Winding number sectors in U(1) quantum link model

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Motivation

- Condensation phenomena of bosonic and fermionic particles exhibit macroscopic quantum phenomena

- The ”silver blaze" problem in QCD

POSSIBLE TO STUDY IN A PURE U(1) GAUGE THEORY IN 2+1 DIM

- The excitations are strings that propagate all over the lattice

- The winding number acts as the "number operator” for those strings

U(1) Quantum link model Hamiltonian

QUANTUM LINK OPERATOR	$U_{x,\mu} = C_{x,\mu} + iS_{x,\mu}$
PLAQUETTE OPERATOR	$U_\square = U_{x,\mu} U_{x+\hat{\mu},\nu} U^\dagger_{x+\hat{\nu},\mu} U^\dagger_{x,\nu}$
HAMILTONIAN	$\mathcal{H} = -J \sum_\square (U_\square + U^\dagger_\square) + \lambda (U_\square + U^\dagger_\square)^2$
GAUGE OPERATOR	$G_x = \sum_\mu \left(E_{x-\hat{\mu},\mu} - E_{x,\mu} \right)$
GAUGE INVARIANCE	$[\mathcal{H}, G_x] = 0$
ZERO CHARGE CONDITION	$G_x \left\| \Psi \right\rangle = 0$

Spin representation

COMMUTATION RELATION

$$[E_{x,\mu}, U_{y,\nu}] = U_{x,\mu}\delta_{\mu,\nu}\delta_{x,y}$$
$$[E_{x,\mu}, U^{\dagger}_{y,\nu}] = -U^{\dagger}_{x,\mu}\delta_{\mu,\nu}\delta_{x,y}$$
$$[U_{x,\mu}, U^{\dagger}_{y,\nu}] = 2E_{x,\mu}\delta_{\mu,\nu}\delta_{x,y}$$

SU(2) REPRESENTATION

$$E_{x,\mu} = S^3_{x,\mu} \quad C_{x,\mu} = S^1_{x,\mu} \quad S_{x,\mu} = S^2_{x,\mu}$$

FINITE HILBERT SPACE

$$U_{x,\mu} = S^x_{x,\mu} + iS^y_{x,\mu} = S^+_{x,\mu}$$
$$U^{\dagger}_{x,\mu} = S^x_{x,\mu} - iS^y_{x,\mu} = S^-_{x,\mu}$$

GAUGE OPERATOR

$$G_x = \sum_{\mu} \left(E_{x-\hat{\mu},\mu} - E_{x,\mu}\right) = \sum_{\mu} \left(S^3_{x-\hat{\mu},\mu} - S^3_{x,\mu}\right)$$

[S Chandrasekharan and U.-J Wiese, Quantum link models: A discrete approach to gauge theories. Nuclear Physics B, Volume 492, Issues 1–2, 12 May 1997, Pages 455-471]

Winding number simmetry

SYMMETRIES OF THE HAMILTONIAN

- U(1) Gauge symmetry
- Point group symmetries:
  - Translation
  - Rotation
  - Parity
- Charge conjugation:
  - $\mathbb{Z}^2$

ADDITIONAL SYMMETRY

- Winding number symmetry: $\mathrm{U}(1) \otimes \mathrm{U}(1)$

WINDING NUMBER OPERATORS

$$W_x = \frac{1}{L_x} \sum_x^{L_x} \sum_y^{L_y} S^3_{(x,y),\hat{x}} \quad W_y = \frac{1}{L_y} \sum_x^{L_x} \sum_y^{L_y} S^3_{(x,y),\hat{y}}$$

