Lattice artefacts on the Landau gauge gluon propagator from hypercubic tensor representations

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Abstract

Lattice tensor representations are used to investigate the lattice Landau gauge gluon propagator for the 4-dimensional, pure SU(3) Yang-Mills gauge theory. Due to the different symmetry structure of hypercubic lattices compared to the continuum space-time, lattice correlation functions are described by different tensor structures. Therefore, form factors describing lattice correlation functions have, in principle, non-trivial relations with the continuum counterparts. The use of several tensor bases respecting lattice symmetries, and the analysis of its completeness allows to quantify the deviations of the lattice results from the continuum theory due to the lattice artefacts, and also estimate the theoretical uncertainty in the propagator. Furthermore, our analysis tests continuum based relations with the lattice data and shows that the lattice Landau gauge gluon propagator is suitably described by a unique form factor, as in the continuum formulation. Additionally, we identified classes of kinematic configurations where these deviations are minimal and the continuum description of lattice tensors is improved.

Lattice and the gauge gluon propagator

The continuum, pure Yang-Mills theory is described by the Lagrangian

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}, \qquad F^a_{\mu\nu} = (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) + g f^{abc} A^b_\mu A^c_\nu, \tag{1}$$

where the A^a_μ are the gluon fields. In the path integral formulation, expectation values are obtained by a weighted average over all field configurations defined by the action

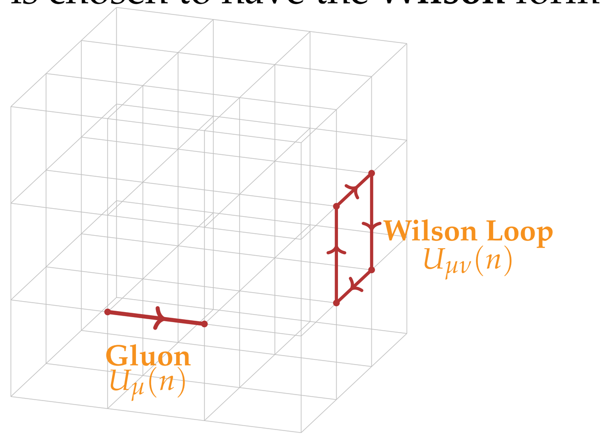
$$\mathcal{Z} = \int \mathcal{D}A_\mu e^{-\int d^4x \mathcal{L}}, \qquad \langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}A_\mu \mathcal{O} e^{-\int d^4x \mathcal{L}}. \tag{2}$$

Together with Euclidean metric, this formulation maps a quantum field theory into a statistical mechanical problem, allowing for the use of computational methods known from the latter.

In the lattice paradigm, expectation values are obtained with an average over a set of representative Monte-Carlo configurations

$$\langle \mathcal{O} \rangle \sim \frac{1}{N} \sum_{\substack{U_n \text{sampled} \\ \text{from } e^{-S[U_n]}}} \mathcal{O}[U_N]. \tag{3}$$

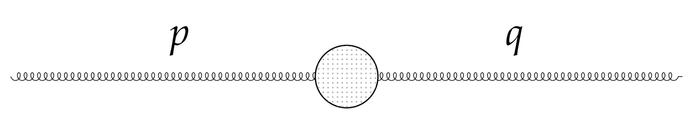
To use the lattice approach it is necessary to consider a discretized version of the action, which is chosen to have the **Wilson** form



$$S_G[U] = \frac{2}{g^2} \sum_n \sum_{\mu<\nu} \text{Re}\,\text{Tr}\left(\mathbb{1} - U_{\mu\nu}(n)\right)$$
$$= \frac{a^4}{2g^2} \sum_n \sum_{\mu,\nu} \text{Tr}\left(F_{\mu\nu}(n)^2\right) + \mathcal{O}\left(a^2\right) \tag{4}$$

$$U_\mu(n) = e^{iagA_\mu(n+a\hat{\mu}/2)} + \mathcal{O}(a). \tag{5}$$

The gluon propagator in the **Landau gauge**, where $p_\mu A^\mu(p) = 0$, is used to study the tensor representations. This is the 2-point correlation function of the gauge field at different points

