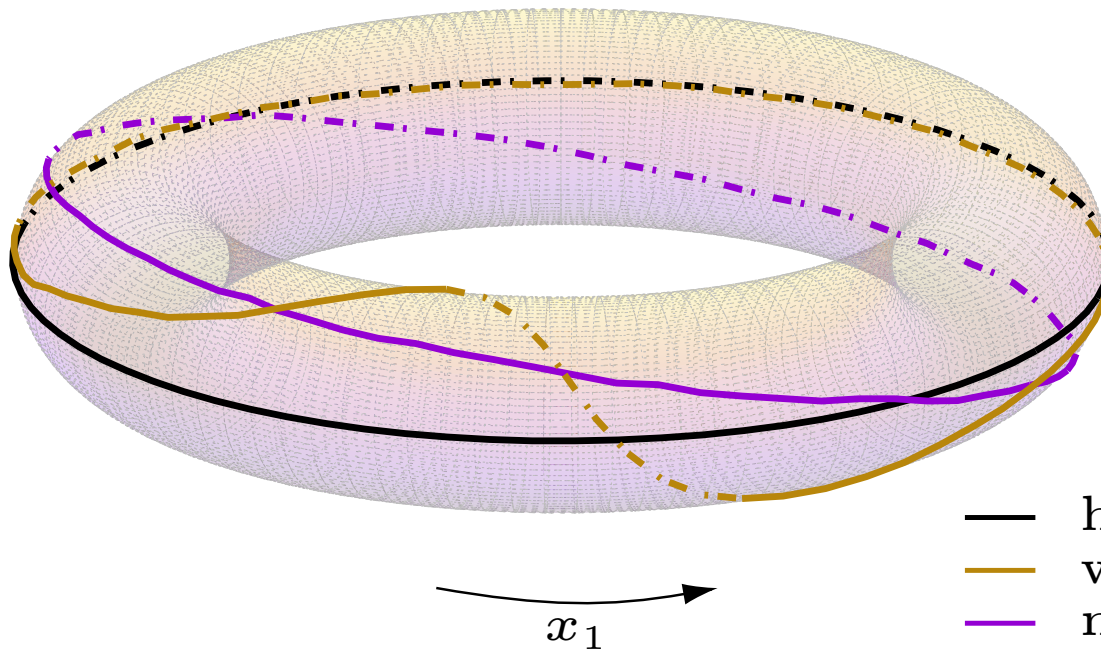
- Spin-wave only in one direction.
- Slice averages

$$\vec{s}(x) = \frac{1}{L^2} \sum_{y,z \in L} \vec{\varphi}(x, y, z).$$

show pronounced inhomogeneities:

Two dominant types of configurations emerge for $\bar{\phi} < \bar{\phi}_{\min}$:

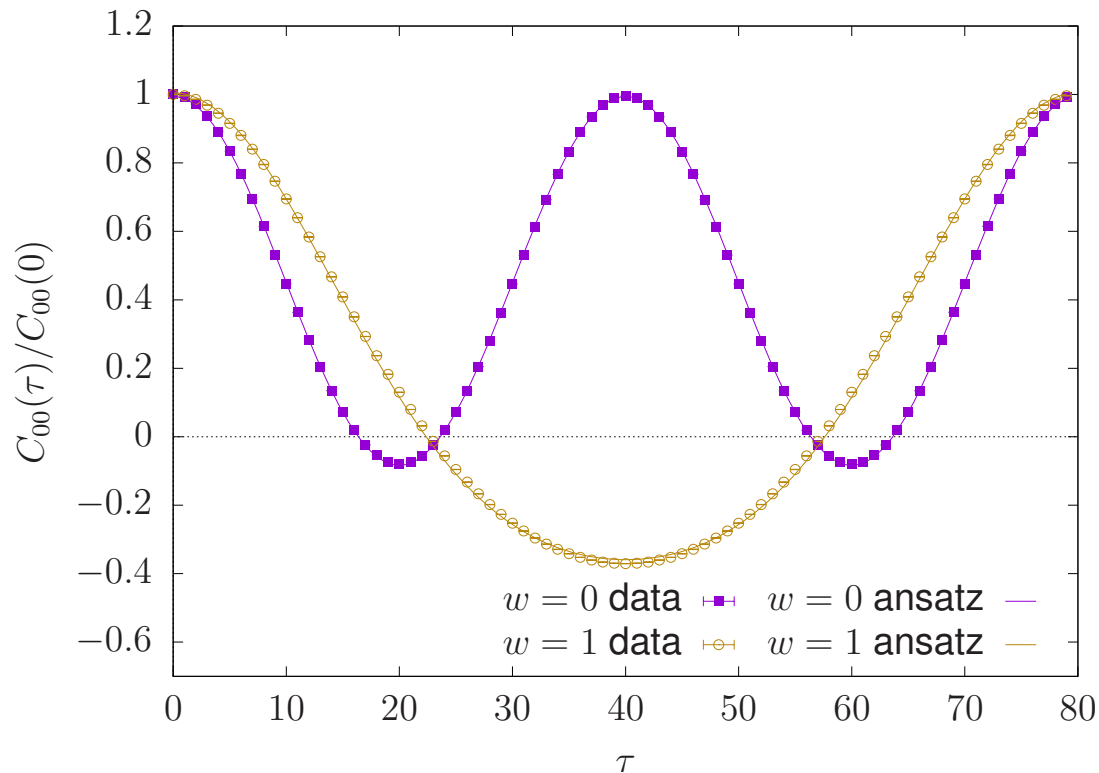
- a **winding**: the field **winds around** the full $O(2)$ space;
- a **non-winding**: the field **oscillates** in $O(2)$ space.



