LATTICE 21
JULY 26-30 2021, ZOOM/GATHER@MIT

Magnetic monopole dominance for Wilson loops in higher representations

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Introduction:: dual superconductivity

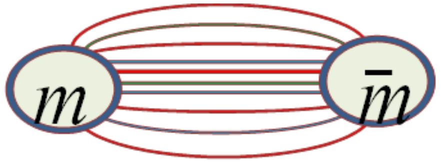
- Dual superconductivity is a promising mechanism for quark confinement.
  [Y.Nambu (1974). G.'t Hooft, (1975). S.Mandelstam, (1976) A.M. Polyakov (1975)]
- In this scenario, QCD vacuum is considered as a dual super conductor.

superconductor

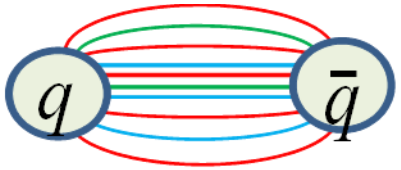
- Condensation of electric charges (Cooper pairs)
- Meissner effect: Abrikosov string (magnetic flux tube) connecting monopole and anti-monopole
- Linear potential between monopoles

dual superconductor

- Condensation of magnetic monopoles
- Dual Meissner effect: formation of a hadron string (chromo-electric flux tube) connecting quark and antiquark
- Linear potential between quarks



Electro- magnetic duality



Evidence for the dual superconductivity :: fundamental rep. (I)

Abelian projection in Maximal Abelian gauge

Extracted the relevant mode for quark confinement as a diagonal part in some gauge

■SU(2) case : Abelian projection $SU(2) \rightarrow U(1)$

✓Abelian Dominance in the string tension by Suzuki-Yotsuyanagi (1990) , by Stack-Tucker-Wensley (2002)

✓Monopole dominance in the string tension (DeGrant-Toussaint) by Stack-Tucker-Wensley (2002)

■SU(3) case; Abelian projection SU(2)→U(1)×U(1)

✓Abelian Dominance by Shiba-Suzuki (1994)

✓perfect dominance by Sakumichi-Suganuma (2016)

✓Monopole dominance by Stack-Tucker-Wensley (2002)

Problem:

Color (global) symmetry and gauge symmetry is broken.

A new formulation of Yang-Mills theory (on a lattice) decomposition method

[Phys.Rept. 579 (2015) 1-226]

<u>Decomposition of SU(N) gauge links</u> For SU(N) YM gauge link, there are sever al possible options of decomposition *discriminated by its stability groups*:

- SU(2) Yang-Mills link variables: unique $U(1) \subset SU(2)$
- SU(3) Yang-Mills link variables: **<u>Two options</u>**

<u>minimal option</u> : $U(2) \cong SU(2) \times U(1) \subset SU(3)$

Minimal case is derived for the Wilson loop, defined for quark in the fundamental representation, which follows from the non-Abelian Stokes' theorem

<u>maximal option :</u> $U(1) \times U(1) \subset SU(3)$

Maximal case is a gauge invariant version of Abelian projection in the maximal Abelian (MA) gauge. (the maximal torus group)

