Looking at the Analytic Structure of the Lattice Landau Propagators

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PRD 102 (2020) 114518 (2008.02615 [hep-lat])

2009.08961 [hep-lat]

LATTICE 2021

Two point correlation functions

QCD Dynamics

Physical Spectra

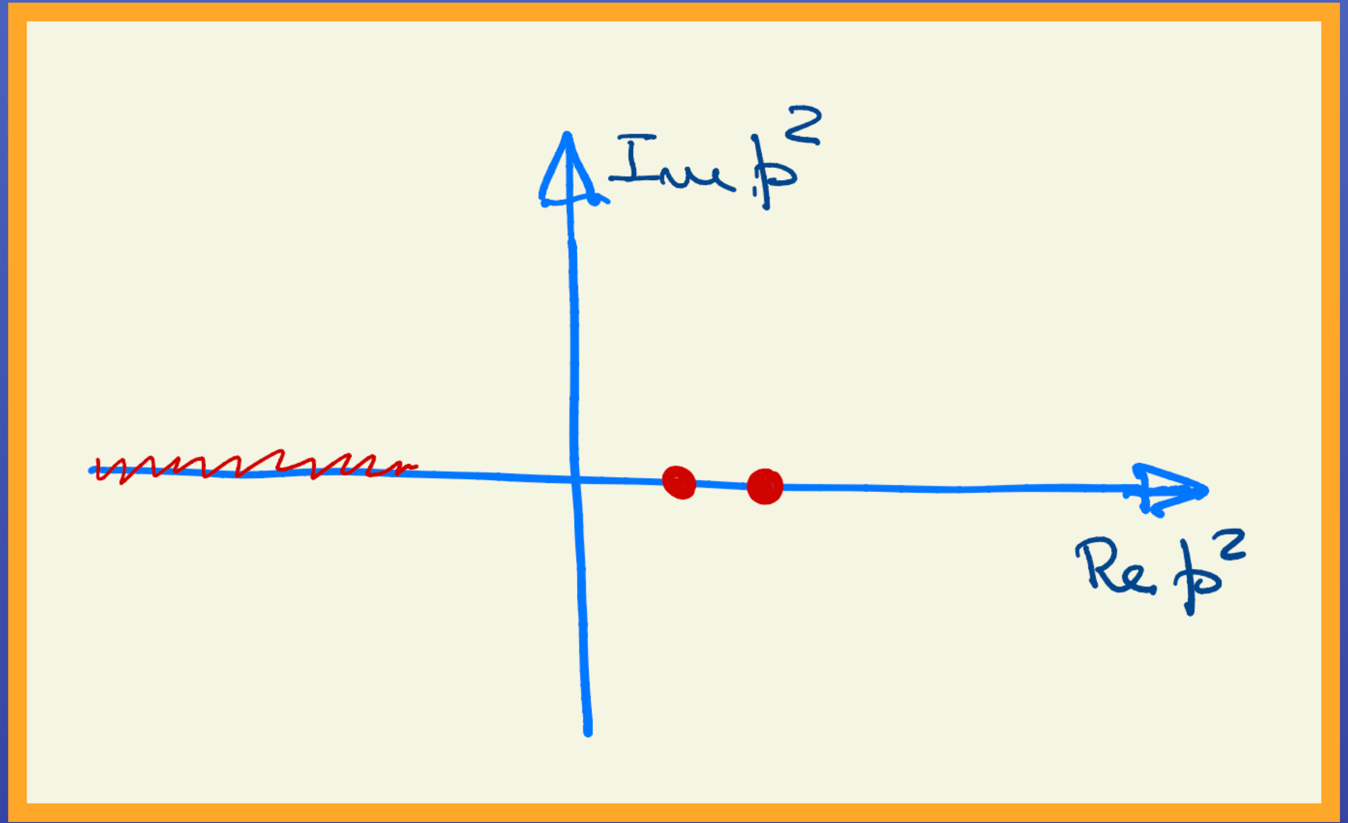
Confinement

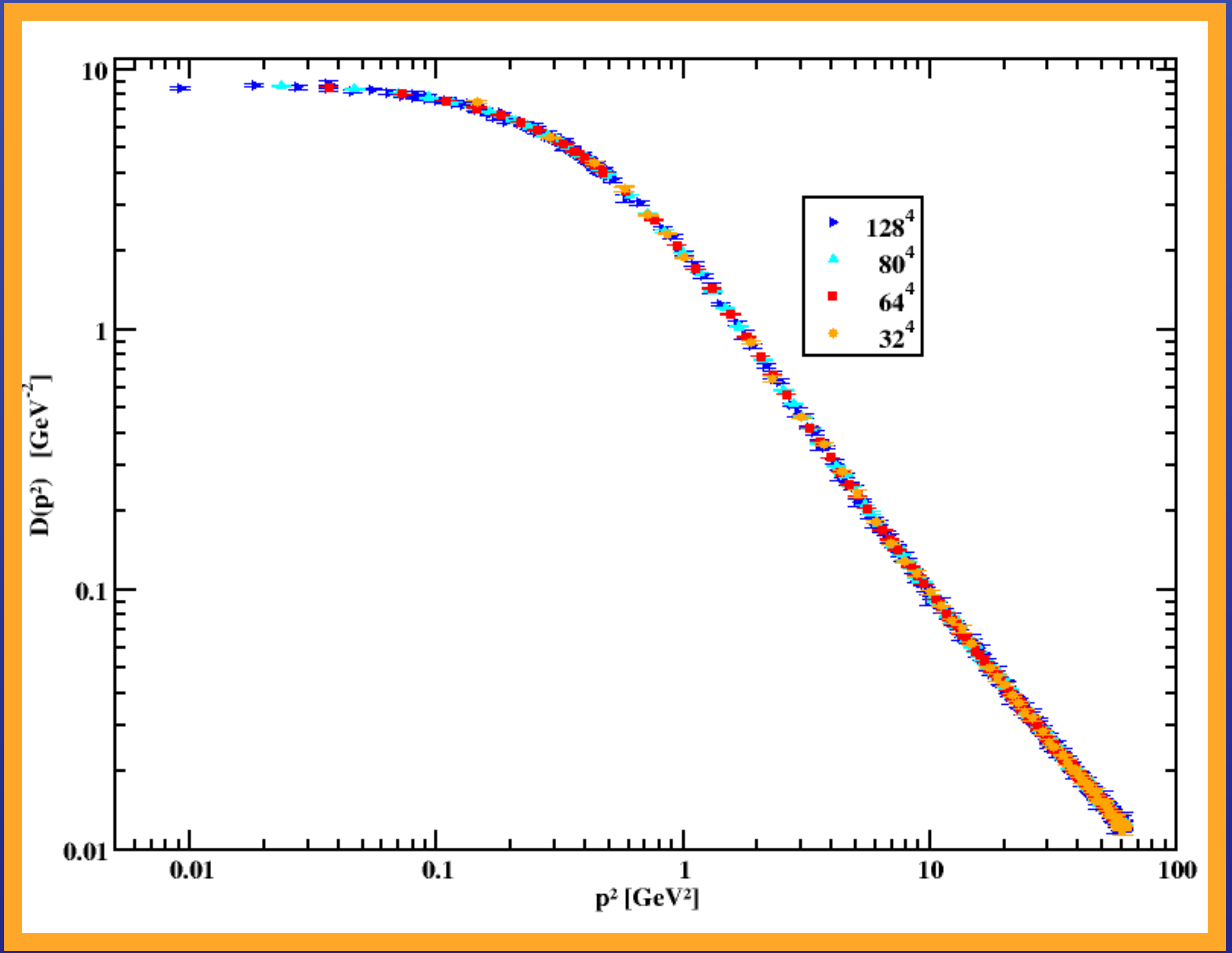
Generation of Mass Scales

Euclidean to Minkowski

What about its Analytic Structure?

Can we get it from lattice simulations?





Landau Gauge !

Pommerenke's Theorem

For a meromorphic function f(z), the Padé sequences [M | M+k], with fixed k, converge to f(z) in any compact set of the complex plane.

In the Padé approximant, single poles of f(z), are sets of zero area, and appear in the [M | N] approximants as stable poles for sufficiently large values of M.

