Complex poles of Landau-gauge QCD propagators and general properties

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in collaboration with Kei-Ichi Kondo (Chiba U.):
based on Y.H. and K.-I. Kondo,
Phys. Rev. D **99**, 074001 (2019) [arXiv:1812.03116];
Phys. Rev. D **101**, 074044 (2020) [arXiv:2001.05987];
Phys. Rev. D **103**, L111504 (2021) [arXiv:2103.14322] + [arXiv:2105.07487].

Introduction

- Analytic structure of a propagator: **states and spectrum**
  Physical case: **Källén-Lehmann spectral representation**

$$D(k^2) = \int_0^\infty d\sigma^2 \frac{\rho(\sigma^2)}{\sigma^2 - k^2},$$

$$\theta(k_0)\rho(k^2) := (2\pi)^d \sum_n |\langle 0|\phi(0)|P_n\rangle|^2 \delta^D(P_n - k),$$

singularities on complex k^2-plane
$\longleftrightarrow$ **states** not orthogonal to $\phi(0)\left|0\right\rangle$

- **Analytic structures of the QCD propagators** would be useful for understanding fundamental aspects of QCD, e.g., **confinement**.

We study analytic structures of the gluon (, quark, and ghost) propagators in the (well-studied) Landau gauge and their implications.

Plan

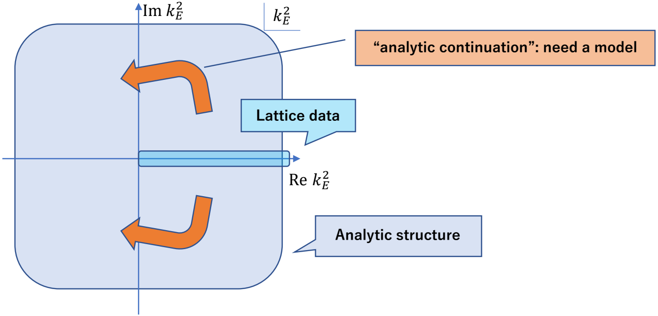
Analytic structures of the QCD propagators by massive Yang-Mills model

[Y.H. and K.-I. Kondo, 1812.03116, 2001.05987]

Modeling the propagators

- We will investigate the analytic structures of the propagators from Euclidean lattice data through "analytic continuation".
- The "analytic continuation" from lattice data is in principle an ill-posed problem: **we need a model with some theoretical background**.

Here, we use **massive Yang-Mills model**, or the Landau-gauge limit of Curci-Ferrari model.



Massive Yang-Mills model: an effective model of the Landau-gauge Yang-Mills theory and QCD

massive Yang-Mills model [Tissier and Wschebor 2011][Peláez et al. 2014]

$$\mathcal{L}_{mYM} = \frac{1}{4} F^A_{\mu\nu} F^A_{\mu\nu} + i B^A \partial_\mu A^A_\mu + \bar{c}^A \partial_\mu \mathcal{D}_\mu [A]^{AB} c^B + \underbrace{\frac{1}{2} M^2 A^A_\mu A^A_\mu}_{Gribov,\ A_\mu A_\mu\ condensate,\ etc.}$$

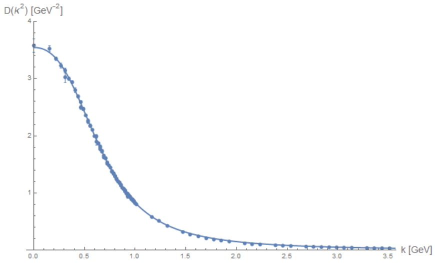
Advantages of this model

(1) **Fitting to lattice data**:

The one-loop gluon and ghost propagators of this model present **striking agreement with lattice results.** (also with $N_F = 2$ quarks)

(2) **"good" perturbation**:

The running coupling can be IR finite in the one-loop RG.

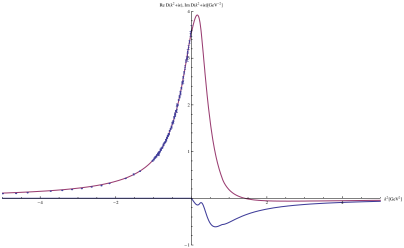


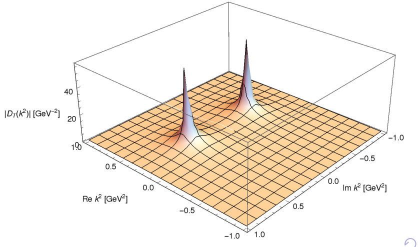
Gluon propagator: lattice data [Duarte, Oliveira, and Silva 2016] and one-loop result for $SU(3)$ YM

Results: analytic structures of the mYM propagators

Results of the propagators modeled by one-loop mYM

- $N_F = 0$ (pure YM): the gluon propagator has a **negative spectral function** and **one pair of complex conjugate poles** for any parameters (g, M).
- $N_F = 2$: near the best-fit parameter (g, M, m_q), both gluon and quark propagators have **one pair of complex conjugate poles**.