$$[\mathcal{H}\,, W_x] = [\mathcal{H}\,, W_y] = 0$$

Hamiltonian with chemical potential

HAMILTONIAN WITH WINDING OPERATOR

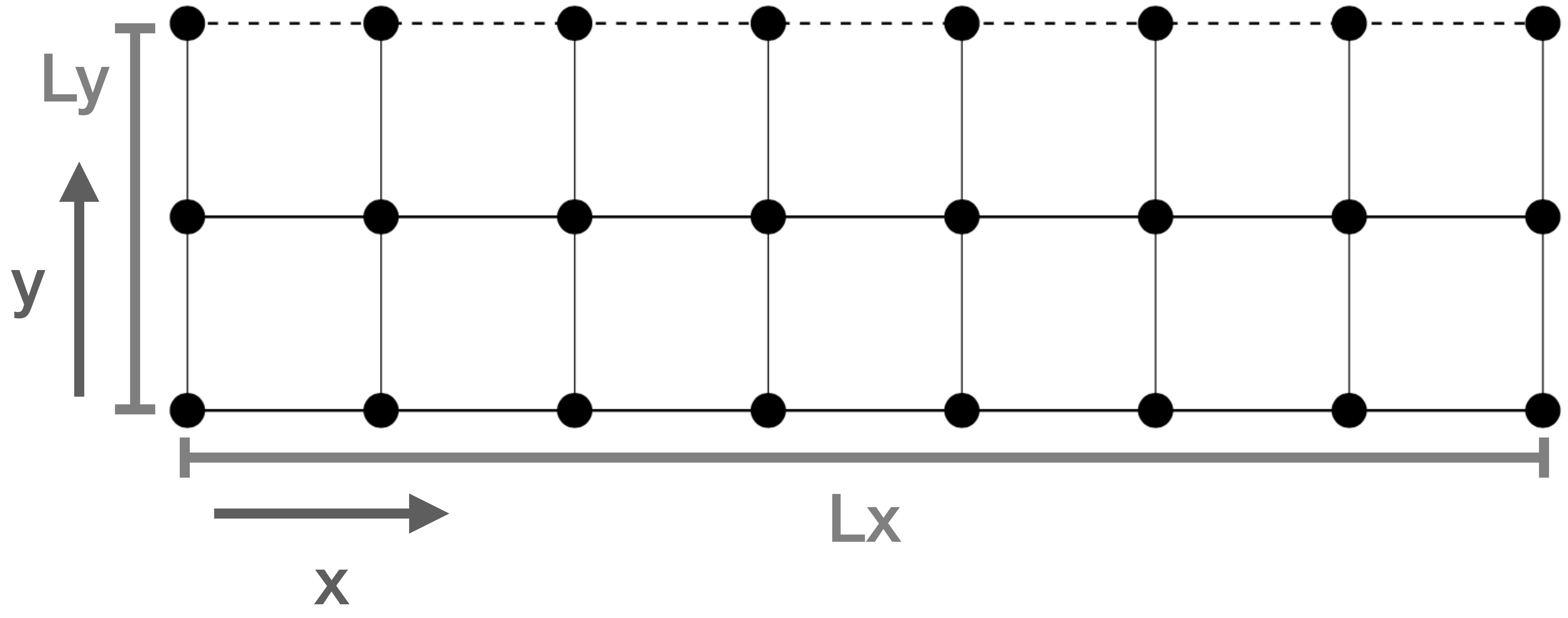
$$\mathcal{H}' = -J\sum_{\square}(U_{\square} + U_{\square}^{\dagger}) + \lambda(U_{\square} + U_{\square}^{\dagger})^2 + \mu_x \sum_x W_x + \mu_y \sum_y W_y$$