[M | N] Padé approximant

$$D(p^2) \approx P^M_N(p^2) = \frac{Q_M(p^2)}{R_N(p^2)}$$

$$Q_M(p^2) = q_0 + \cdots + q_N \ \left(p^2\right)^M$$

$$R_N(p^2) = 1 + \cdots + r_N \ \left(p^2\right)^N$$

Near diagonal **M = N, N + 1, N - 1** sequences

$$f(z) = \int_0^{+\infty} \frac{d\mu(t)}{1 + z\,t} \, , \qquad |Arg(z)| \; < \; \pi$$

Stieltjes Function

$$f(z) = \int_0^{+\infty} \frac{d\mu(t)}{1 + z\,t}\,, \qquad |Arg(z)| < \pi$$

Stieltjes Function

Positive defined

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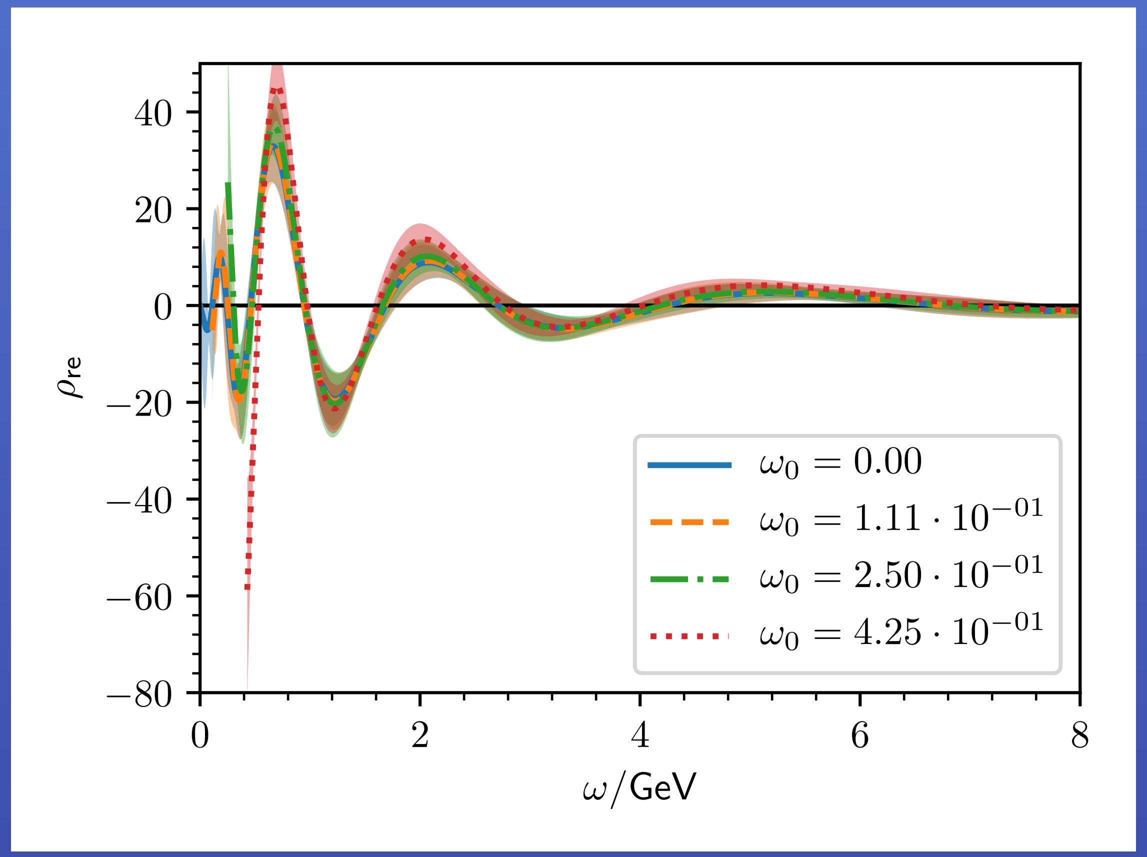
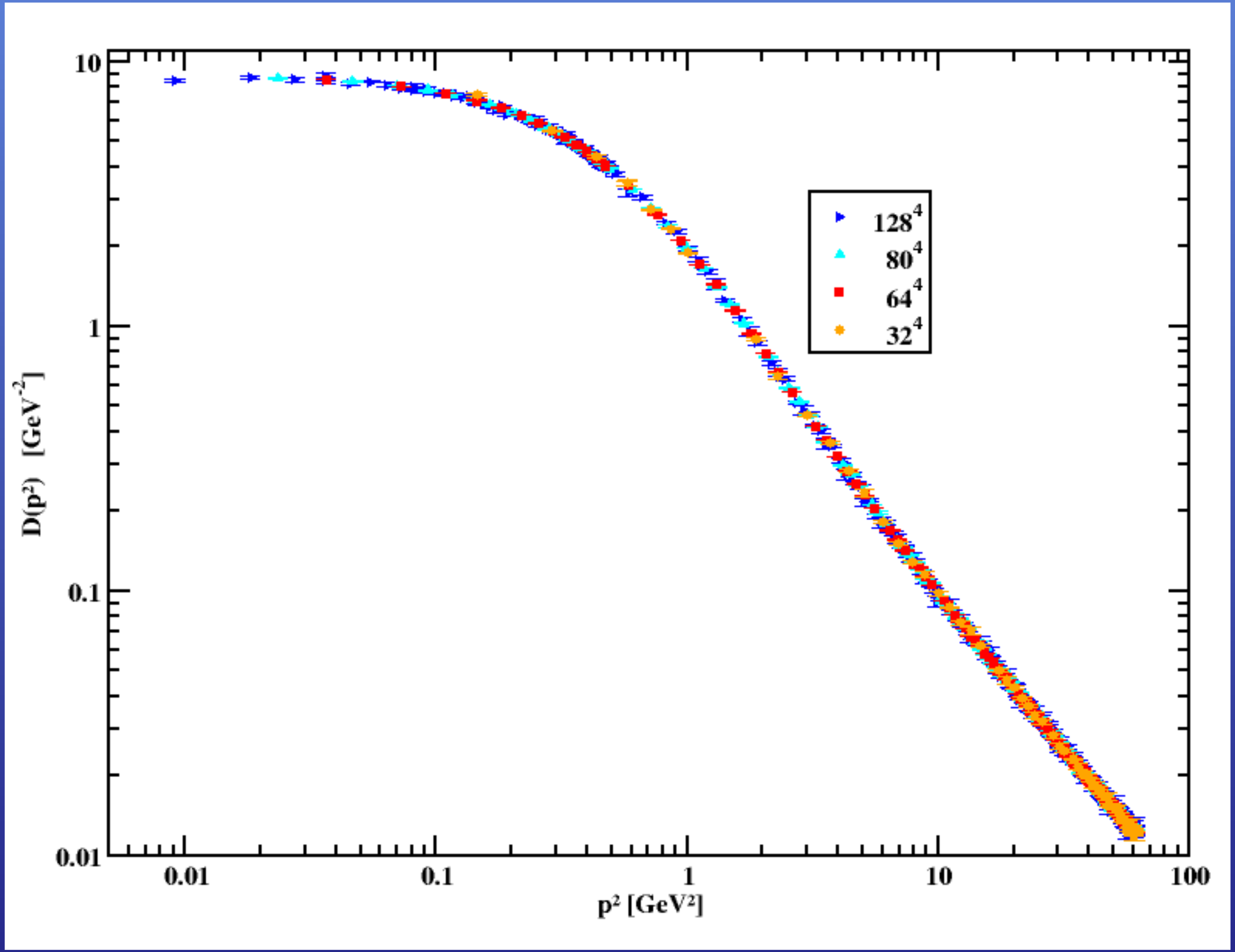
Stieltjes Function

Positive defined



D Dudal, O O, M Roelfs, P Silva, Nucl Phys B 952, 114912 (2020)

Gluon Propagator



From 3.25 fm up to 13.01 fm

α = 0.101 fm

Define Padé approximant by minimising

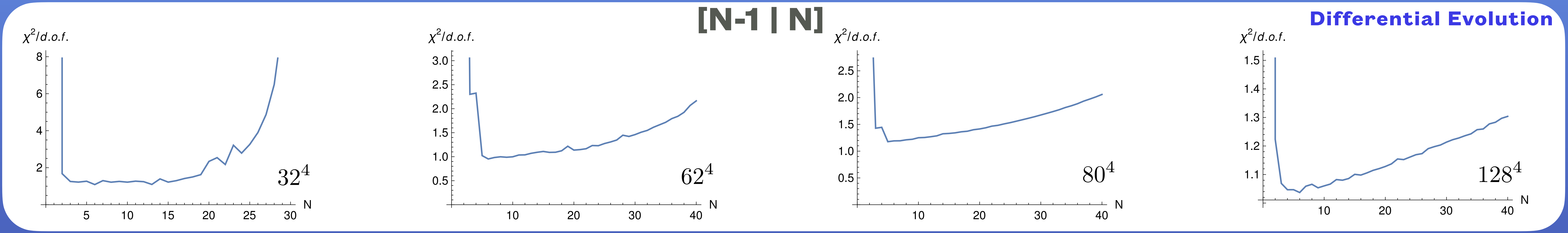
$$\chi^2 = \sum_{j=1}^{N} \left(\frac{D(p_j^2) - D_i(p_j^2)}{\sigma(p_j^2)} \right)^2$$

Global optimisation methods:

Differential Evolution

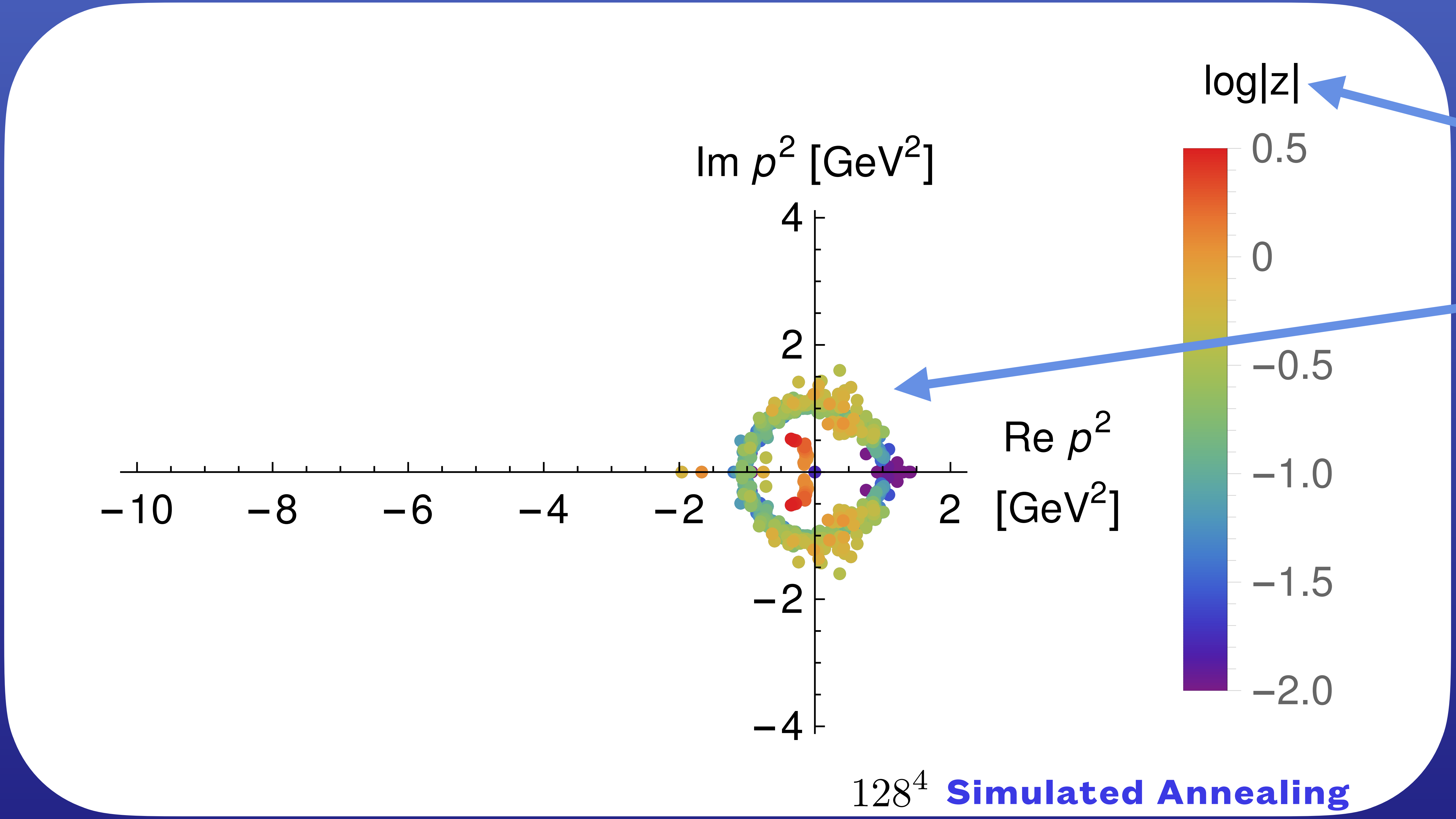
Simulated Annealing

Gluon Propagator



[N-1 | N]