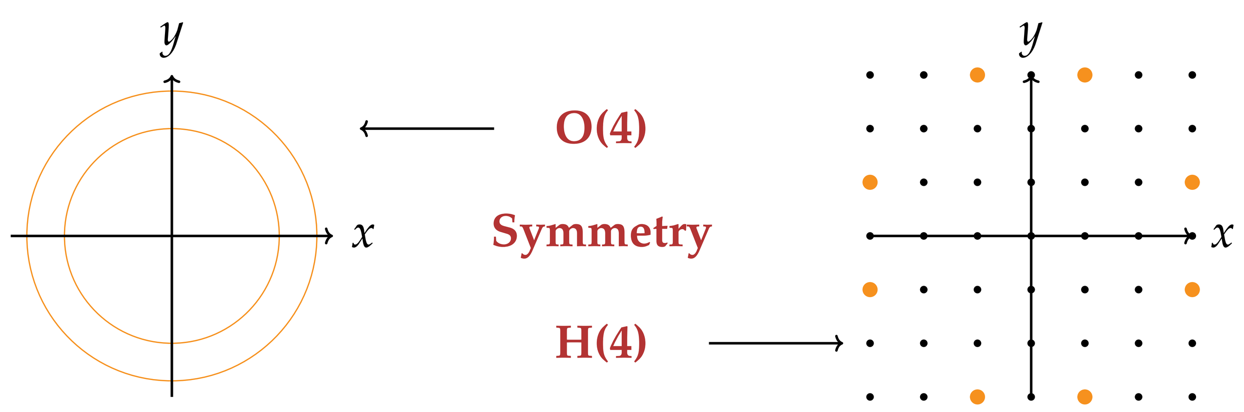


$$\left\langle A^a_\mu(p) A^b_\nu(q) \right\rangle = D^{ab}_{\mu\nu}(p) V \delta(p+q)$$

In this work we study the representations of the $D^{ab}_{\mu\nu}$ tensor according to the symmetries of the space.

Lattice Tensors

Continuum O(4) symmetry reduces to the hypercubic, H(4) symmetry on the lattice



Tensors T in a vector space are defined by its transformation with respect to a given transformation M

$$T'_{\mu_1 \dots \mu_k} = M_{\mu_1\nu_1} \dots M_{\mu_k\nu_k} T_{\nu_1 \dots \nu_k}. \tag{6}$$

On the lattice, only $\pi/2$ rotations and parity transformations around the coordinate axes correspond to valid symmetry transformations. Under this symmetry, scalars/H(4) invariants, vectors, and second rank tensors depending on a single momentum are generalized to

$$p^2 \to p^{[2n]} = \sum_\mu p^{2n}_\mu, n \in \mathbb{N}, \qquad p_\mu \to v^n_\mu = p^{2n+1}_\mu, n \in \mathbb{N}, \tag{7}$$

$$\delta_{\mu\nu}, \quad p_\mu p_\nu \to d^{n,m}_{\mu\nu} = p^{2n+1}_\mu p^{2m+1}_\nu, n, m \in \mathbb{N}. \tag{8}$$

The continuum propagator is built by the most general tensor under the O(4) symmetry,

$$D_{\mu\nu}(p) = A(p^2)\delta_{\mu\nu} + B(p^2) p_\mu p_\nu \tag{9}$$

($A(p^2) = -B(p^2) = D(p^2)$ in the Langau gauge). In general, an increased number of **tensor structures** is required to describe lattice tensors, thus this form may lead to loss of information. With eq. (8), a minimal and extended (non-complete) tensor bases to describe the lattice propagator read,

$$\begin{cases} D^{ab}_{\mu\mu} = \delta^{ab}\left(J(p^2)\delta_{\mu\mu} + K(p^2)p^2_\mu\right), \\ D^{ab}_{\mu\nu} = \delta^{ab} L(p^2) p_\mu p\nu \end{cases} \quad \begin{cases} D^{ab}_{\mu\mu} = \delta^{ab}\left(E(p^2)\delta_{\mu\mu} + F(p^2)p^2_\mu + G(p^2)p^4_\mu\right), \\ D^{ab}_{\mu\nu} = \delta^{ab}\left(H(p^2)p_\mu p\nu + I(p^2) p_\mu p_\nu (p^2_\mu + p^2_\nu)\right). \end{cases} \tag{10}$$