$$E_{GS} = E_{\mathcal{H}} - \mu_x N^x - \mu_y N^y$$

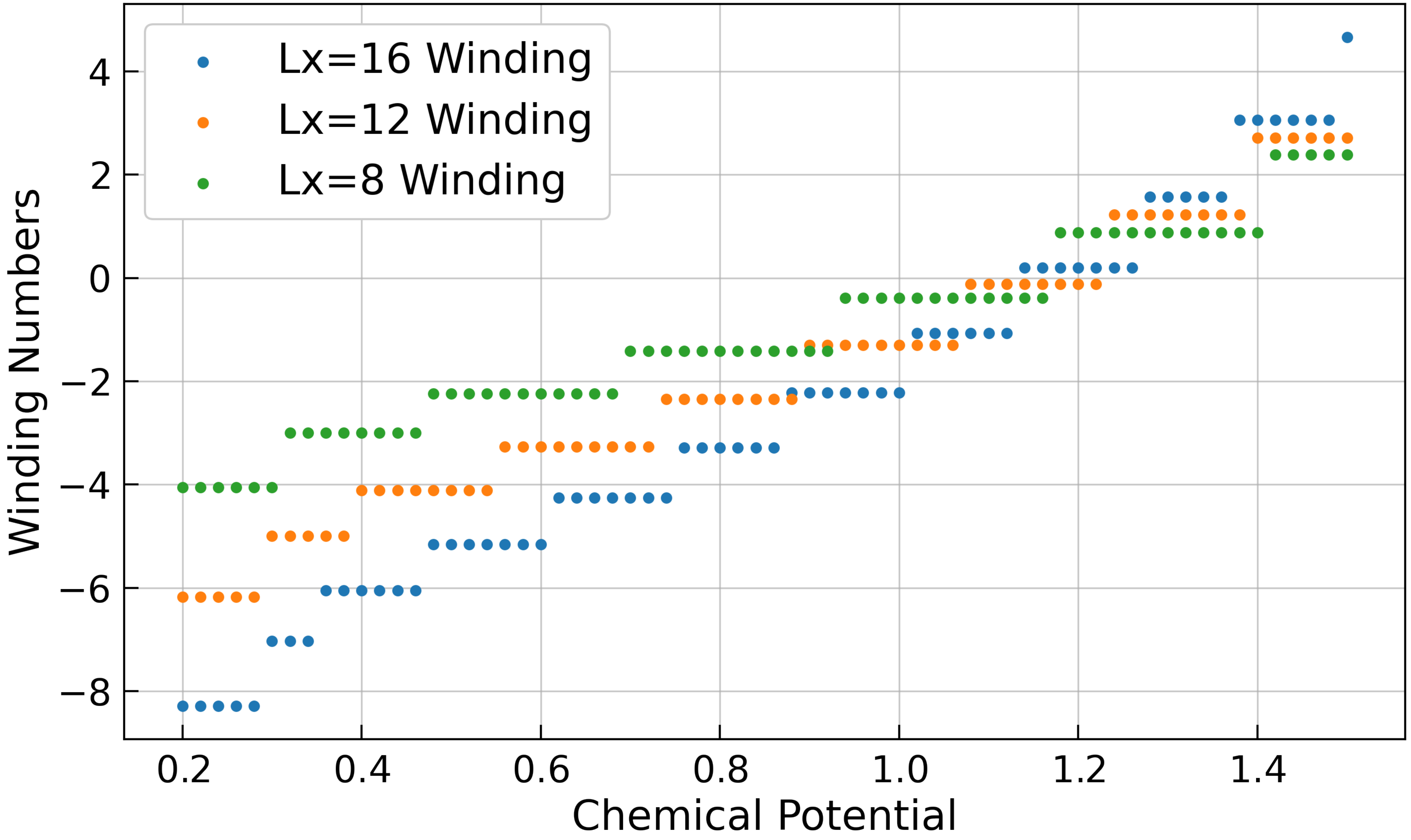
NUMERICAL SETUP FOR THE 2+1 DIMENSIONAL QLM

LATTICE CONFIGURATION

- 2 dimensional ladder
- Open boundary conditions in y direction
- Periodic boundary conditions in x direction