Differential Evolution



128^4 **Simulated Annealing**

All N

Residua

Froissart Doublets

Gluon Propagator

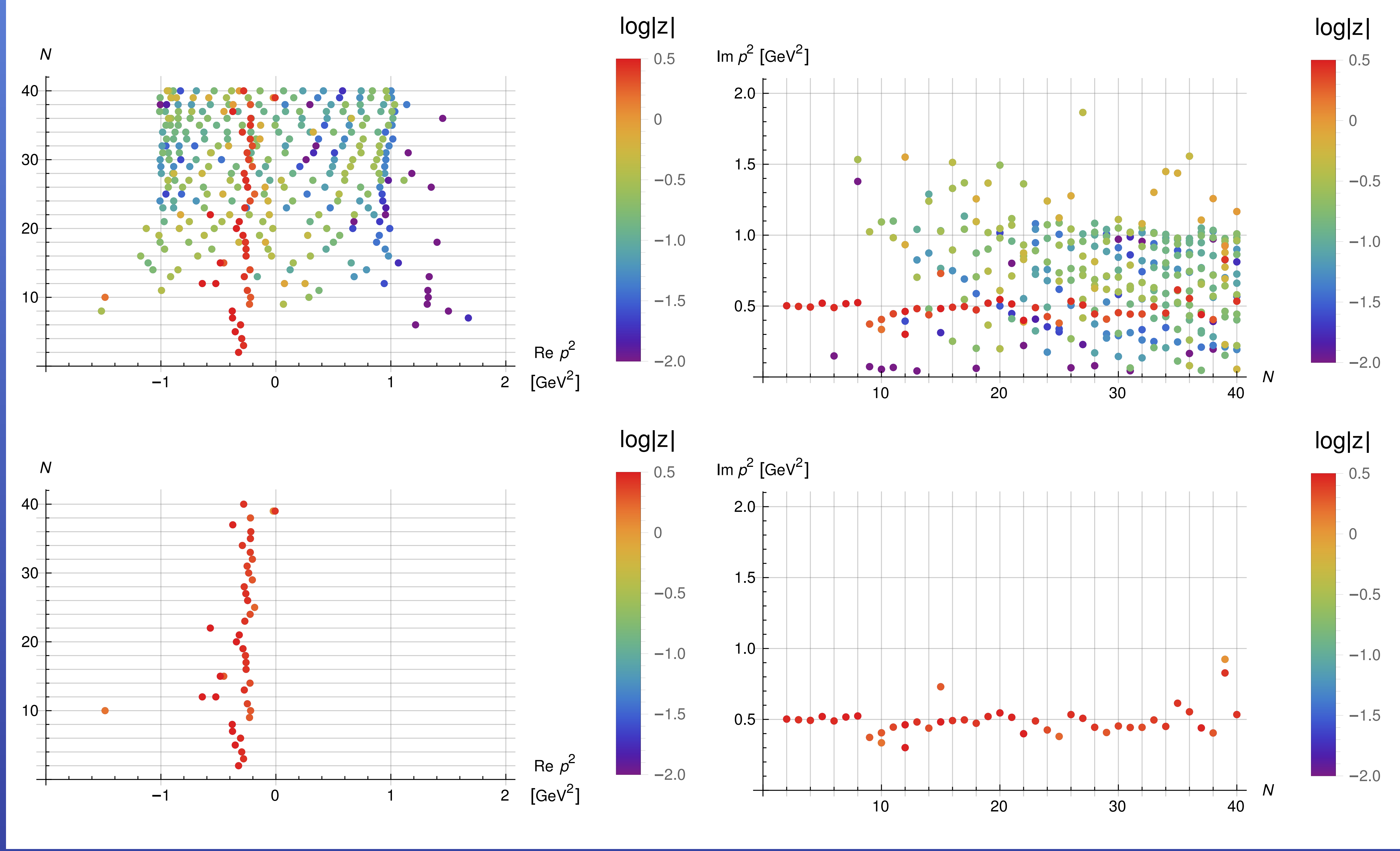
80^4

Differential Evolution

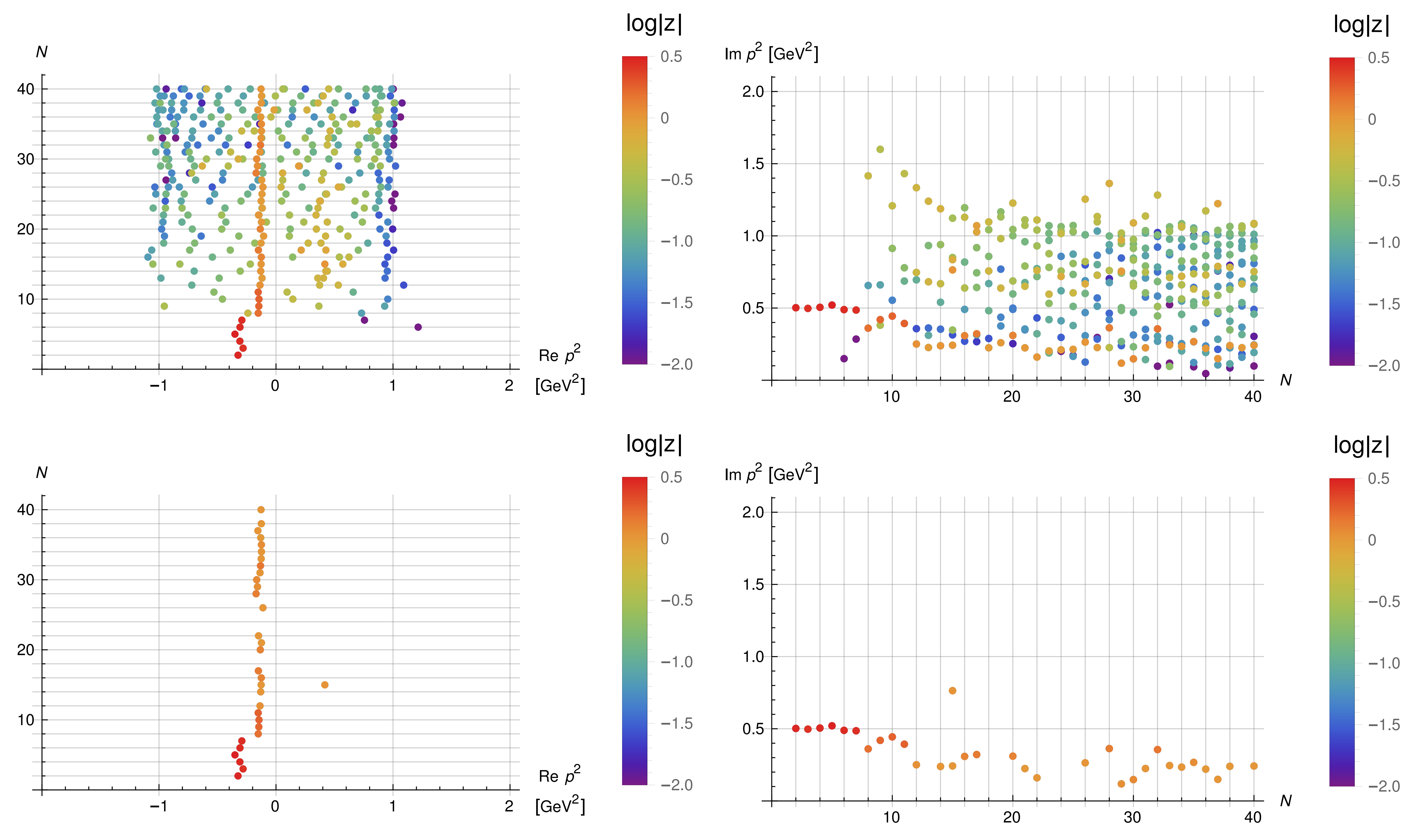
Simulated Annealing

Gluon Propagator

128^4



Differential Evolution



Simulated Annealing

Gluon Propagator

128^4