The decomposition of SU(3) link variable: maximal option

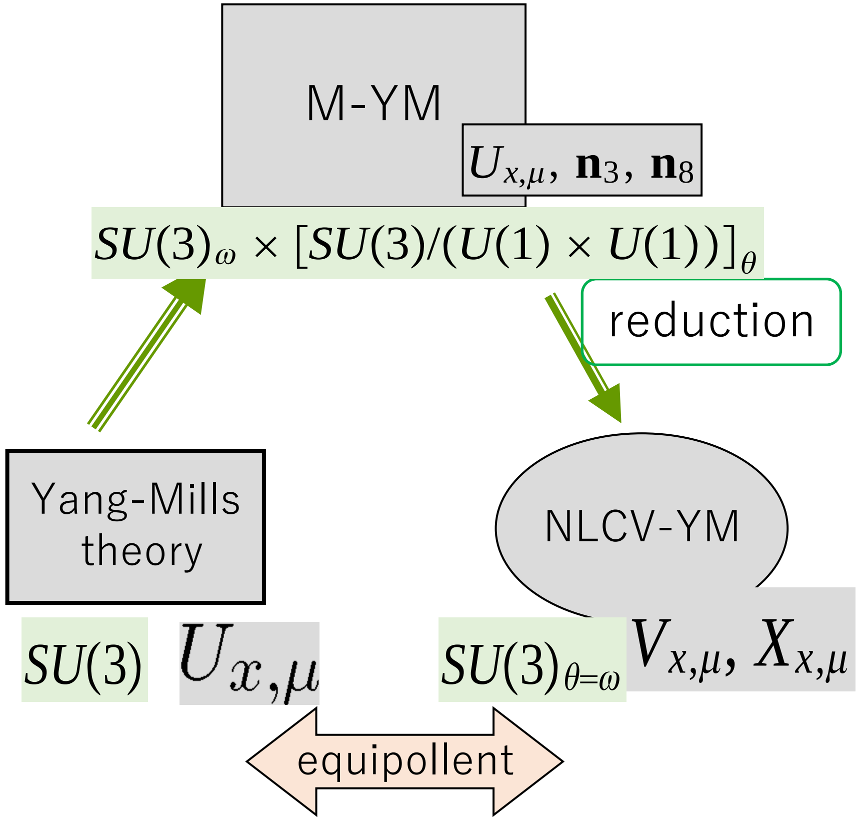
$$U_{x,\mu} = X_{x,\mu} V_{x,\mu}$$

$$U_{x,\mu} \to U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$V_{x,\mu} \to V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega^\dagger_{x+\mu}$$

$$X_{x,\mu} \to X'_{x,\mu} = \boxed{\Omega_x X_{x,\mu} \Omega^\dagger_x}$$

$$\Omega_x \in G = SU(N)$$



Gauge invariant construction of the Abelian projection to maximal torus group U(1) x U(1) in MA gauge.

maximal option: Defining equation for the decomposition

By introducing color fields $\mathbf{n}_x^{(3)} = \Theta_x(\lambda^3/2)\Theta^\dagger$, $\mathbf{n}_x^{(8)} = \Theta_x(\lambda^8/2)\Theta^\dagger$ $\in SU(3)_\omega \times [SU(3)/(U(1)\times U(1))]_\theta$, a set of the defining equation for the decomposition $U_{x,\mu} = X_{x,\mu}V_{x,\mu}$ is given by

$$D_\mu^\varepsilon[V]n_x^{(k)} = \frac{1}{\varepsilon}(V_{x,\mu}n_{x+\mu}^{(k)} - n_x^{(k)}V_{x,\mu}) = 0, \ (k = 3,8)$$

$$g_x = \exp(2\pi i n/N)\exp(i\sum_{j=3,8} a^{(j)}n_x^{(j)}) = 1$$

Coressponding to the continuum version of the dexomposition $\mathcal{A}_\mu(x) = V_\mu(x) + \mathcal{X}_\mu(x)$

$$D_\mu[V_\mu]\mathbf{n}^{(k)}(x) = 0, \quad tr(\mathbf{n}^{(k)}(x)\mathcal{X}_\mu(x)) = 0, \ (k = 3,8)$$

$$X_{x,\mu} = \hat{K}_{x,\mu}^\dagger \det(K_{x,\mu})^{1/3}g_x^{-1}, \quad V_{x,\mu} = g_x\hat{K}_{x,\mu}\det(K_{x,\mu})^{-1/3}$$