From lattice perturbation theory, instead of the $p_\mu = 2\pi n_\mu / aL$ momentum, the **improved momentum** $\hat{p}_\mu = 2\sin(p_\mu/2)$ may be used instead to correct for some lattice artefacts.

The **completeness of a basis** can be measured by the computation of the following quantity

$$\mathcal{R} = \frac{\sum_{\mu,\nu} |D^{\text{orig}}_{\mu\nu}|}{\sum_{\mu,\nu} |D^{\text{rec}}_{\mu\nu}|}. \tag{11}$$

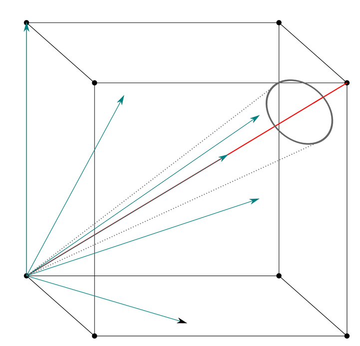
Form factors from simulations

The propagator is obtained from lattice simulations [1, 2], using all H(4) orbits to improve the signal. 550 gauge field configurations are used for a lattice with 80^4 sites, $\beta = 6.0$, $a = 0.1016(25)$ fm.

To reduce lattice artefacts, two procedures are considered:

- **Momentum cuts**:

  Momenta closer to the diagonal of the lattice have less artefacts [5]. In this method, all momenta falling out of a chosen conical plus cylindrical region are discarded.



- **H4 Extrapolation:**
  For the H(4) symmetry there are $D = 4$ independent scalars that define any form factor, $S(p^2) \to S_L(p^2, p^{[4]}, p^{[6]}, p^{[8]})$. A single O(4) orbit labelled by p^2 splits into different H(4) orbits [4]. By considering an expansion in the invariants,

$$F\left(p^{[n]}\right) \approx F\left(p^2\right) + p^{[4]} \frac{\partial F}{\partial p^{[4]}}\left(p^2, 0\right) + \mathcal{O}\left(a^2\right). \tag{12}$$

  it is possible to perform a $p^{[4]} \to 0$ extrapolation for each p^2 that estimate an $\mathcal{O}\left(a^2\right)$ correction.

