The behavior of Topological objects above the chiral crossover transition in QCD

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Overview

- How do we identify Topological object on the lattice:
  - Find fermionic zero-modes with Overlap operator
- How does the zero modes looks like
- Which Topological object create the zero modes
  - Comparison with analytic formula for Caloron and Instanton-dyons
- Information extracted from fits with analytic formula to lattice zero modes
  - Polyakov loop
  - Dyon separation
- Eigenvalue distribution of overlap operator

How do we see topological object on the lattice

- Indirect detection
  - Find fermionic zero-modes
  - Compare shape from lattice with zero-modes from analytic formula
  - Pros:
    - No need for cooling
  - Problem:
    - Works best for zero-modes or near-zero-modes with smallest eigenvalues

- Needs exact zero-modes $->$ overlap Dirac operator

$$D_{ov} = 1 - \gamma_5 sign(H_W) \;, H_W = \gamma_5 (M - aD_W) \tag{1}$$

- Obey the Ginsparg-Wilson relation within numerical precision (10^{-9})

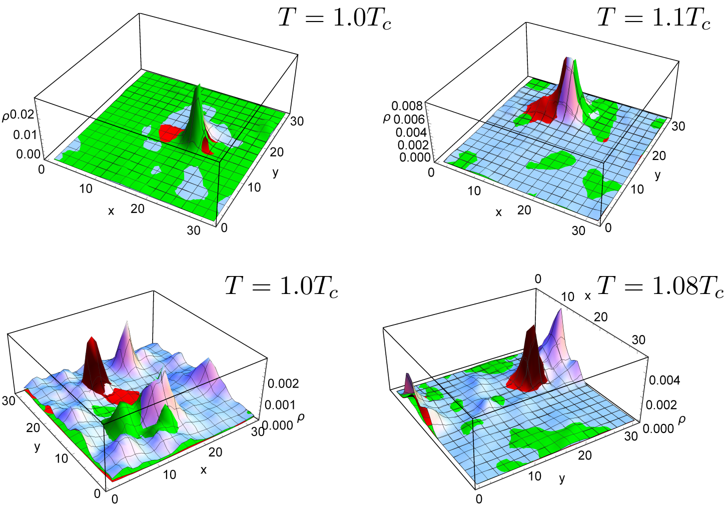
$$\gamma_5 D_{ov}^{-1} + D_{ov}^{-1} \gamma_5 \quad = \quad \gamma_5 \tag{2}$$

Lattice Setup

- We explore the range $T = 1 - 1.2T_c$
- Configurations was generated with Physical masses using domain wall fermions
- Size: $N_s = 32$ and $N_\tau = 8$
- We find the zero-modes using the overlap operator with zero fermion mass
- Zero-modes appear alone
- near zero-modes (λ around 10^{-6}) appears in pairs
- $N_c = 3$
- Will explore 3 boundary conditions:
  - $\psi(\tau + 1/T) = \exp(i\phi)\psi(\tau)$
  - The 3 chosen ϕ's are centered in region of suspected dyon
  - $\phi/(2\pi) = 1/2$ (L dyon) , $1/6$ (M_1 dyon), $-1/6$ (M_2 dyon)

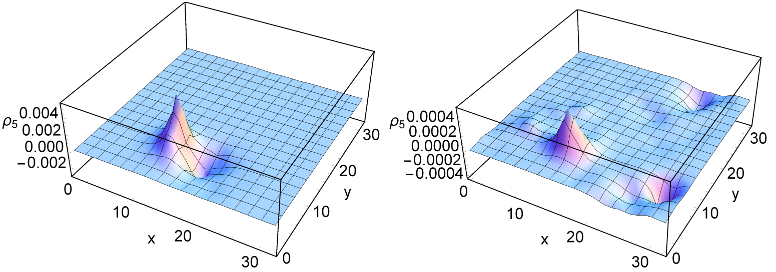
Zero-modes of the overlap Dirac operator