where

$$\hat{K}_{x,\mu} := \left(\sqrt{K_{x,\mu}K_{x,\mu}^\dagger}\right)^{-1}K_{x,\mu}, \quad \hat{K}_{x,\mu}^\dagger = K_{x,\mu}^\dagger\left(\sqrt{K_{x,\mu}K_{x,\mu}^\dagger}\right)^{-1}$$

$$K_{x,\mu} = 1 + 6\mathbf{n}_x^{(3)}U_{x,\mu}\mathbf{n}_{x+\mu}^{(3)}U_{x,\mu}^\dagger + 6\mathbf{n}_x^{(8)}U_{x,\mu}\mathbf{n}_{x+\mu}^{(8)}U_{x,\mu}^\dagger$$

Evidence for the dual superconductivity :: fundamental rep. (II)

Gauge decomposition method (our new formulation)

- Extracting the relevant mode *V* for quark confinement by solving the defining equation in the gauge independent way (gauge-invariant way)
- SU(2) case: a lattice compact representation of the Cho-Duan-Ge-Faddeev-Niemi-Shabanov (CDGFNS) decomposition.

we have showed that

- ➢ almost perfect V-field dominance, magnetic monopole dominance in string tension
- ➢ chromo-electric flux tube and dual Meissner effect.
- ➢ The vacuum of dual superconductor is of Type I

[Phys.Rev. D91 (2015) 3, 034506]

Evidence for the dual superconductivity :: fundamental rep. (III)

Gauge decomposition method SU(3) case

✓Extension of SU(2) case and two options

- **Maximal option** (Cho-Faddev-Niemi decomposition also N Cundy, Y.M. Cho et.al])
- **Minimal option** (our proposed non-Abelian dual superconductivity)

➔for minimal option that we have showed in the series works

➢ V-field dominance, non-Abalian magnetic monopole dominance in string tension,

➢ chromo-flux tube and dual Meissner effect.

➢The first observation on quark confinement/deconfinement phase transition in terms of dual Meissner effect at finite temperature

➔for minimal option

➢The same with the minimal option , [ours][N Cundy, Y.M. Cho et.al]

To establish dual superconductivity

- We must show that monopole plays a dominant role for the Wilson loops in higher representations as well as in the fundamental representation.
- In the previous studies, these sometimes made naïve replacement of Wilson loop operator between the Yan-Mills field and Abelian projected filed

  E.g., recentry in order to test the mechanism of quark confinement, J.Greensite and R.Hollwieser compares the double winding Wilson loop in SU(2) Yang-Mills theory made of Yang-Mills field, Abelian projected field in the MAG, and the center in the maximal center gauge. [PRD91 054509 (2015)]

- We investigate the Wilson loop by using our presented new formulation of the Lattice Yang-Mills theory based on the non-Abelian Stokes theorem.

Non-Abelian Stokes theorem

Kondo and Matsudo RRD92 125083 (2015)

Non-Abelian theorem in the presentation R can be given by

$$W_C[A] = \int [d\mu(g)]_C \exp\left(ig \oint \langle \Lambda | A^U | \Lambda \rangle\right) = \int [d\mu(g)]_\Sigma \exp\left(ig \int_{\Sigma:\partial\Sigma=C} d(\langle \Lambda | A^U | \Lambda \rangle)\right)$$

where $[d\mu(g)]_C$ and $[d\mu(g)]_\Sigma$ are the product of the Haar measure over the loop and a surface, respectively. $A^{U^\dagger} := UAU^\dagger + ig^{-1}UdU$, and $|\Lambda\rangle$ the highest weight state of the representation R.