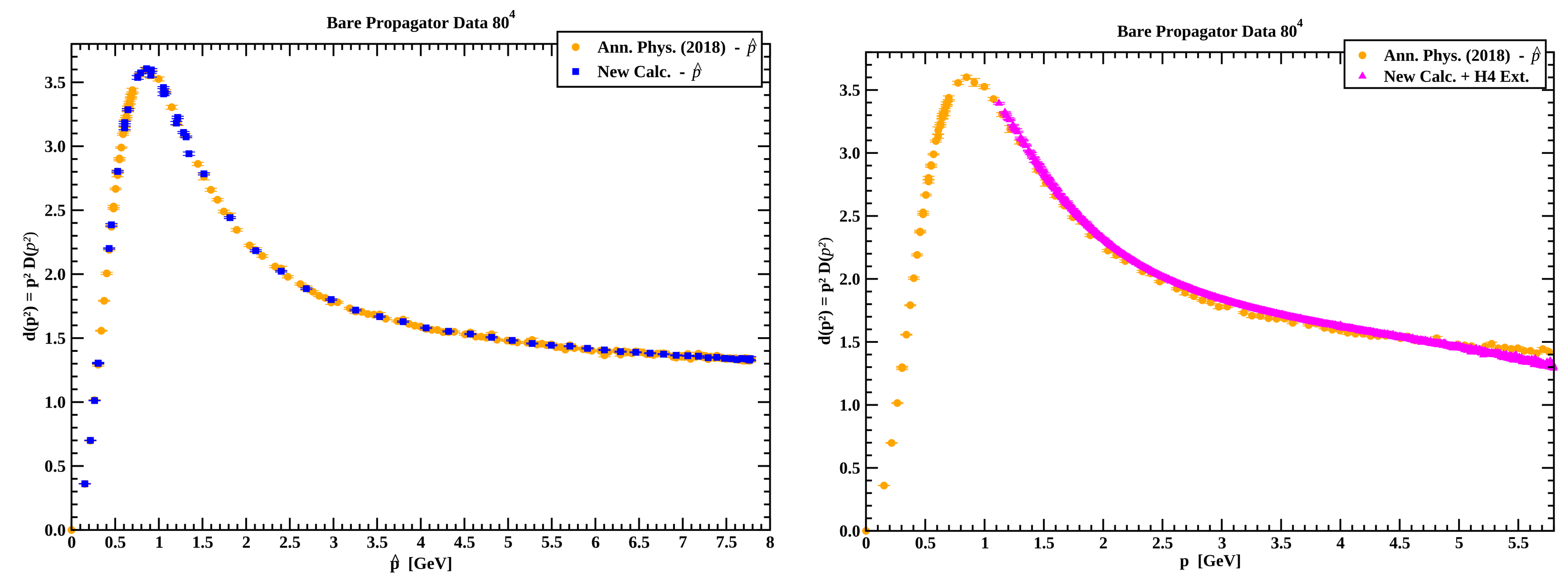


Figure 1: Landau gluon dressing function using the continuum tensor basis with the improved momentum after momentum cuts (left) and after performing the H4 extrapolation (right), compared to reference data (yellow) [3].