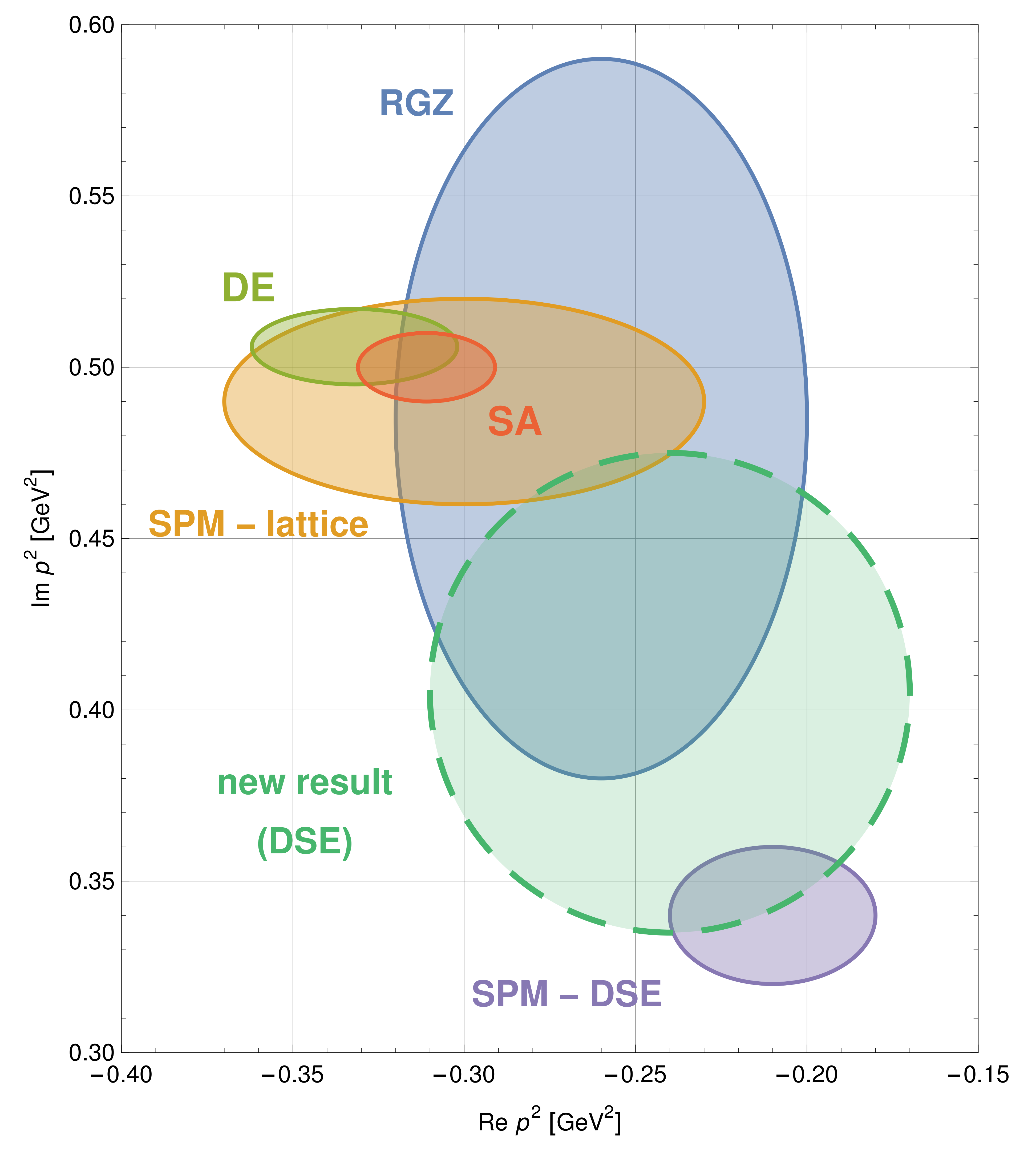
Differential Evolution

Simulated Annealing

$$p^2 = -(0.343 - 0.220) \pm i\,(0.301 - 0.546)\ \mathrm{GeV}^2$$

$$p^2 = -(0.220 - 0.150) \pm i\,(0.227 - 0.444)\ \mathrm{GeV}^2$$

Gluon Propagator



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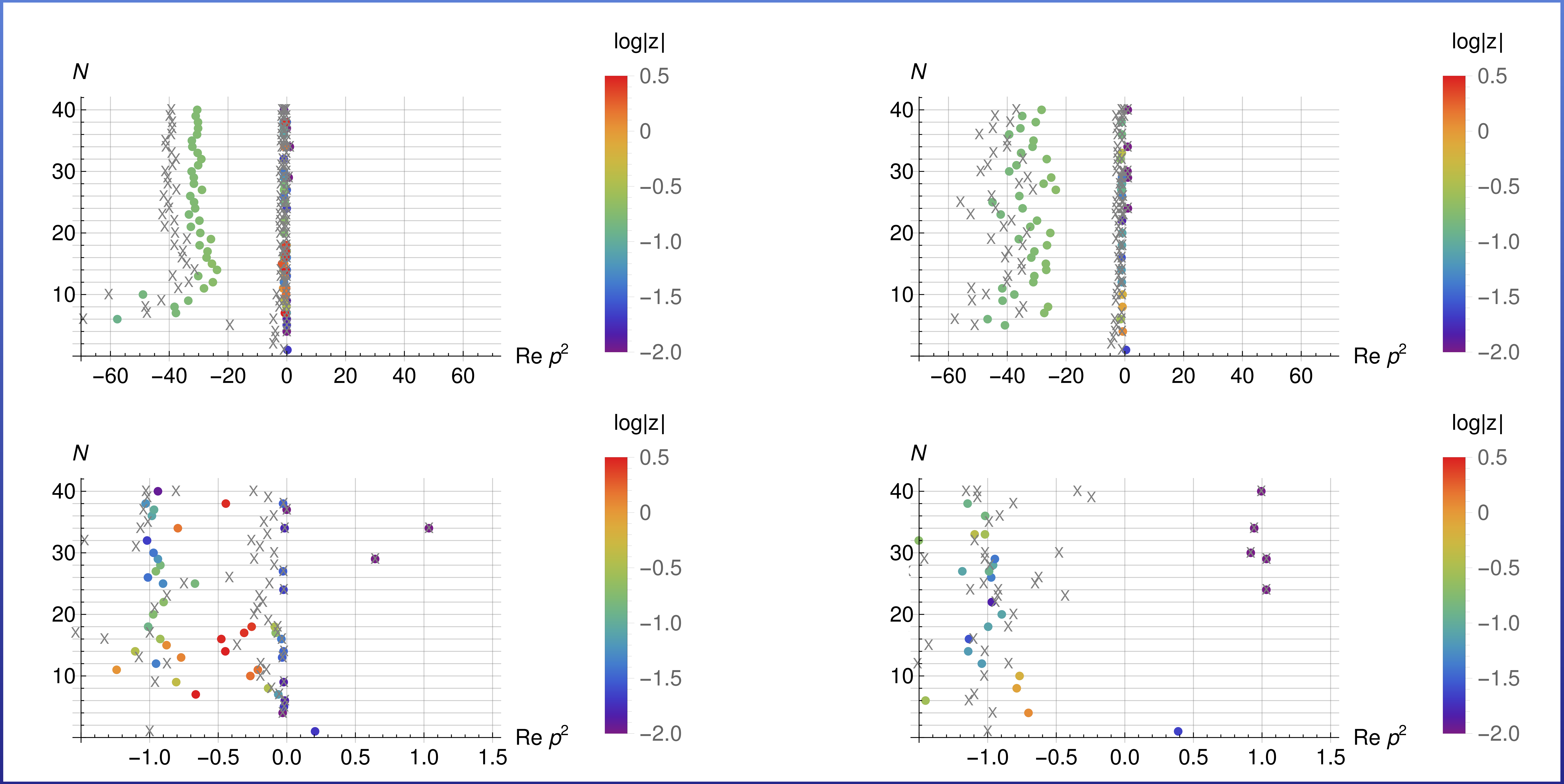
Complex pole associated with IR momentum