OR

$$W_C[\mathscr{A}] = \int [d\mu(g)]_\Sigma \exp\left[-ig_{\rm YM} \int_{\Sigma:\partial\Sigma=C} F^g\right] \qquad F^g := \frac{1}{2} f^g_{\mu\nu}(x) dx^\mu \wedge dx^\nu,$$

$$F^g_{\mu\nu}(x) = \Lambda_j \{\partial_\mu [n^A_j(x) \mathscr{A}^A_\nu(x)] - \partial_\nu [n^A_j(x) \mathscr{A}^A_\mu(x)] - g^{-1}_{\rm YM} f^{ABC} n^A_j(x) \partial_\mu n^B_k(x) \partial_\nu n^C_k(x)\},$$

$$\boldsymbol{n}_j(x) = g(x) H_j g^\dagger(x) = n^A_j(x) T_A (j = 1, \ldots, r).$$

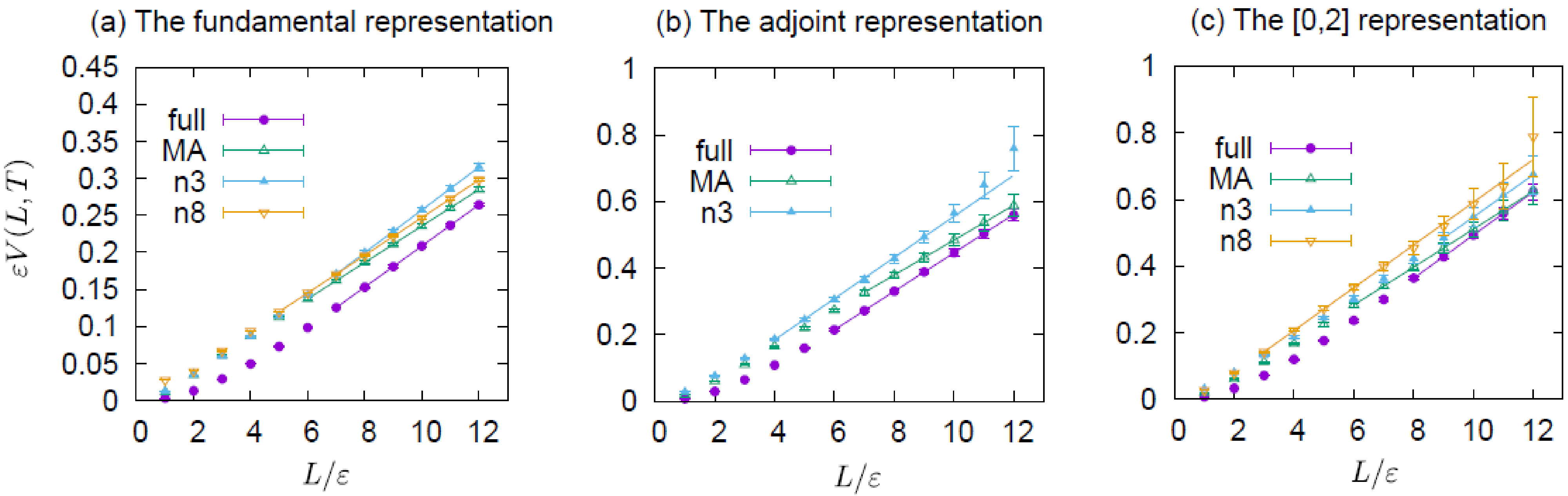
Note that the F is nothingbut the field strength of the restricted field V extracted by using the decomposition method.

SU(3) case

Phys.Rev.D 100 (2019) 1, 014505

$$V_C := \prod_{<x,\mu>\in C} V_{x,\mu}$$

$$W_{[m,n]}[V;C] = \frac{1}{6}\left\{\mathrm{tr}(V_C)^m \mathrm{tr}(V_C^\dagger)^n - \mathrm{tr}\Big((V_C)^m (V_C^\dagger)^n\Big)\right\}$$