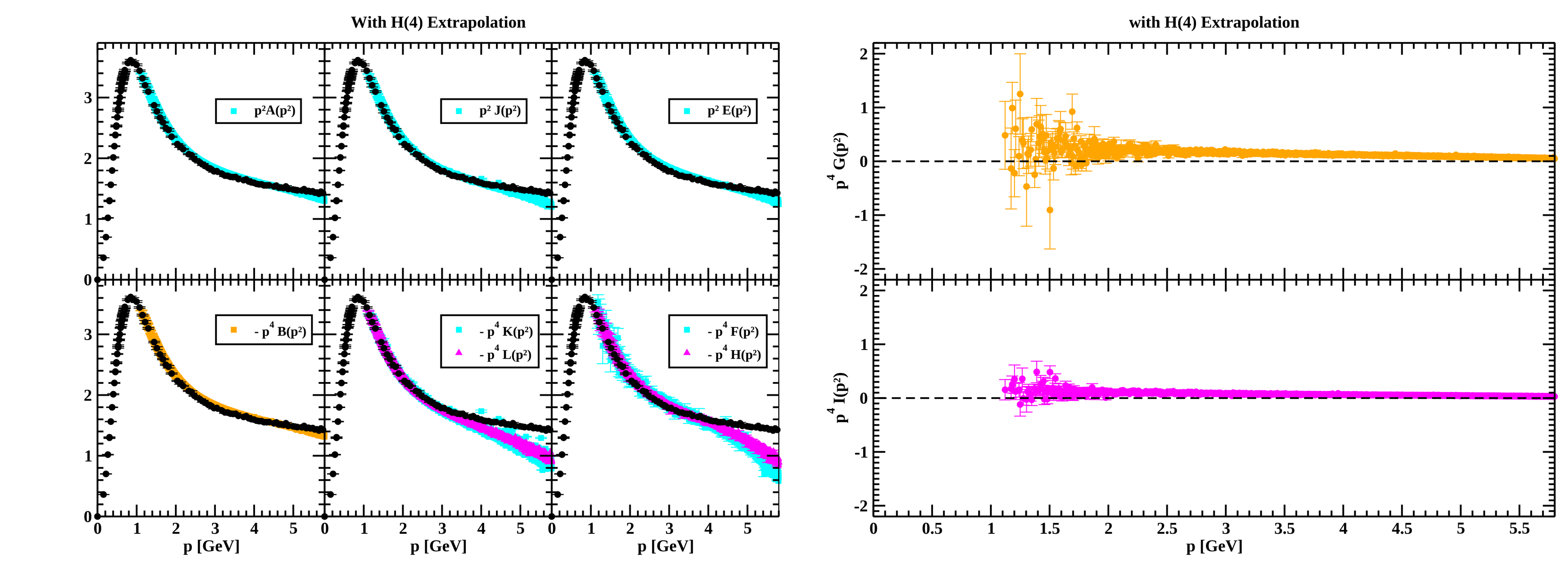


Figure 2: Form factors from the continuum, eq. (9) and lattice tensor bases, eq. (10) after the H4 extrapolation.

Basis Completeness and Orthogonality

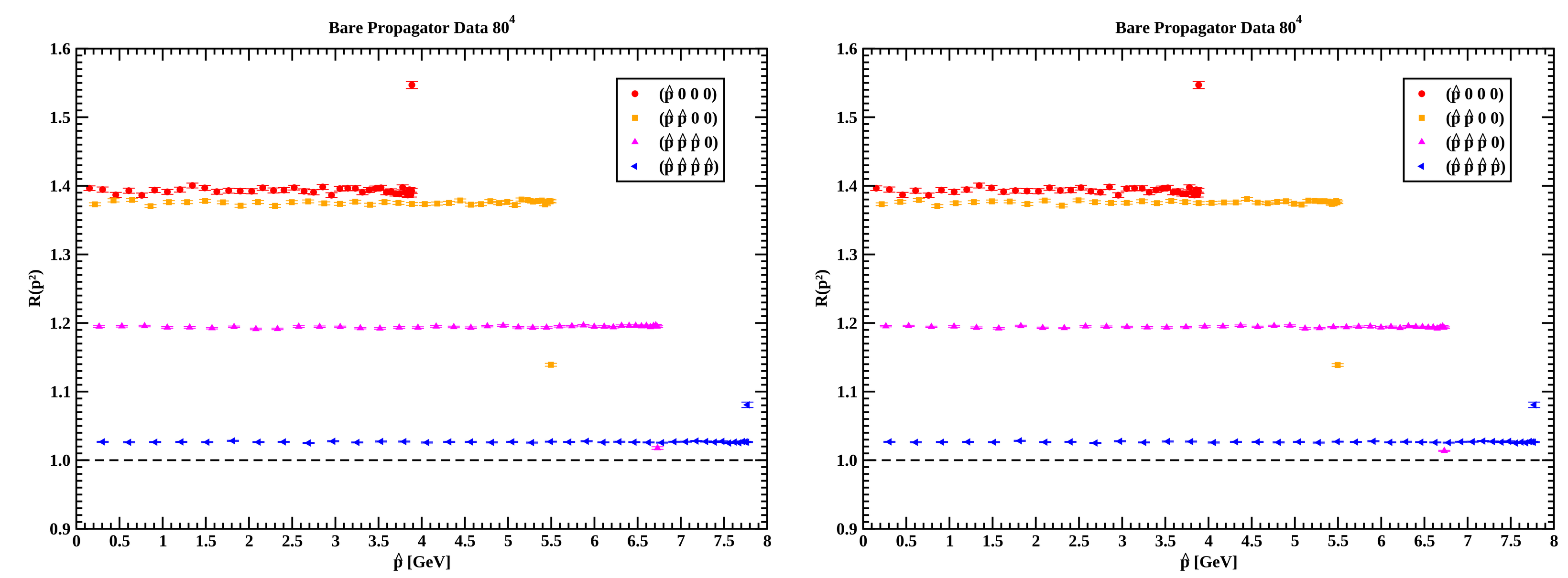


Figure 3: Reconstruction ratio for different classes of single scale improved momenta for the continuum basis (left) and for the extended basis (right).