- homogeneous
- winding
- non-winding

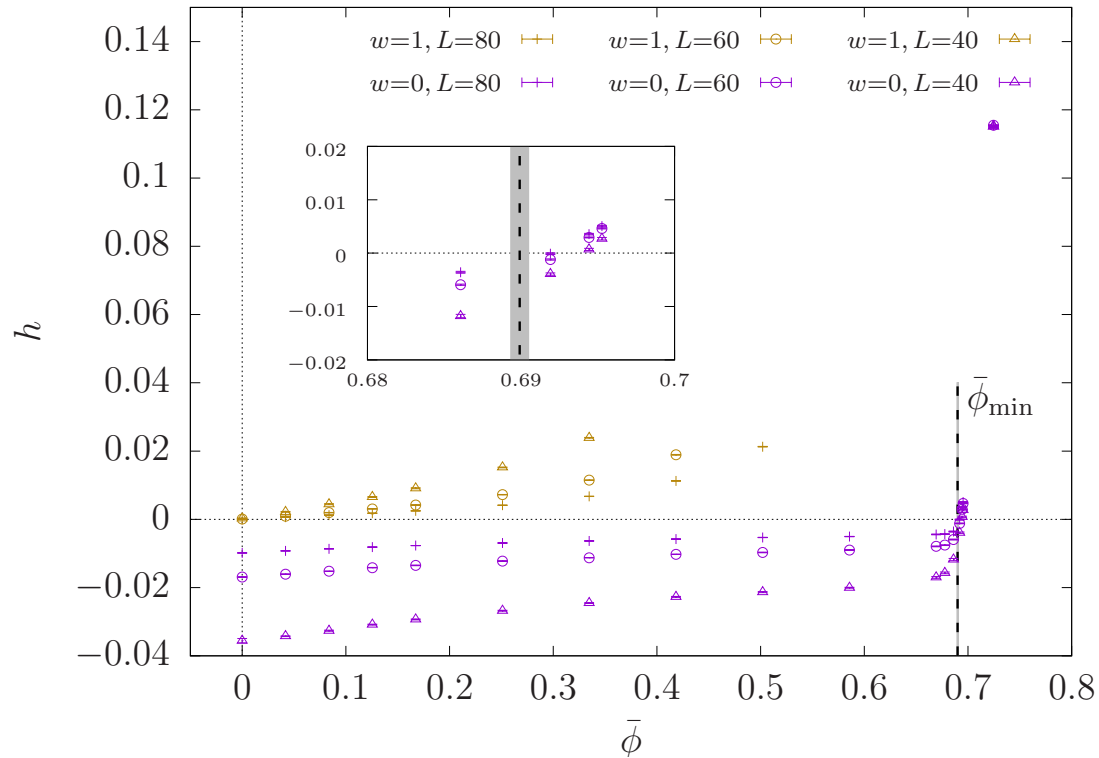
Constrained simulations

Ensemble averaged slice **correlators**: $C_{ij}(\tau) = \frac{1}{L} \left\langle \sum_{x \in L} s_i(x + \tau) s_j(x) \right\rangle_{\bar{\phi}}$.



- **Non-winding** cfgs **continuously connect** to homogeneous cfgs and dominate for $\bar{\phi} \gtrsim \bar{\phi}_{\min}$.
- **Winding** cfgs **do not exist** for $\forall \bar{\phi}$, but dominate for small $\bar{\phi}$.
- Sharp transition between the two ($\bar{\phi}_c$).
- Classically motivated ansatz.
- Can be loosely interpreted as **connecting different vacua**.
- Trivial volume dependence: length scale proportional to L to minimize kinetic energy.

Constrained simulations



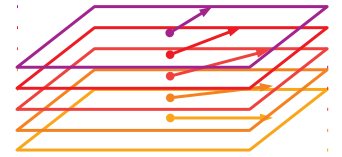
- At infinite volume both sets of curves tend to zero.
- Obtain $\Omega_w(\bar{\phi})$ by the integral method.
- Setting the integration constants is non-trivial.
- Use analogue of surface tension from bubble formation!

$$\Omega_w(\bar{\phi}) = \int_0^{\bar{\phi}} d\phi h_w(\phi) + c_w ,$$

(Differential?) Surface tension

- Different vacua \equiv different points on the circle.
- We can think of **rotating cfgs** as **connecting** different vacua.

- \sum **kinetic energy** between roughly homogeneous slices



$$E_{\text{kin}}(\bar{\phi}) = \frac{1}{2L} \sum_x \langle [\partial_x \vec{s}(x)]^2 \rangle_{\bar{\phi}}.$$