General properties of propagators with complex poles

[Y.H. and K.-I. Kondo, 2103.14322, 2105.07487]

Review: recent approaches to analytic structure

Recent analyses of independent approaches agree that the gluon propagator seems to have complex singularities, e.g.,

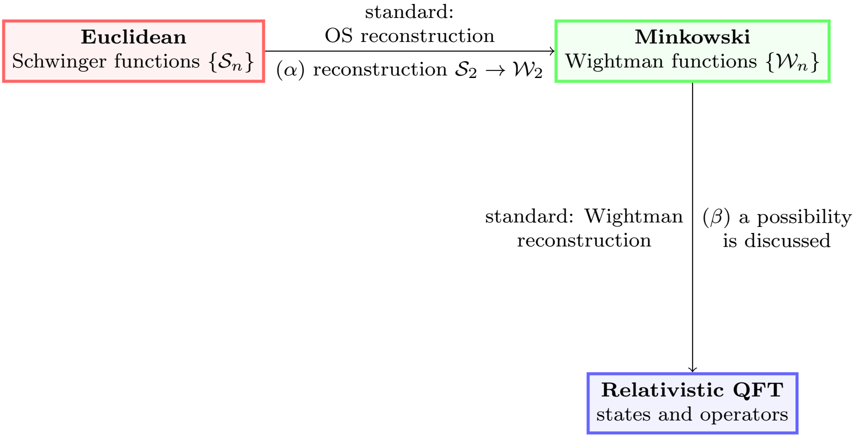
1. Modeling gluon propagator to fit lattice results
   - (refined-)Gribov-Zwanziger model [Dudal et. al. 2008]......
   - Massive-type modeling [Siringo 2016] [this presentation]
   - Padé approximation [Falcão, Oliveira, and Silva 2020]
   - (A variant of) Schlessinger-point method [Binosi and Tripolt 2019]
2. Dyson-Schwinger equation on the complex momentum plane

   [Strauss, Fischer, and Kellermann 2012] [Binosi and Tripolt 2019] [Huber and Fischer 2020]

$\rightarrow$ Let us consider general properties of complex singularities.

Where do we start? Euclidean field theory and QFT

[Osterwalder and Schrader 1973, 1975]



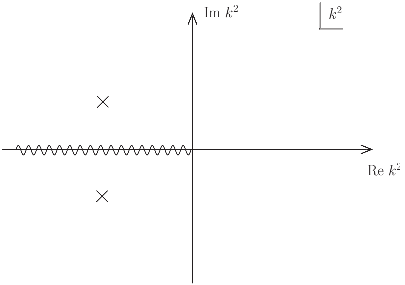
Complex poles: definition

Definition: Suppose that **a two-point function in a Euclidean field theory** (Schwinger function) in the momentum space has the following analytic structure, after an analytic continuation,

$$D(k_E^2) = \sum_i \frac{Z_i}{w_i + k_E^2} + \int_0^\infty d\sigma^2 \frac{\rho(\sigma^2)}{\sigma^2 + k_E^2},$$

The poles except for timelike ($k_E^2 < 0$) singularities are called **complex poles**. (More generally, **complex singularities**).

For an interpretation, one has to **reconstruct** a QFT from the Euclidean field theory.



General properties of complex poles

Wightman function $W(t, \vec{x})$ is reconstructed from Schwinger function $S(\tau, \vec{x})$ by identifying $S(\tau, \vec{x}) = W(-i\tau, \vec{x}) \ \ (\tau > 0)$.
In the presence of complex singularities (bounded in k_E^2-plane), we rigorously prove:

List of properties

- Holomorphy of $W(t, \vec{x})$ in the tube $\mathbb{R}^4 - iV_+$
- Existence of the boundary value $W(t, \vec{x}) = \lim_{\tau \to +0} W(t - i\tau, \vec{x})$ as a distribution.
- $W(t, \vec{x})$ satisfies Lorentz symmetry and locality (i.e. spacelike commutativity).
- ◦ Non-temperedness of the boundary value $W(t, \vec{x}) \notin \mathscr{S}'(\mathbb{R}^4)$
- ◦ Violation of the positivity of $W(t, \vec{x})$ (and the reflection positivity of $S(\tau, \vec{x})$).