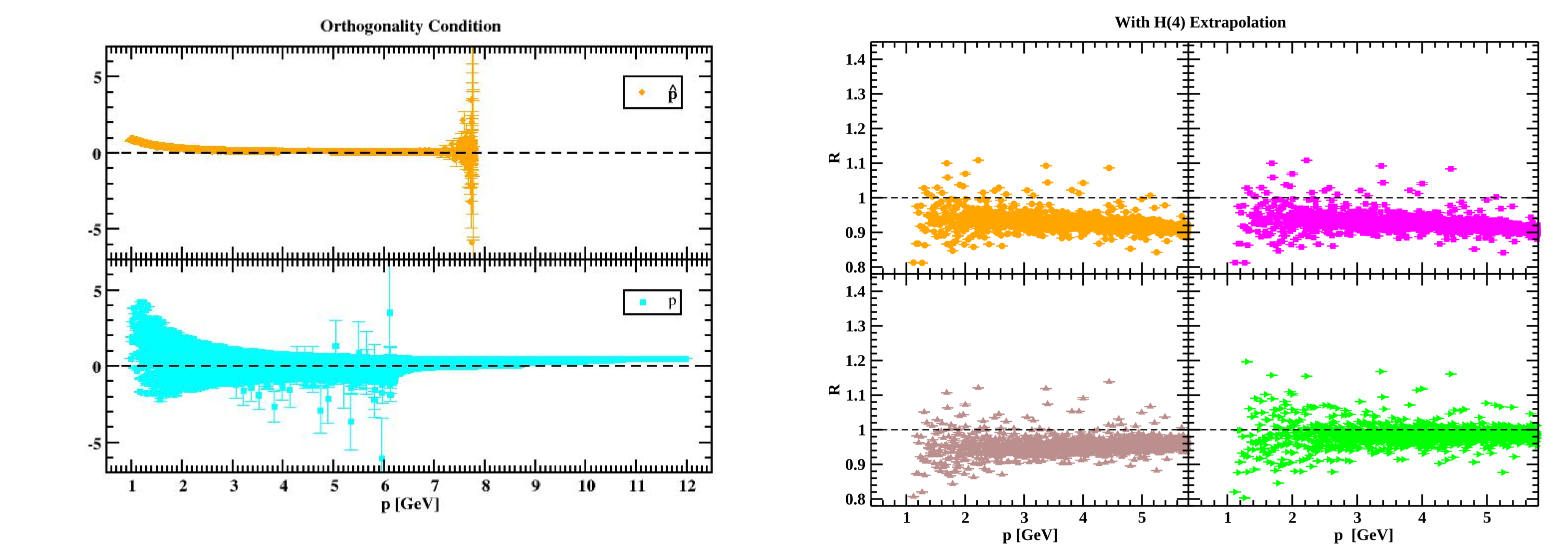


Figure 4: Left plot: Orthogonality condition, $p_\mu D_{\mu\nu}(p)$ for the improved (top) and lattice momentum (bottom); Right plot: $\mathcal{R}$ for the single form factor continuum basis (top left), for the general continuum basis (top right), and the minimal and extended lattice tensor bases (bottom left and right, respectively).

Conclusions

- The use of momentum $\hat{p}$ reduces lattice artefacts with the cost of discarding data;
- H4 extrapolation takes into account all lattice data, removes lattice artefacts for a limited range of momenta, and improves the signal to noise ratio;
- Lattice tensors are needed for a more complete description of lattice correlation functions;
- For single scale momentum configurations, e.g. (p, p, p, p), the continuum tensor basis and the use of lattice tensors produce equivalent results. The estimation of the propagator for this class of momenta is more reliable;
- Orthogonality conditions verified for the lattice gluon propagator.

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