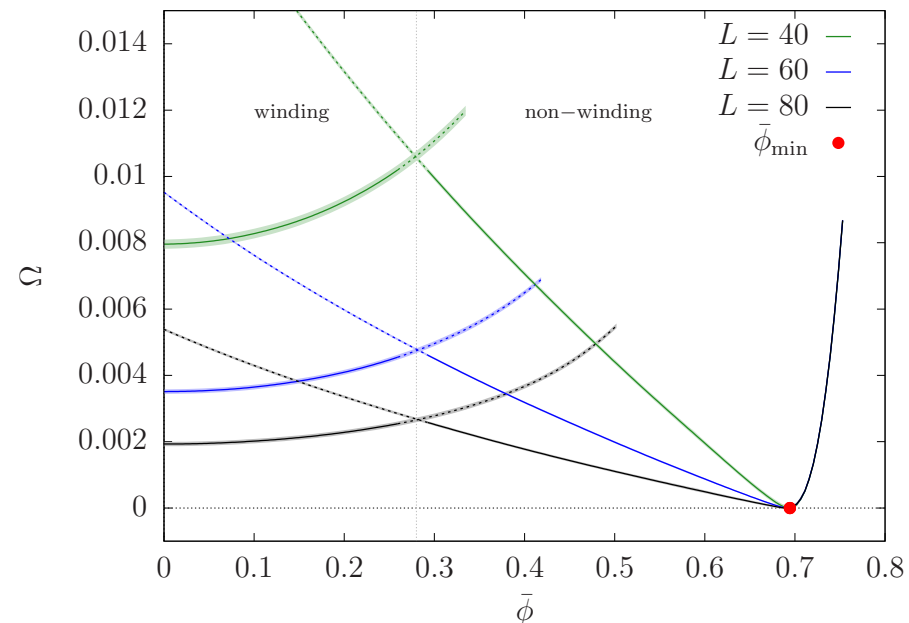
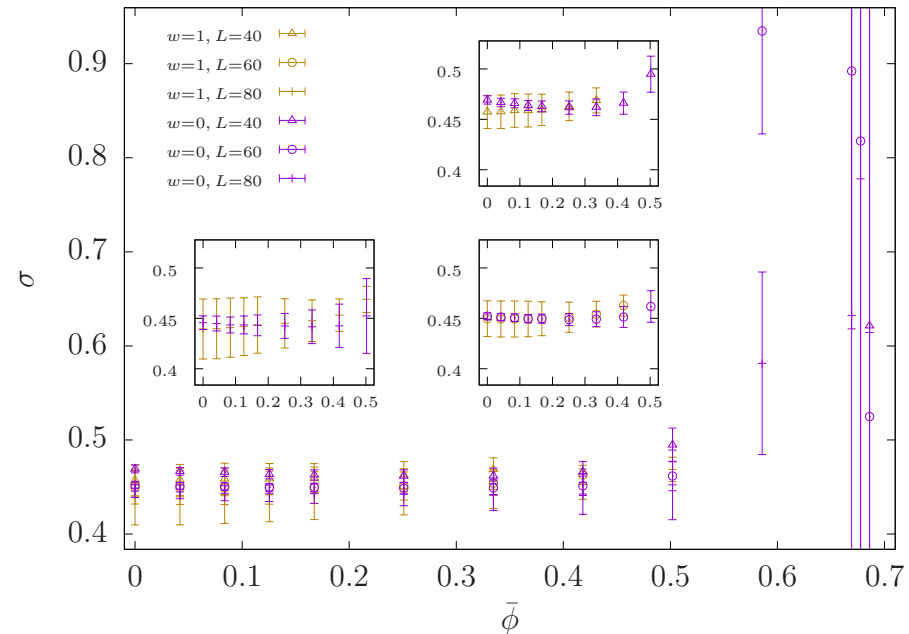
- Assume and later check that the potential energy is proportional to this kinetic term:

$$\Omega_{\text{inhom}} - \Omega_{\text{hom}} = \frac{\sigma}{\bar{\phi}_{\text{min}}^2} [E_{\text{kin}}(\bar{\phi}) - E_{\text{kin}}(\bar{\phi}_{\text{min}})].$$

- **Non-winding** cfgs become homogeneous at $\bar{\phi}_{\text{min}}$, set $\Omega_{w=0}(\bar{\phi}_{\text{min}}) = 0$ to obtain c_0 .
- **Winding** cfgs do not connect to $\bar{\phi}_{\text{min}}$ \rightarrow require topology independence of σ !

(Differential?) Surface tension

- Consistent with a constant.
- Independent of topology.
- Volume dependence also scales out.
- Approaching $\bar{\phi}_{\min}$ requires a $\frac{0}{0}$ type limit
 \Rightarrow errors blow up.
- Constraint effective potential can be reconstructed almost everywhere.
- Except around $\bar{\phi}_c$ (transition $w = 0 \rightarrow 1$).
- Ω becomes flat as $V \rightarrow \infty$ indeed.



Summary

- One can construct and simulate the constraint potential which coincides with the effective potential in the $V \rightarrow \infty$ limit.
- The flat region is dominated by inhomogeneous spin wave configurations.
- The dominant configurations can be topologically classified (**winding** or **not**).
- We extended the definition of the surface tension to spontaneously broken continuous symmetry systems \rightarrow applications?
- Reconstructed the constraint potential across the different topological sectors.