Centre Vortices in the Presence of Dynamical Fermions

James Biddle
In collaboration with: Prof. Derek Leinweber and Dr. Waseem Kamleh
Introduction

- It is an open question how the QCD Standard Model gives rise to key non-perturbative features of QCD, namely:
  - Confinement
  - Chiral symmetry breaking ($\chi_{SB}$), resulting in dynamical generation of mass.
- Is there a single, unifying property of QCD that gives rise to these features?
- Many models (Abelian monopoles, instantons, etc...).
- Most successful is the centre vortex model.
What are we aiming to explain?

- The lattice provides an excellent framework in which to test the performance of the centre vortex model.
- Confinement can be probed by studying the
  - String tension (P. Bowman et. al. Phys. Rev. D 84 (2011) 034501)
  - Gluon propagator (J. C. Biddle, W. Kamleh and D. B. Leinweber, Phys. Rev. D 98 (2018))
- $\chi_{SB}$ can be probed by studying the
Previous centre vortex work

- Current understanding of centre vortices comes primarily from studies in pure Yang-Mills theory.
- In $SU(2)$, the centre vortex model is nearly flawless.
- In pure gauge $SU(3)$, things aren’t so simple.

Figure: J. Biddle et. al. Phys. Rev. D 102, 034504
Despite the aforementioned discrepancy, the vortex model has proved remarkably capable of reproducing the salient features of QCD.

Naturally, one might then want to see what happens when dynamical fermions are introduced.
Configurations

- Centre vortex projection allows us to define 3 sets of configurations:
  - Untouched - $U_\mu$
  - Vortex Only - $Z_\mu$
  - Vortex Removed - $R_\mu = Z_\mu^\dagger U_\mu$
Configurations

- Centre vortex projection allows us to define 3 sets of configurations:
  - Untouched - $U_\mu$
  - Vortex Only - $Z_\mu$
  - Vortex Removed - $R_\mu = Z_\mu^+ U_\mu$

  - $32^3 \times 64$ pure gauge (PG), spacing $a = 0.100$ fm
  - $32^3 \times 64$ dynamical, spacing $a = 0.1022$ fm, $m_\pi = 701$ MeV
  - $32^3 \times 64$ dynamical, spacing $a = 0.0933$ fm, $m_\pi = 156$ MeV
Static Quark Potential
Static Quark Potential

- Measures the potential energy between two massive, static quarks at separation $r$.
- Serves as an indicator of confining behaviour in the form of a linear long-range potential.
- Typically described via the Cornell potential

$$V(r) = V_0 - \frac{\alpha}{r} + \sigma r$$
On the lattice, the static quark potential is calculated by considering Wilson loops $W(r, t)$ with spatial extent $r$ and temporal extent $t$. The static quark potential $E_0$ is obtained through the relationship:

$$\langle W(r, t) \rangle = \sum_i a_i e^{-E_i(r) t}$$

$E_0$ is extracted via a variational analysis.
Static Quark Potential in pure-gauge QCD

- Original:
  \[ V(r) = V_0 - \frac{\alpha}{r} + \sigma r \]

- Vortex-only:
  \[ V(r) = V_0 + \sigma r \]

- Vortex-removed:
  \[ V(r) = V_0 - \frac{\alpha}{r} \]

- Vortex-only reproduces about 62% of the static quark potential.
Introducing Dynamical Fermions \( (m_\pi = 701\text{MeV}) \)

- In the presence of dynamical fermions, this issue seems to vanish!
  \[
  \sigma_{PG} = 0.0562(9) \\
  \sigma_{\text{dynamical}} = 0.054(1)
  \]
- Even at a very heavy pion mass, centre vortices now capture the full string tension.
- The vortex removed also now has no residual strength.
Lighter pion \((m_\pi = 156\text{MeV})\)

- This trend continues at lighter pion masses.
- Lighter pion masses lead to overall suppression of the potential, \(\sigma = 0.040(1)\).
- The vortex-only results slightly overestimate the original.
- Could be residual excited state contamination.
- Regardless, results overlap well and are considerably closer than the pure-gauge values.
Gluon Propagator
Centre Vortices and the Landau-Gauge Gluon Propagator

• $D(q^2)$ is characterised by an infrared peak, with a UV tail

• The nonperturbative scalar gluon propagator in momentum space is

\[ D(q^2) \equiv \frac{Z(q^2)}{q^2} \to \frac{1}{q^2}. \]

• Consider the renormalisation function

\[ Z(q^2) = q^2 D(q^2). \]

• Renormalise by setting $Z(q^2) = 1$ at $q = 5.5$ GeV.
Gluon Propagator – Pure Gauge Sector

- Vortex Removal (VR) suppresses infrared enhancement whilst preserving UV perturbative behaviour.

- Vortex-Only (VO) configurations capture the long-distance physics.

- Reconstruction of the propagator as a linear combination of the vortex-modified parts recovers full propagator.

- Residual infrared enhancement in the vortex-removed result is undesirable.
Gluon Propagator – Dynamical Fermions $m_\pi = 701$ MeV

- **Dynamical fermions** (UT) suppress the overall infrared strength.
- **Vortex Removal** (VR) almost eliminates infrared enhancement.
- **Vortex-Only** (VO) configurations capture the long-distance physics.
- **Reconstruction** is less perfect.
Gluon Propagator – Dynamical Fermions $m_\pi = 156$ MeV

• Lighter dynamical $u$ and $d$ quarks further suppress the infrared enhancement.

• Centre Vortex degrees of freedom are able to capture the effects of dynamical fermions in QCD.
Conclusions

- Dynamical fermions, even at heavy pion masses, radically alter the behaviour of the vortex vacuum.
- Vortex removal has an amplified effect.
- The vortex-only gluon propagator captures a greater fraction of the infrared strength.
- The static quark potential is fully reproduced by vortices.

Further exploration is needed to fully explore how other quantities are impacted. Searching for signs of string-breaking with other operators would also be of interest.
Conclusions

- Dynamical fermions, even at heavy pion masses, radically alter the behaviour of the vortex vacuum.
- Vortex removal has an amplified effect.
- The vortex-only gluon propagator captures a greater fraction of the infrared strength.
- The static quark potential is fully reproduced by vortices.
- Further exploration is needed to fully explore how other quantities are impacted.
- Searching for signs of string-breaking with other operators would also be of interest.
**Table:** The static quark potential as extracted from the original and vortex-only ensembles.

<table>
<thead>
<tr>
<th>$m_\pi$ (MeV)</th>
<th>$\sigma_{UT}$</th>
<th>$\sigma_{VO}$</th>
<th>$\sigma_{VO}/\sigma_{UT}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure gauge</td>
<td>0.0562(9)</td>
<td>0.0349(4)</td>
<td>0.62(1)</td>
</tr>
<tr>
<td>701</td>
<td>0.054(1)</td>
<td>0.0563(7)</td>
<td>1.04(3)</td>
</tr>
<tr>
<td>155.8</td>
<td>0.040(1)</td>
<td>0.0499(7)</td>
<td>1.24(4)</td>
</tr>
</tbody>
</table>