SU(2) case

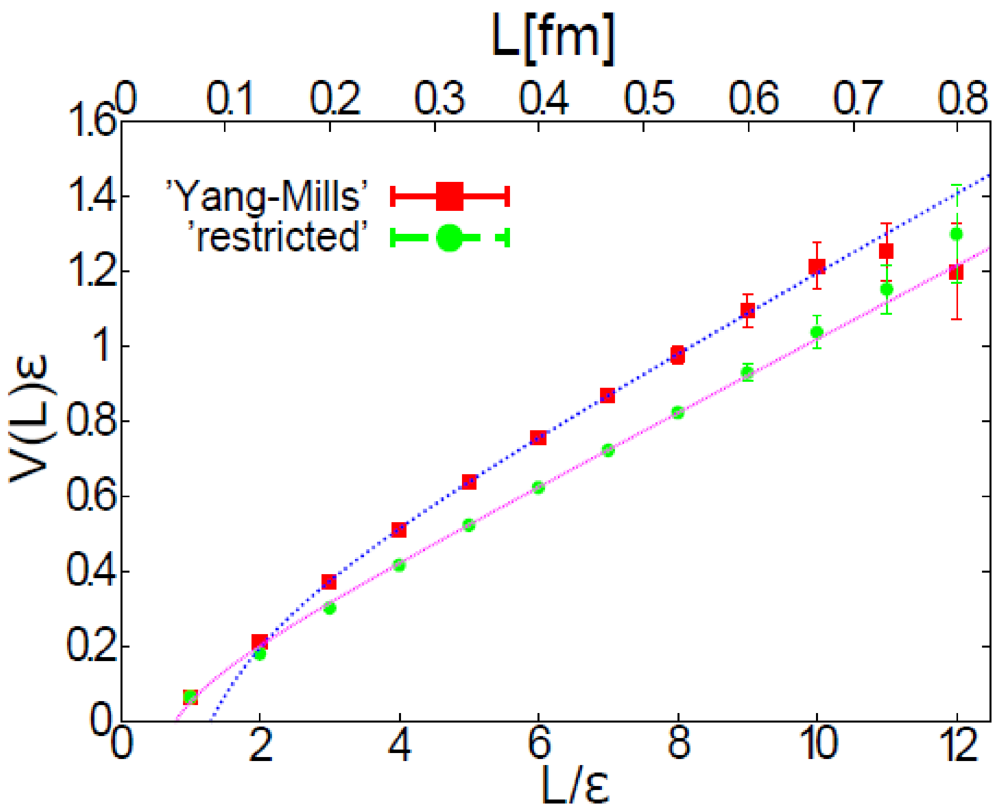
Phys.Rev.D 100 (2019) 1, 014505

$$V_C := \prod_{<x,\mu>\in C} V_{x,\mu}$$

$$W_J[V,C] = \mathrm{tr}((V_C)^{2J})$$

J=1 (adjoint representation)

Perfect restricted field dominance ("abelian dominance") in string tension.



Magnetic Monopoles

By using the Hodge decomposition Wilson loop operator is rewritten into

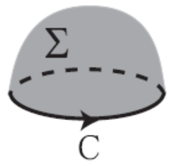
$$
\boxed{\begin{aligned} W_C[\mathcal{A}] &= \int [d\mu(g)] \exp\left[-ig_{YM}\int_{\Sigma_c:\,\partial\Sigma=C} F^g\right] \\ &= \int [d\mu(g)] \exp\left\{-ig_{YM}\sqrt{\frac{N-1}{2N}}\,[(\omega_{\Sigma_C},k)+(N_{\Sigma_C},j)]\right\} \end{aligned}}
$$

we have introduced an antisymmetric tensor $\Theta_\Sigma^{\mu\nu}$ of rank two which has the support only on the surface Σ_C spanned by the loop C:

$$
\Theta_{\Sigma_C}^{\mu\nu} := \int_{\Sigma_c:\,\partial\Sigma=C} d^2S^{\mu\nu}(x(\sigma))\delta^{(D)}(x-x(\sigma))
$$

and we have defined the (D − 3)−form ω_{Σ_C} and one−form N_{Σ_C} using the Laplacian $\Delta$by

$$
\omega_{\Sigma_C} := {}^*d\Delta^{-1}\Theta_{\Sigma_C}^{\mu\nu} = \delta\Delta^{-1}{}^*\Theta_{\Sigma_C}^{\mu\nu}, \qquad N_{\Sigma_C} := \delta\Delta^{-1}\Theta_{\Sigma_C}^{\mu\nu}
$$