- Zero-mode density ρ examples (sum over t)
- Boundary condition is, red $\phi = 1/2$, blue $\phi = 1/6$, green $\phi = -1/6$



Near-zero-modes of the overlap Dirac operator

- Chiral density ρ_5 of near-zero-modes
- Smallest eigenvalues expected to be dominated by topological objects
- Left: $\phi = 1/2$, right $\phi = 1/6$
- Both chiral densities are the same configuration for $T = 1.1T_c$



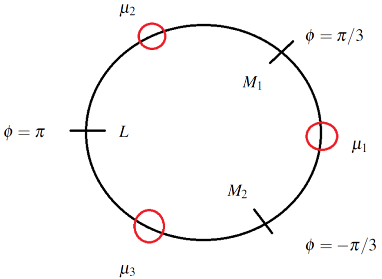
- Shows difference in distribution of topological objects

How does Instanton-dyons look like

- The generalized Caloron is constructed using ADHM, by requiring the Polyakov loop to be able to take any value [Thomas C. Kraan and Pierre van Baal arXiv:hep-th/$9805168v1$]

$$P = \exp(2\pi i * diag(\mu_1, \mu_2, ..., \mu_{Nc}))$$

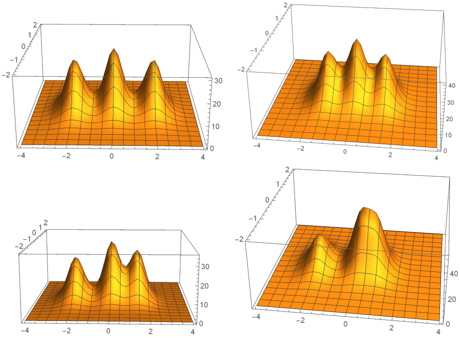
$$\psi(\tau + 1/T) = \exp(i\phi)\psi(\tau)$$



- Each region on the circle corresponds to an instanton-dyon
- Caloron therefore can be seen as being composed of N_c objects, which we call Instanton-dyons
- Changing boundary condition ϕ, changes shape and position of zero-mode

How does Instanton-dyons look like 2

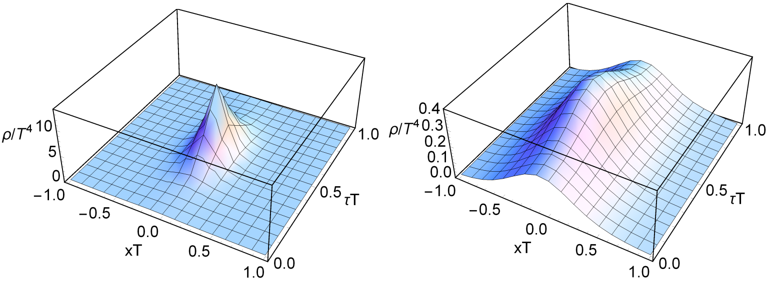
- We therefore get a picture of N_c object inside one Caloron Solution
- Figures: Density of the instanton-dyons in x-y plane



- The sum of Instanton-dyons Add up to 1 Caloron
  - $Q_t = 1$
  - $S = 8\pi^2/g^2$
  - 1 fermionic zero mode
  - When well separated, each dyon holds a fraction proportional to $\mu_{i+1} - \mu_i$

Zero-modes of Instanton-dyons

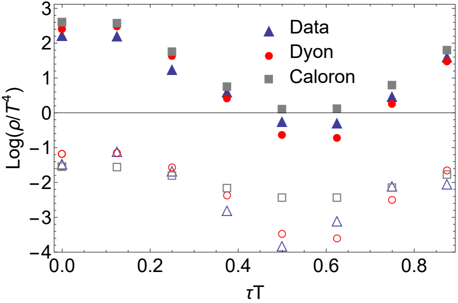
- zero-mode: Short range behavior dependent on distance between dyons
- zero-mode: Long range behavior dependent on $\mu - \phi$
- Left: Dyons close to each other
- Right: Dyons far away from each other