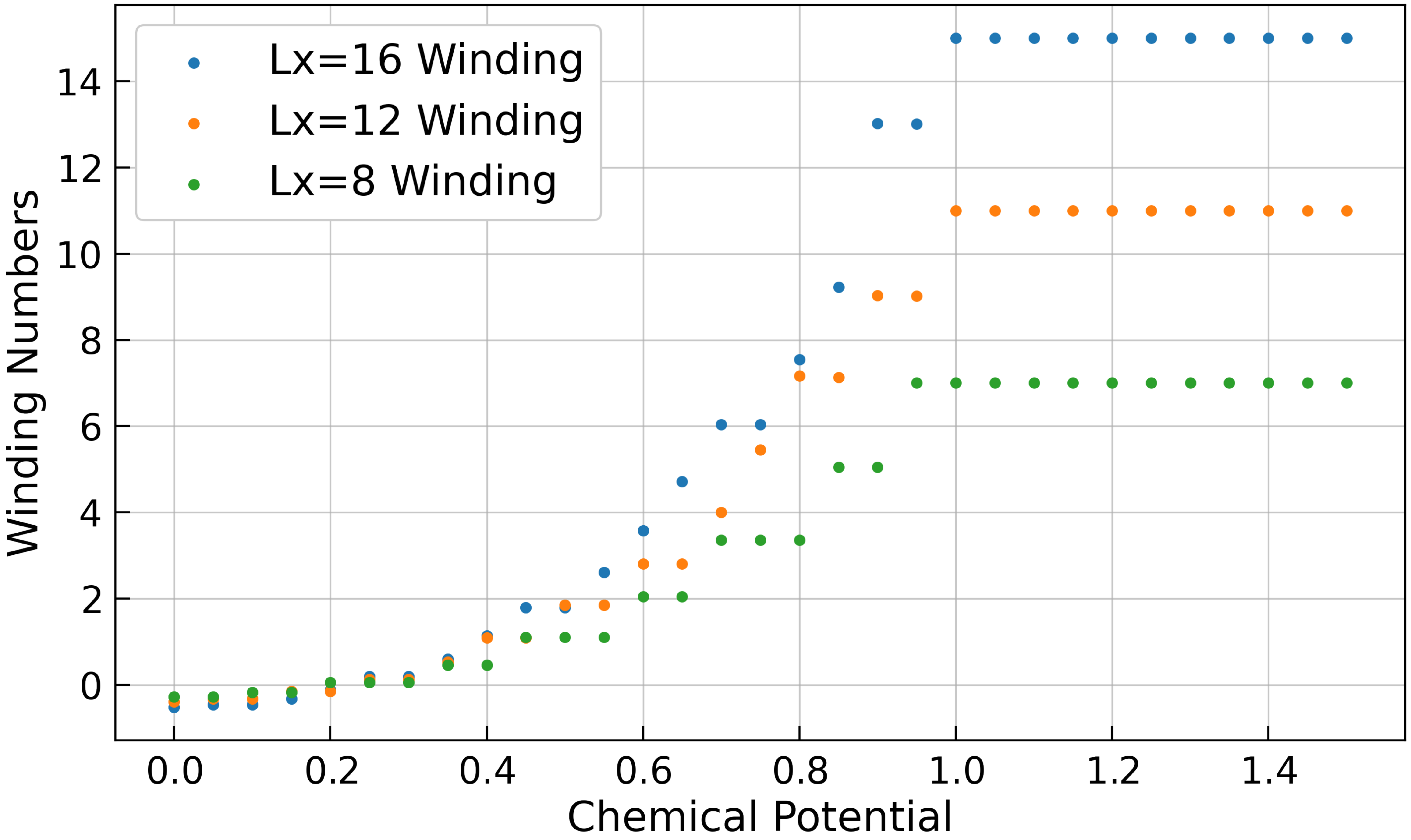


Numerical results



WINDING NUMBER SECTORS AS A FUNCTION OF THE CHEMICAL POTENTIAL FOR $J = 1$ $\lambda = -1$

Numerical results



WINDING NUMBER SECTORS AS A FUNCTION OF THE CHEMICAL POTENTIAL FOR $J = 1 \ \lambda = -2$

Summary

- Investigation of the winding number sector in U(1) quantum link model
- Increase of chemical potential associated with the winding number causes condensation of string flux

Outlook

- Thermodynamic limit
- Study of other observables
- Study of real-time dynamic of strings