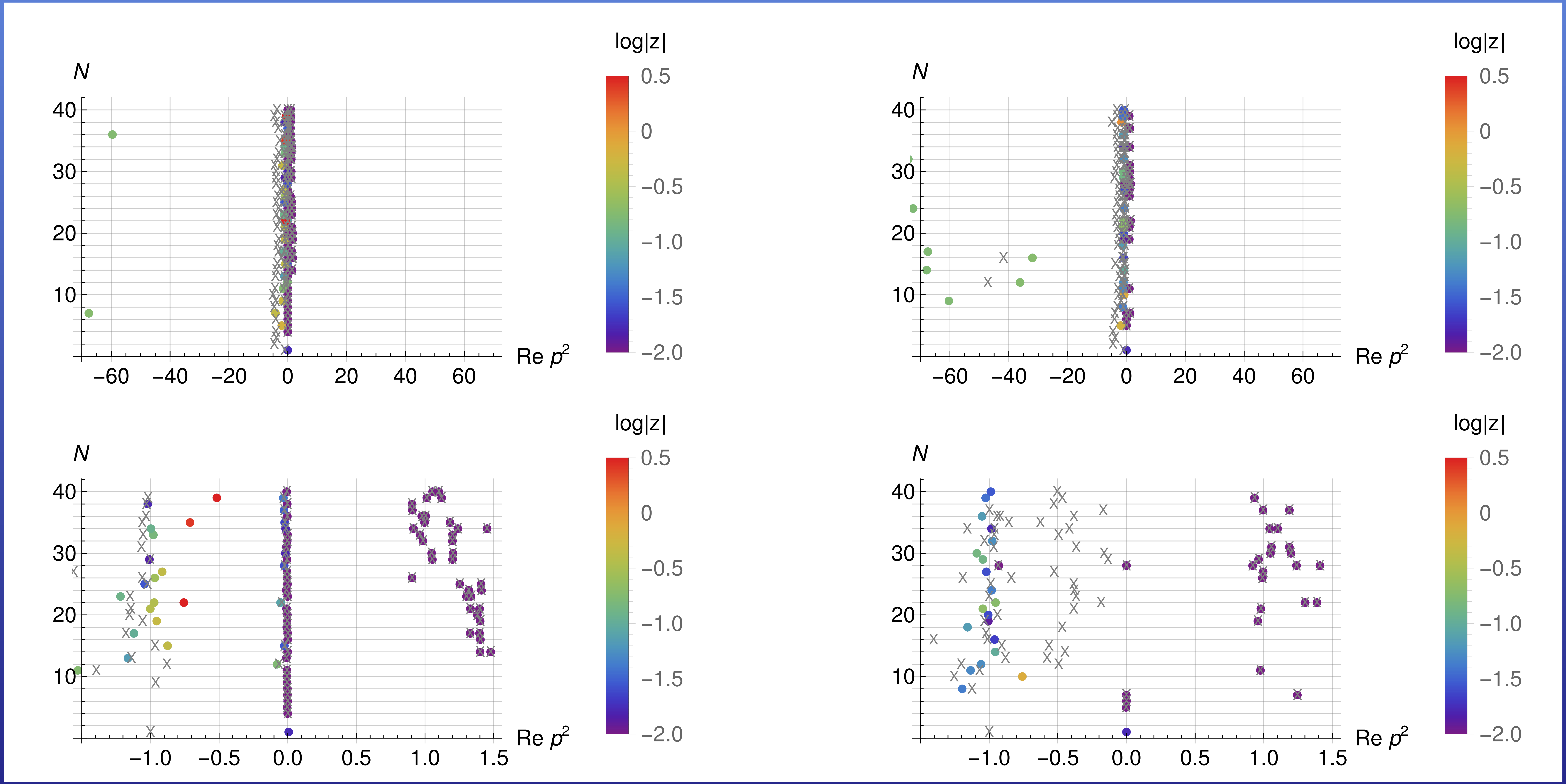
Landau Gauge Gluon Propagator

80^4



Landau Gauge Gluon Propagator

128^4



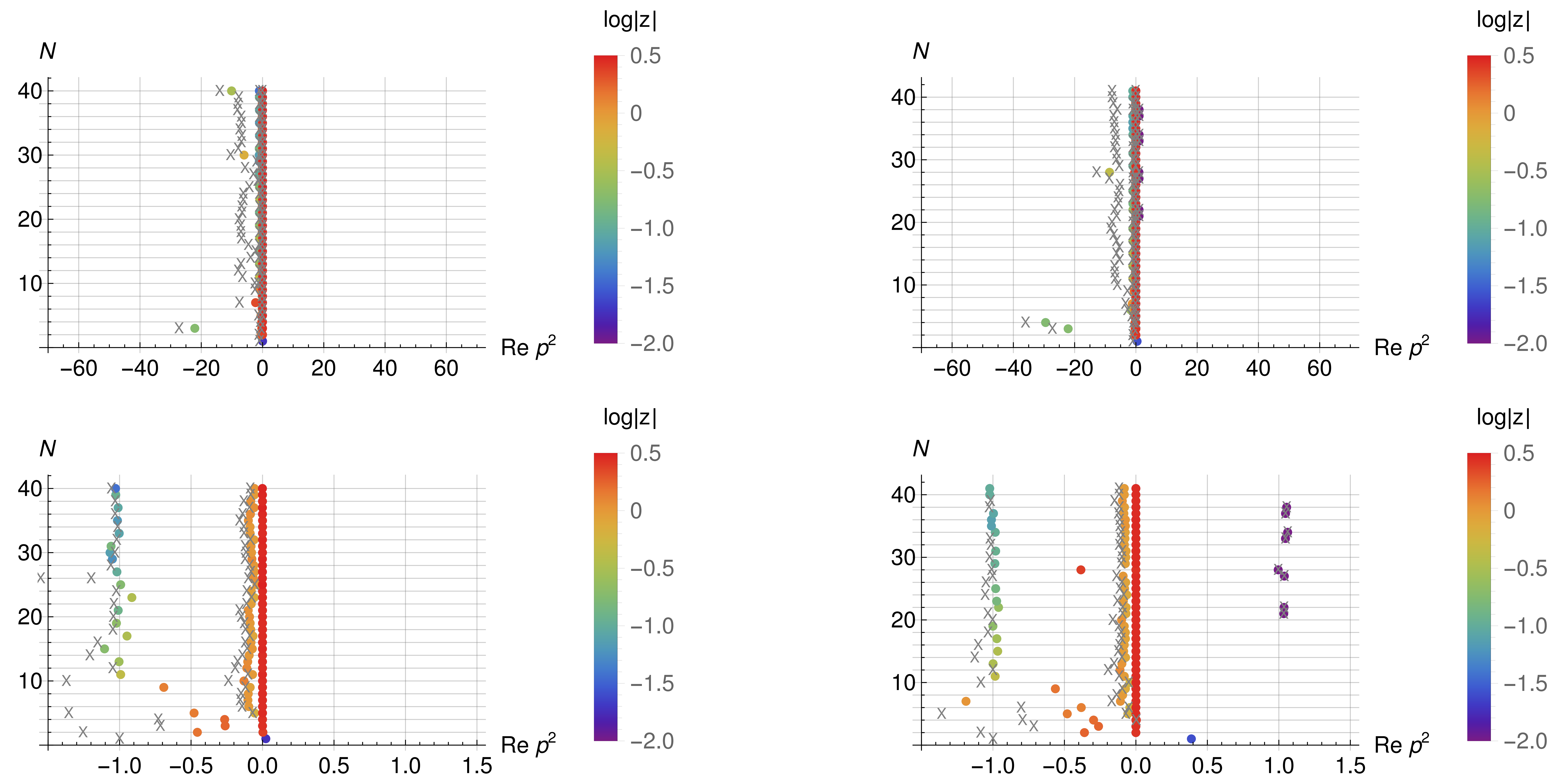
Gluon Propagator

Poles ⟶ $p^2 = -0.31 \pm i\, 0.5 \ \mathrm{GeV}^2 = 0.59\, e^{\pm\, i\, 0.68\, \pi} \ \ \mathrm{GeV}^2$