Remarks on the non-temperedness

- Non-temperedness: "exponential growth" of $W(\xi)$ in ξ^0.
  e.g.) Gribov-type propagator in $(0+1)$-dim.

$$D(p) = \frac{p^2}{p^4 + \gamma^4} \to W(t) = \frac{i}{2\gamma} e^{-i\frac{\gamma t}{\sqrt{2}}} \sinh\left(\frac{\gamma t}{\sqrt{2}} + \frac{i\pi}{4}\right)$$

  grows exponentially due to "complex energies" $E = \frac{\gamma}{\sqrt{2}} \pm \frac{i\gamma}{\sqrt{2}}$.
  This exponential growth suggests that the corresponding **asymptotic states will be ill-defined** ('confined').
- The **spectral condition is violated** since the spectral condition requires the temperedness as a prerequisite.
- The **positivity condition is violated** since the positivity ($\to$unitarity) implies boundedness.
- The non-temperedness implies that "a native inverse Wick rotation" in momentum space $k_E^2 \to -k^2$ cannot be applied in the presence of complex singularities.

Interpretation in an indefinite metric state space

- The Wightman function $W(t, \vec{x})$ grows exponentially as $t \to \pm\infty \to$ $^{\exists}$states with complex conjugate energies.
- Such states with complex conjugate eigenvalues of a hermitian Hamiltonian can be realized by zero-norm pairs in an indefinite metric state space:

$$(|E\rangle, |E^*\rangle) \begin{cases} H|E\rangle = E|E\rangle, \quad H|E^*\rangle = E^*|E^*\rangle \\ \langle E|E\rangle = \langle E^*|E^*\rangle = 0, \quad \langle E|E^*\rangle \neq 0 \end{cases}$$

  e.g.) the Lee-Wick model

- **Complex singularities correspond to zero-norm pairs, which should be confined**. In the Kugo-Ojima scenario, they should be in BRST quartets.
  $\to$ Both complex-conjugate-energy states in the "one-gluon state" $A_\mu^A(0)|0\rangle$ should contain BRST-parent states.
  [$\to$ complex singularities in ghost-gluon bound states?]

Summary

We have investigated analytic structures of the QCD propagators using the massive Yang-Mills model and considered their general properties and interpretation.

- The gluon (and quark) propagators, modeled by the one-loop massive YM model, have **one pair of complex conjugate poles** near the best-fit parameter.
- Complex singularities lead to **non-temperedness** and **violation of the positivity** of the Wightman function, while they are **consistent with Lorentz symmetry and locality**.
- Complex singularities in a propagator can be understood as **pairs of zero-norm confined states**.

Backup

Details: the massive YM model for SU(3) pure YM

[1812.03116][Kondo et. al., 1902.08894]

- Renormalization scheme (IR-safe scheme [Tissier and Wschebor 2011])

$$\begin{cases} Z_A Z_C Z_{M^2} = 1, \quad Z_g \sqrt{Z_A} Z_C = 1 \\ \Gamma_A^{(2)}(k_E = \mu) = \mu^2 + M^2, \quad \Gamma_{gh}^{(2)}(k_E = \mu) = \mu^2 \end{cases}$$

- Best-fit parameters at one-loop

$$g = 4.1, \quad M = 0.45 \text{ GeV}, \quad \text{renormalized at } \mu = 1 \text{ GeV}$$

- Overall scale of the propagators is needed to fit lattice results due to difference of the renormalization schemes.
- Positions of poles of the gluon propagator

$$-k_E^2 = 0.23 \pm 0.42i \text{ GeV}^2$$

Sketches of proofs: (0)holomorphy

Reconstruction: $S(\tau,\vec{x}) \to W(\xi = (t,\vec{x}))$

$$S(\tau,\vec{x}) \overset{\tau\geq 0}{=} W(-i\tau,\vec{x}) \;\to\; W(\xi - i\eta) \text{ in } \mathbb{R}^4 - iV_+$$
$$\to\; W(\xi) = \lim_{\eta\to 0,\ \eta\in V_+} W(\xi - i\eta)$$