We can define the gauge−invariant magnetic−monopole current k from the field strength through the non−Abelian Stokes theorem.

$$
k := \delta^* f^g = {}^*df^g\ , \qquad j =: \delta f^g
$$

Magnetic Monopoles (2)

$$k^{\mu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\partial_{\nu}f^{g}_{\rho\sigma}, \qquad f^{g}_{\mu\nu} := \sum_{j=1}^{r}\Lambda_j f^{(j)}_{\mu\nu}$$

for SU(3), we have $\vec{\Lambda} = (\Lambda_3, \Lambda_8) = \left(\frac{m}{2}, \frac{m+2n}{2\sqrt{3}}\right)$

- On the lattice SU(3)

The magnetic monopole on the lattice is defined by using the restriced field V which obtained from the decomposition method

$$\Theta^{(1)}_{\mu\nu} := \arg Tr\left[\left(\frac{1}{3}\mathbf{1} - \frac{2}{\sqrt{3}}\mathbf{n}^{(8)}_{x}\right)V_{x,\mu}V_{x+\mu,\nu}V^{\dagger}_{x+\nu,\mu}V^{\dagger}_{x,\nu}\right]$$

$$\Theta^{(2)}_{\mu\nu} := \arg Tr\left[\left(\frac{1}{3}\mathbf{1} + \mathbf{n}^{(3)} + \frac{1}{\sqrt{3}}\mathbf{n}^{(8)}_{x}\right)V_{x,\mu}V_{x+\mu,\nu}V^{\dagger}_{x+\nu,\mu}V^{\dagger}_{x,\nu}\right]$$

$$k^{(i)}_{\mu} := \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\partial_{\nu}\Theta^{(i)}_{\alpha\beta}$$

This definition is gauge invariant.
The appearance of magnetic monopoles means breaking the Bianchi identity, because of
$k := {}^{*}dF$ and $F = dA$

Lattice data SU(2)

$$W_C[\mathcal{A}] = \int [d\mu(g)] \exp\{-ig_{YM} J[(\omega_{\Sigma_C}, k) + (N_{\Sigma_C}, j)]\}$$

- The Wilson loop for higher representation is given by multiplied J . For fundamental representation J=1/2 and for adjoint representation J=1.
- In the lattice measurement , instead of performing the integration over the measure μ, we use the restricted field which is obtained by solving the reduction condition.