Branch Cut ⟶ $\Re(p^2) \sim \ -0.5 \ \mathrm{GeV}^2$ or smaller

Ghost Propagator

Ghost Propagator

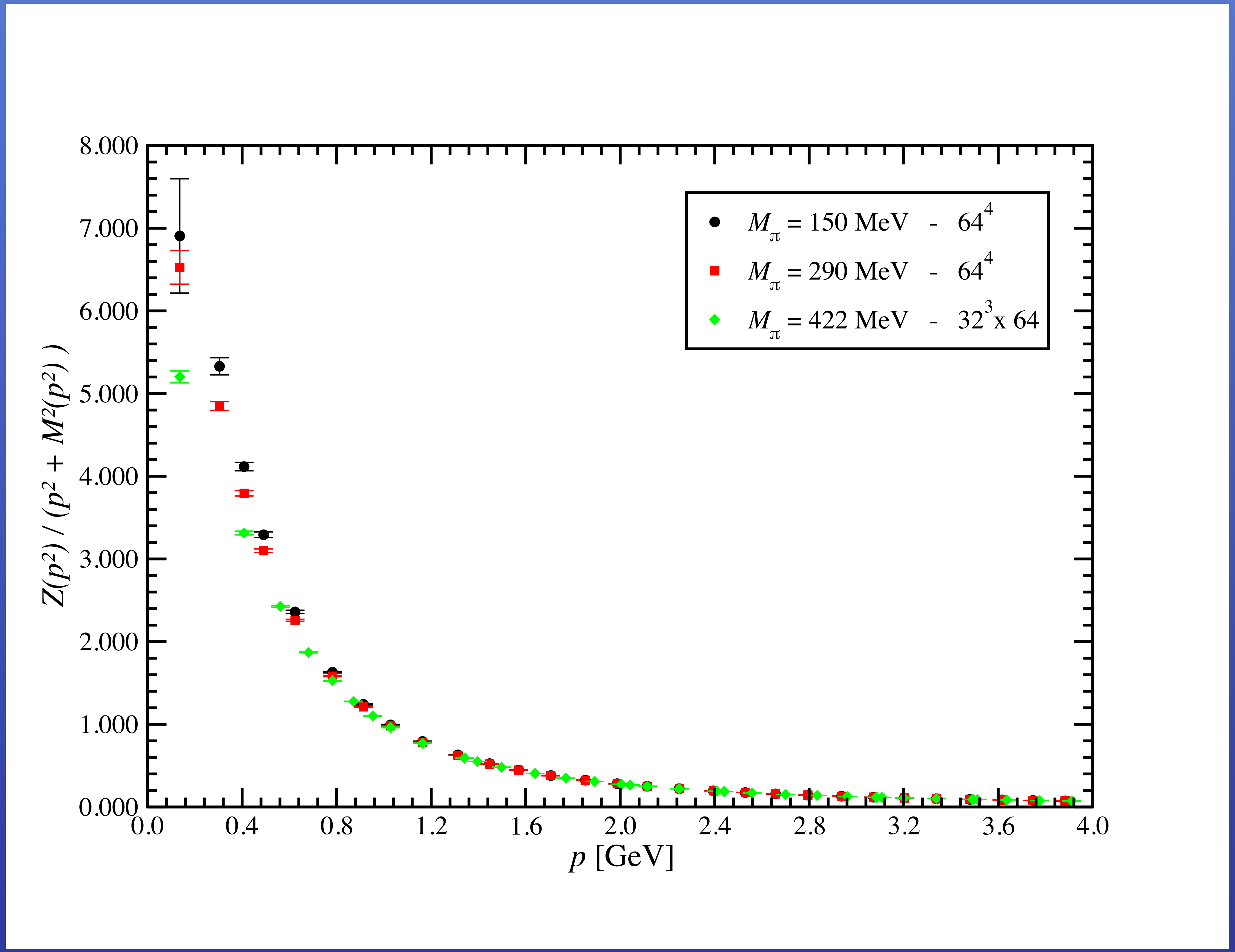


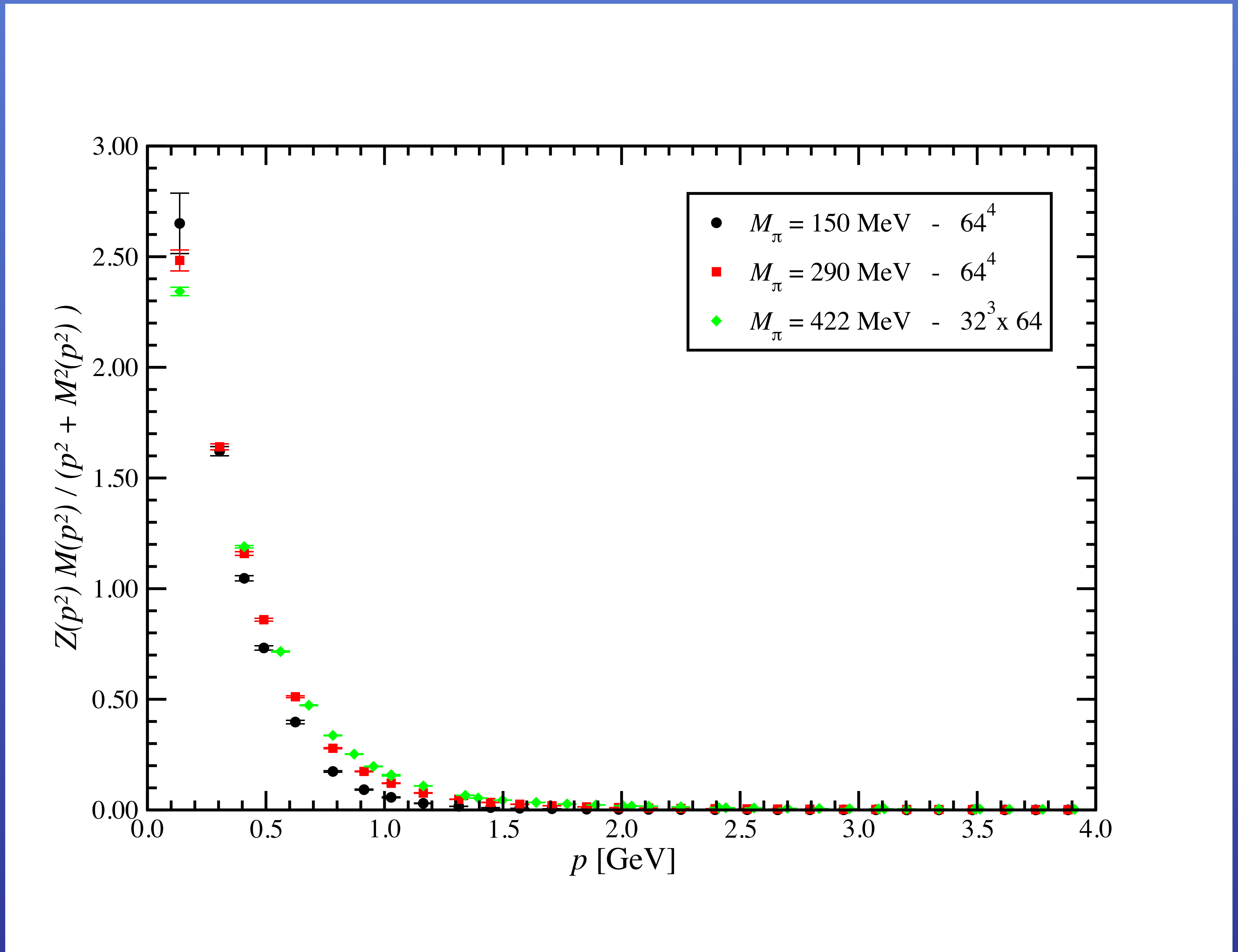
Ghost Propagator

$p^2 = 0 \quad \mathrm{GeV}^2$

Branch Cut $\longrightarrow$ $\Re(p^2) \sim -0.1 \ \mathrm{GeV}^2$

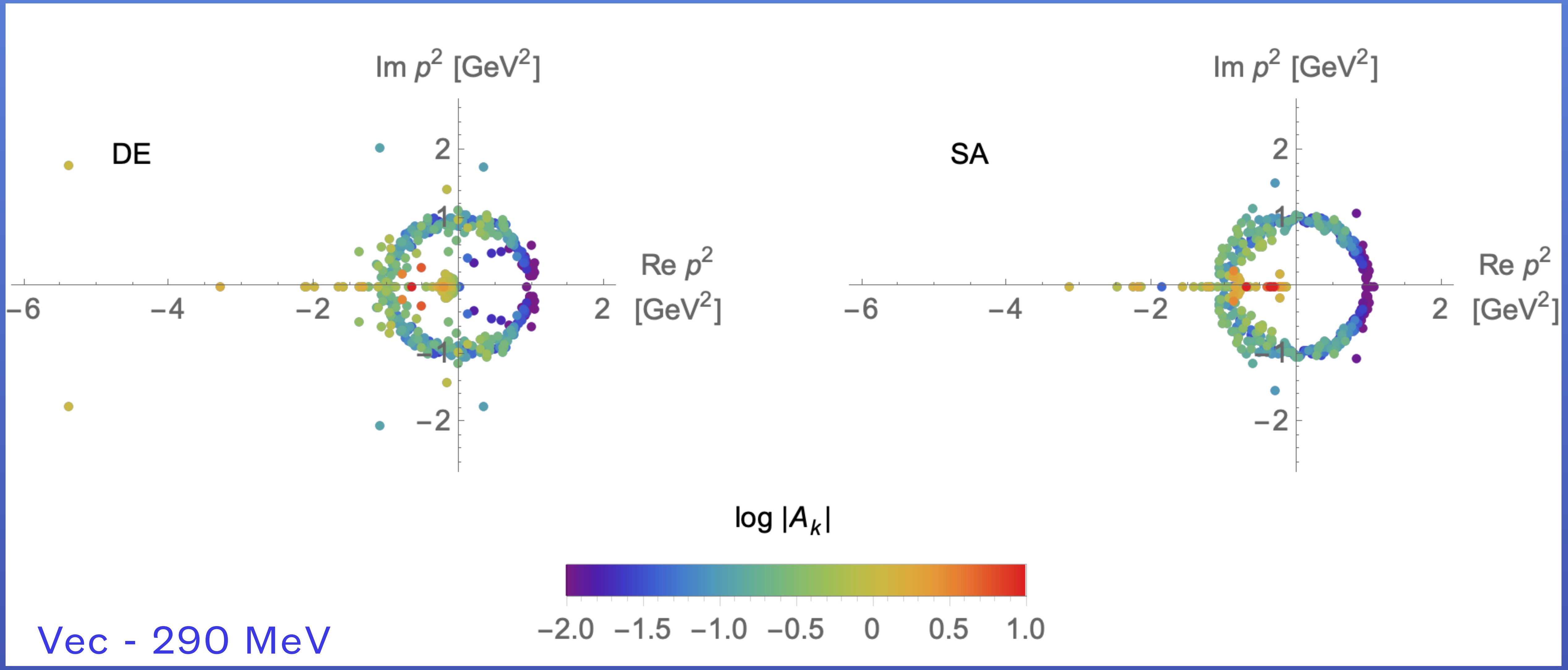
Quark Propagator



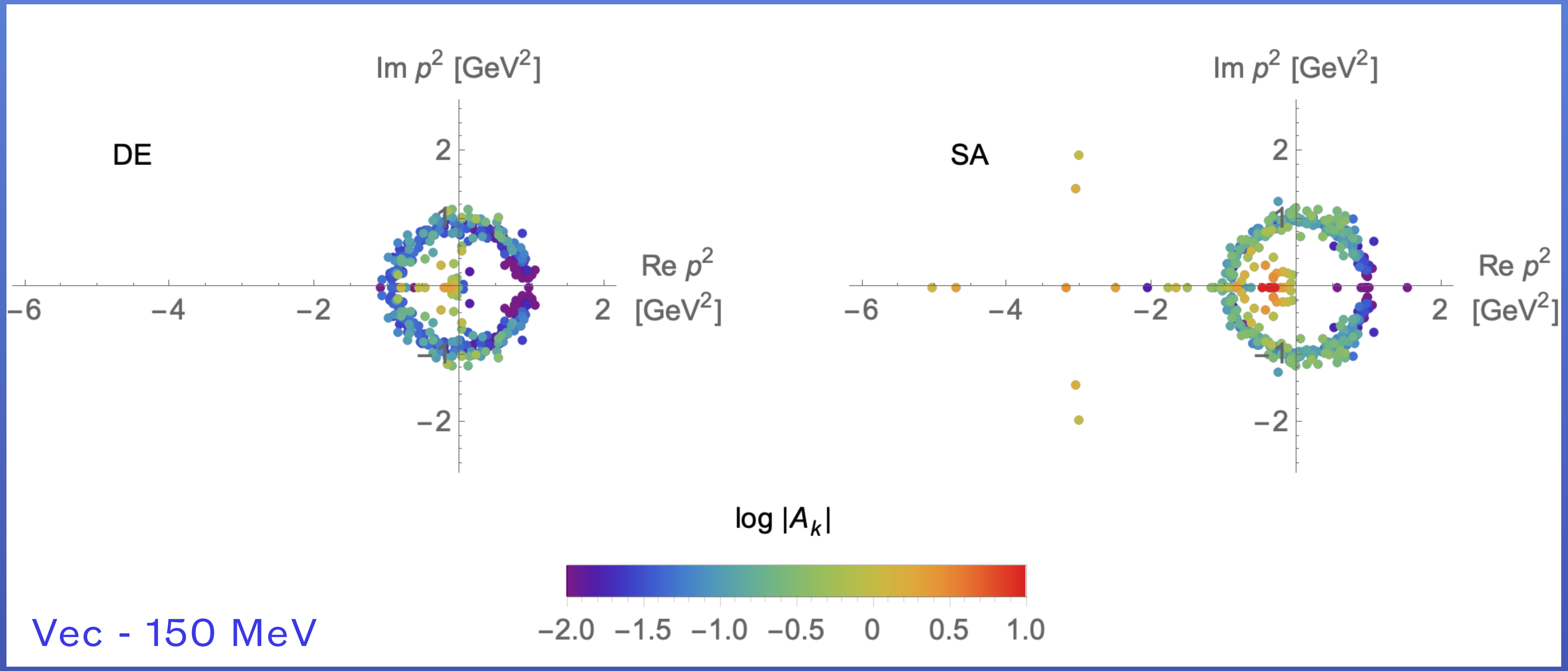


Data from PRD 99, 094506 (2019)