e.g.) Gribov-type propagator ($E_{\vec{p}} := \sqrt{\vec{p}^2 + i\gamma^2}$ with Re $E_{\vec{p}} > 0$)

$$D(p) = \frac{p^2}{p^4+\gamma^4} = \frac{1}{2}\left(\frac{1}{p^2 - i\gamma^2} + \frac{1}{p^2 + i\gamma^2}\right)$$
$$\to S(\tau,\vec{x}) = \int \frac{d^3\vec{p}}{(2\pi)^3} e^{i\vec{p}\cdot\vec{x}} \left[\frac{e^{-E_{\vec{p}}\tau}}{2E_{\vec{p}}} + \frac{e^{-E^*_{\vec{p}}\tau}}{2E^*_{\vec{p}}}\right]$$
$$\to W(\xi - i\eta) = \int \frac{d^3\vec{p}}{(2\pi)^3} e^{i\vec{p}\cdot(\vec{\xi}-i\vec{\eta})} \left[\frac{e^{-iE_{\vec{p}}(\xi^0 - i\eta^0)}}{2E_{\vec{p}}} + \frac{e^{-iE^*_{\vec{p}}(\xi^0 - i\eta^0)}}{2E^*_{\vec{p}}}\right]$$

converges (and is holomorphic in $\xi - i\eta$) for $\eta^0 > |\vec{\eta}|$, i.e., $\eta \in V_+$.

Sketches of proofs:
(1) boundary value and non-temperedness

Reconstruction: $S(\tau,\vec{x}) \to W(\xi = (t,\vec{x}))$

$$S(\tau,\vec{x}) \overset{\tau\geq 0}{=} W(-i\tau,\vec{x}) \;\to\; W(\xi - i\eta) \text{ in } \mathbb{R}^4 - iV_+$$
$$\to\; W(\xi) = \lim_{\eta\to 0,\ \eta\in V_+} W(\xi - i\eta)$$

e.g.) Gribov-type propagator (cont'd)

$$W(\xi - i\eta) = \int \frac{d^3\vec{p}}{(2\pi)^3} e^{i\vec{p}\cdot(\vec{\xi}-i\vec{\eta})} \left[\frac{e^{-iE_{\vec{p}}(\xi^0 - i\eta^0)}}{2E_{\vec{p}}} + \frac{e^{-iE^*_{\vec{p}}(\xi^0 - i\eta^0)}}{2E^*_{\vec{p}}} \right]$$

- If smeared by a smooth compactly-supported function of ξ, $W(\xi - i\eta)$ has a limit $\eta \to 0$ ($\eta \in V_+$): ${}^{\exists}W(\xi) = \lim_{\eta\to 0,\ \eta\in V_+} W(\xi - i\eta)$ as a distribution.
- Since $E_{\vec{p}}$ is complex, $W(\xi)$ grows exponentially for $\xi^0 \to \pm\infty$. $W(\xi)$ is **not tempered**.

Sketches of proofs: (2) violation of positivity

The positivity is violated due to the non-temperedness.

For this, we show

$$\text{positivity} \Longrightarrow \text{temperedness}$$

Rough idea:

- Positivity of 2pt.-function $\to$ the sector $\{\phi(x)\,|0\rangle\}_{x\in\mathbb{R}^4}$ has a positive metric.
- translational invariance $\to$ translation operator $U(a)$: $U(a)\phi(x)\,|0\rangle = \phi(x+a)\,|0\rangle$ is unitary.

Therefore, the Wightman function $W(a) = \langle 0|\phi(0)U(-a)\phi(0)|0\rangle$ will be bounded above $\Rightarrow$ tempered.

Sketches of proofs:
(3) Lorentz symmetry and locality

Lorentz symmetry

Euclidean rotation $SO(4)$ invariance of $S(\tau, \vec{x})$
$\rightarrow$ Complex Lorentz $L_+(\mathbb{C})$ invariance of $W(\xi - i\eta)$
$\rightarrow$ Restricted Lorentz invariance of $W(\xi)$

Spacelike commutativity

For general cases[a], the spacelike commutativity follows from

- permutation symmetry of Schwinger function,
- single-valued continuation $W(\xi - i\eta)$ in the 'extended tube' $L_+(\mathbb{C})[\mathbb{R}^4 - iV_+]$ including spacelike points.
- $-1 \in L_+(\mathbb{C}) \Rightarrow W(z) = W(-z)$.

[a] For a single scalar field, this immediately follows from Lorentz symmetry.