- Periodic in τ direction

How does the Shape of zero-modes compare

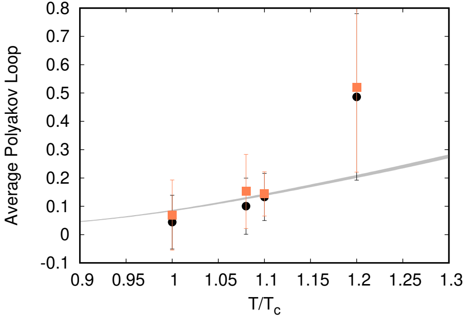
- We fit the analytic formula to the lattice **data** zero-modes by minimizing χ^2
- Figure: 1D slice along τ direction of 4D density fit
- Filled points: (x,y,z) at the center of the zero-mode
- Open points: (x,y,z) far away from the center of the zero-mode



- Short distance (upper): Dyon and Caloron able to explain reasonably
- Long distance (lower): Dyon describes behavior around minimum better

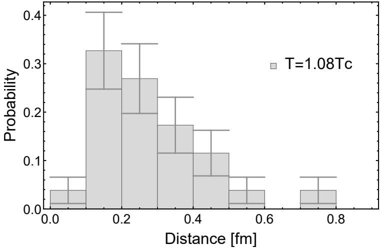
How is the predicted Polyakov Loop

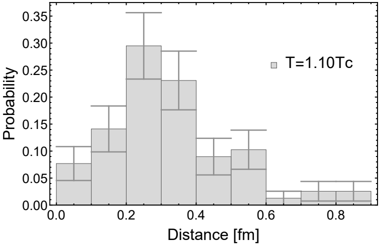
- From the χ^2 fit we obtain position of dyons and eigenvalue of Polyakov loop for the specific configuration
- Can recreate Polyakov loop expectation value from fit
- Low statistics for $T = 1.2T_c$
- Different points are for different starting conditions in χ^2 fit



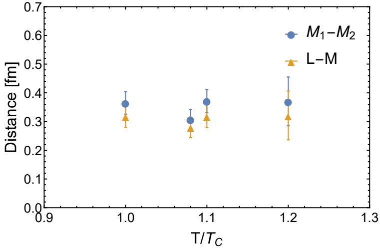
Other results from fits

- Distance distribution to other dyons inside the caloron



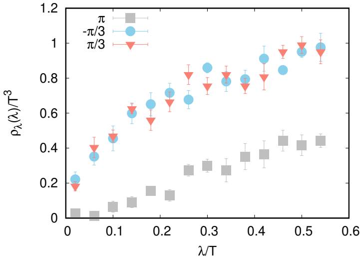


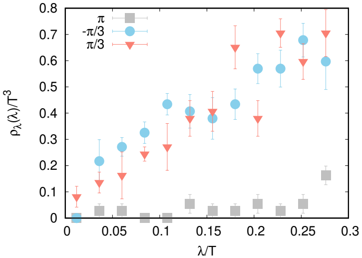
- Average distance between dyons inside the caloron



Spectral density for different boundary conditions

- Chiral condensate comes from eigenvalue distribution at zero
- left $T = 1.1T_c$, right $T = 1.2T_c$





- $\phi/(2\pi) = 1/2$ shows different behavior from other boundaries
- Indicates different species of dyons, since only the dyon in sector $\phi/(2\pi) = 1/2$ was affected by zero-modes during the generation of the configuration

Summary

- Instanton-dyons comes from the need to generalize to finite temperature and Polyakov loop different from 1
- Instanton split into N_c fractions, each fraction is a dyon
- Zero-modes shape depend on dyon position and Polyakov loop
- Lattice zero-modes are in good agreement with dyon description
  - Shape well described, no obvious differences, though fluctuation of size 20% observed
  - Spectral function shows chiral symmetry restored only for anti-periodic case
  - Polyakov loop reproduced, more statistics needed, especially at higher temperature