Lattice Data: for SU(2) adjoint representation

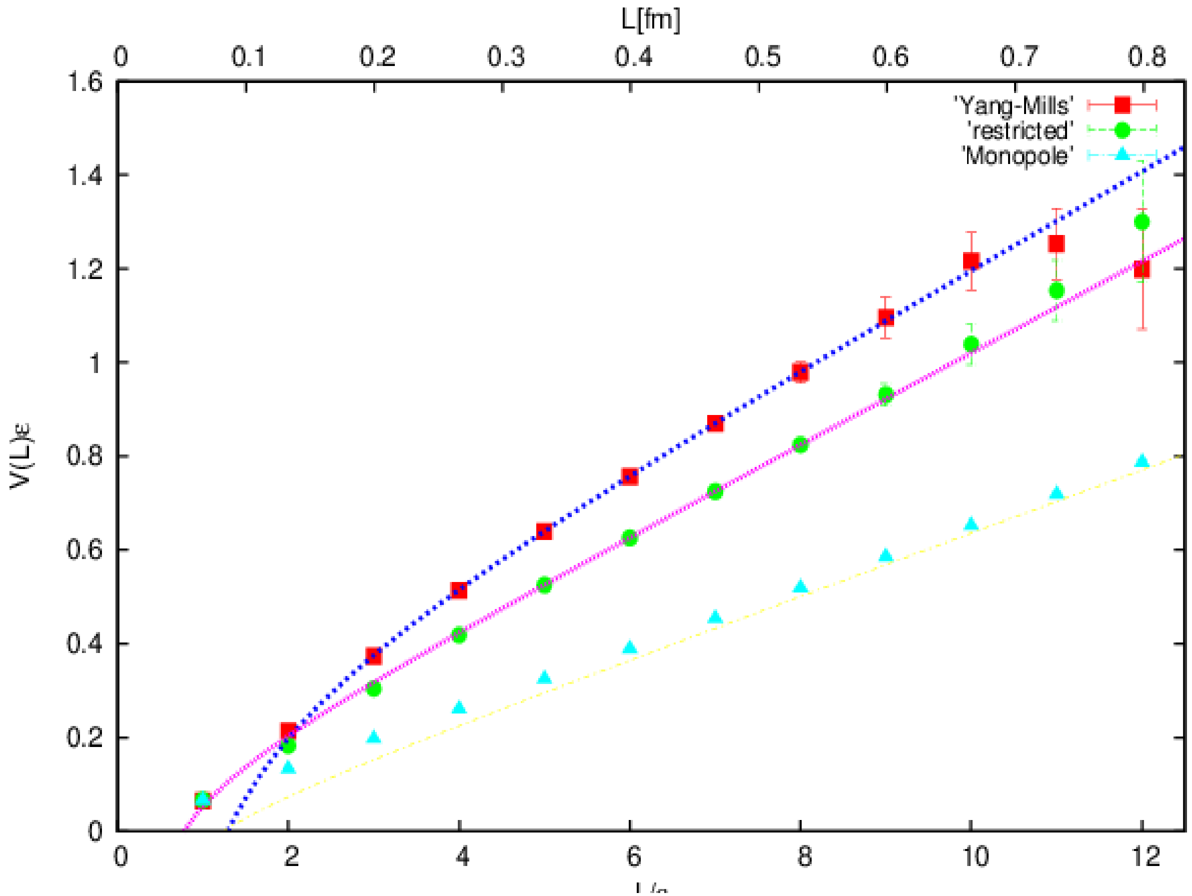
$$\langle W_M \rangle = \langle \exp\{-igJ(\omega_{\Sigma_C}, k)\} \rangle$$

Preliminary

- The string tension estimated only by $\langle W_M \rangle$ can not be correct, since we cannot separate the electric part

$$\sigma_{mono}/\sigma_{full} \simeq 68\%$$

$$\langle \exp\{J[(\omega_{\Sigma_C}, k) + (N_{\Sigma_C}, j)]\} \rangle$$
$$\neq \langle \exp J(\omega_{\Sigma_C}, k) \rangle \langle \exp\{J[(N_{\Sigma_C}, j)]\} \rangle$$



Summary and outlook

- Recently, by the non-Abelian Stokes theorem (NAST) for the Wilson loop operator, we have proposed suitable gauge-invariant operators constructed from the restricted field to reproduce the correct behavior of the original Wilson loop averages for higher representations.
- we focus on the magnetic monopole. According to this picture, magnetic monopoles causing the dual superconductivity are regarded as the dominant degrees of freedom responsible for confinement.
- With the help of the NAST, we can define the magnetic monopole and the string tension extracted from the magnetic-monopole part of the Wilson loop in a gauge-invariant manner.
- First, we have performed lattice simulations to measure the static potential for quarks in the adjoin representation for SU(2) gauge using the proposed operators and examined the magnetic monopole dominance in the string tension. We obtain preliminary result.

- We will further investigate the magnetic monopole dominance in the string tension in several representations for SU(2) and SU(3) case.