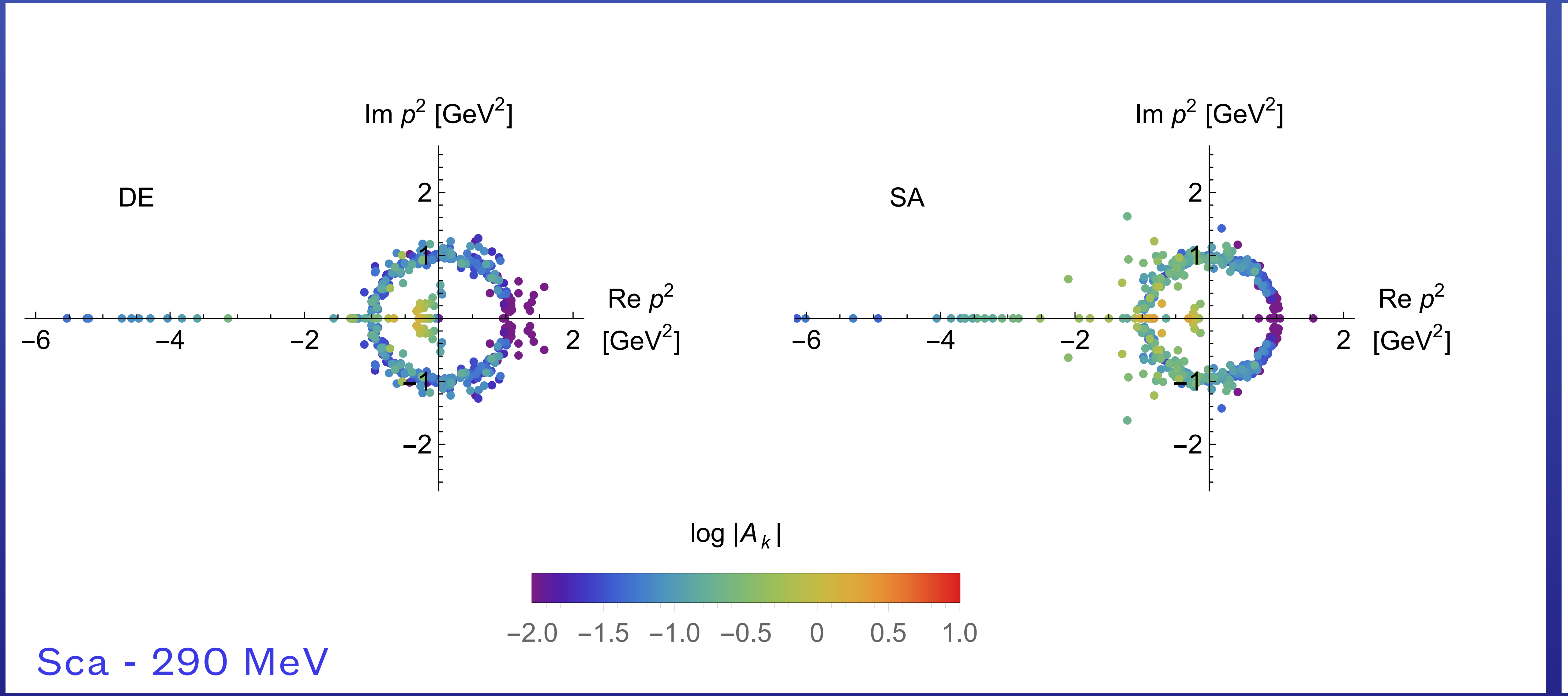
Quark Propagator



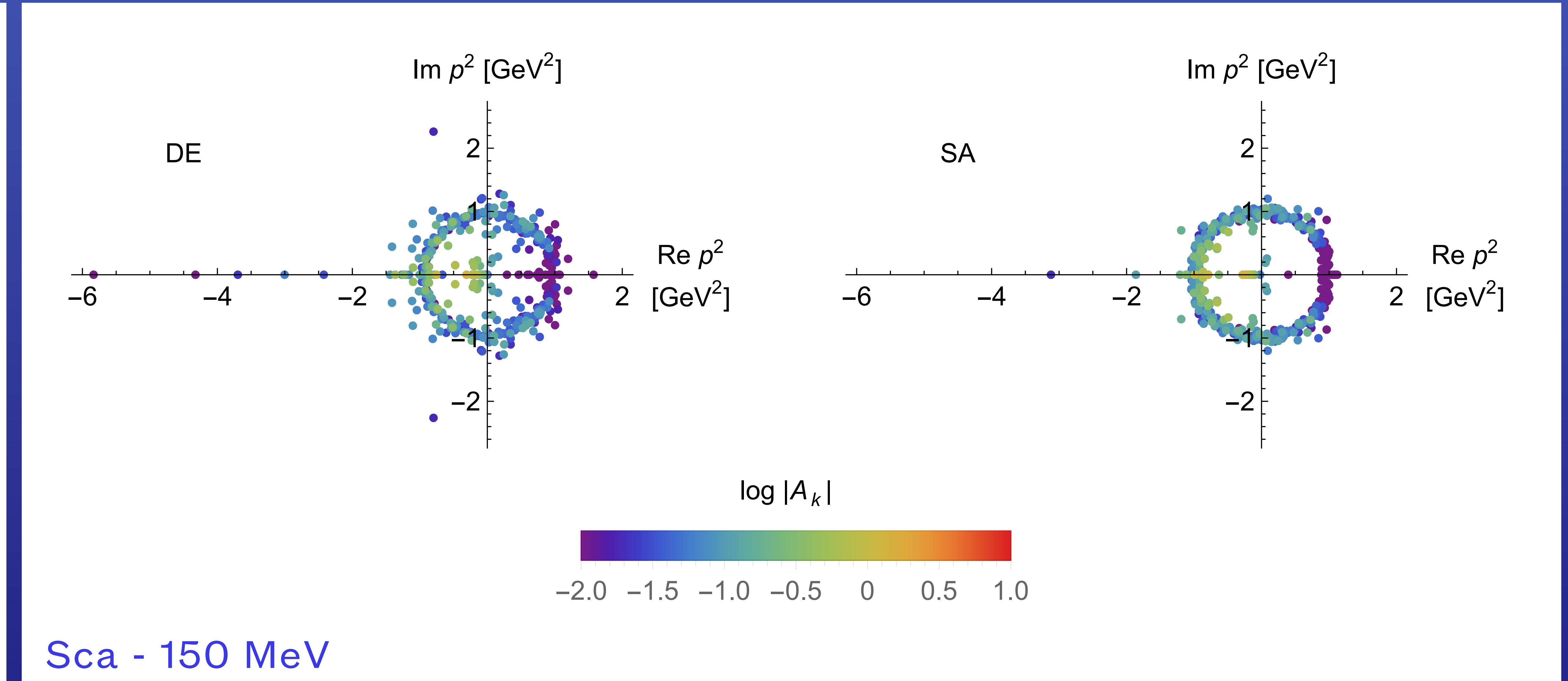
Vec - 290 MeV



Vec - 150 MeV

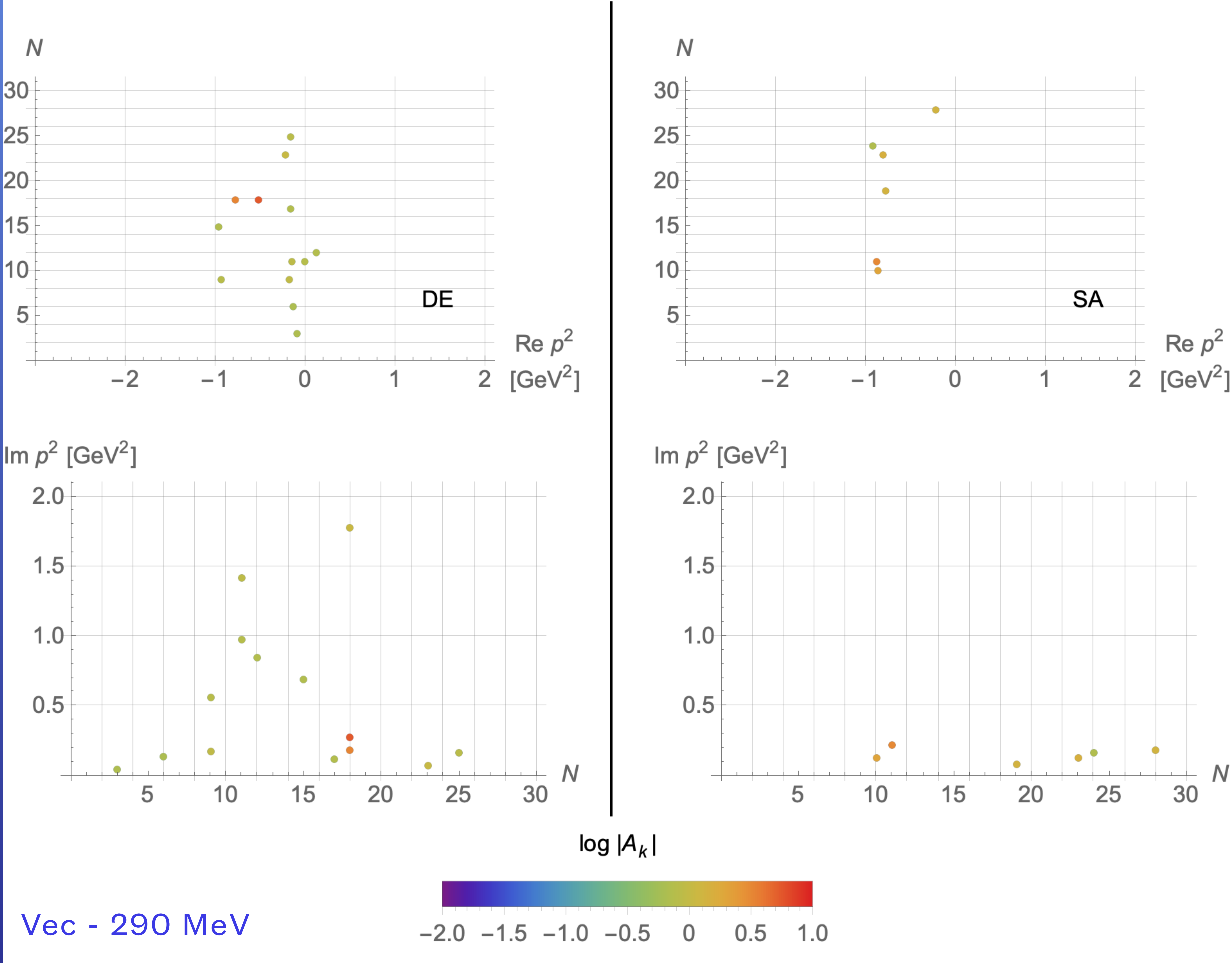


Sca - 290 MeV

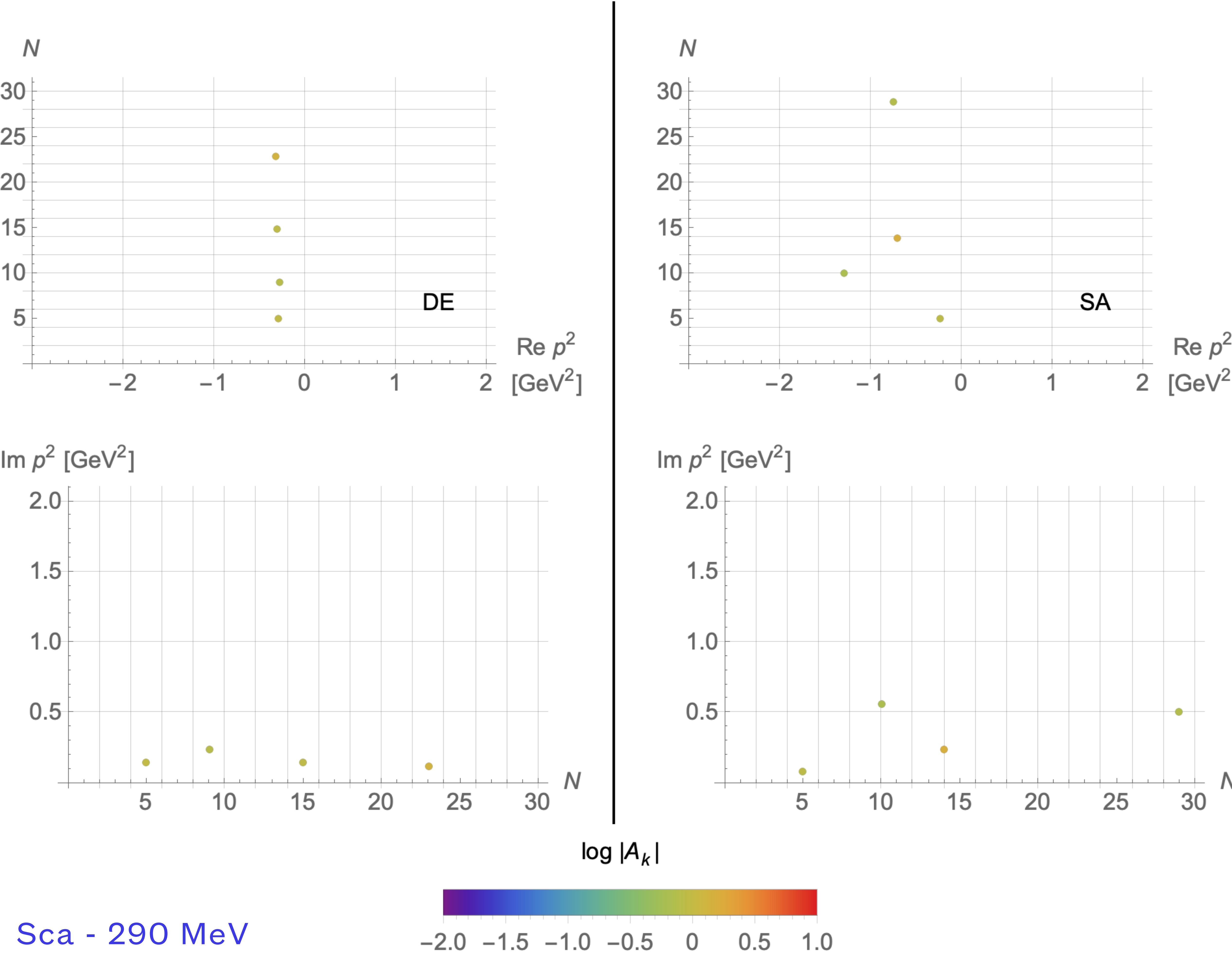


Sca - 150 MeV

Quark Propagator



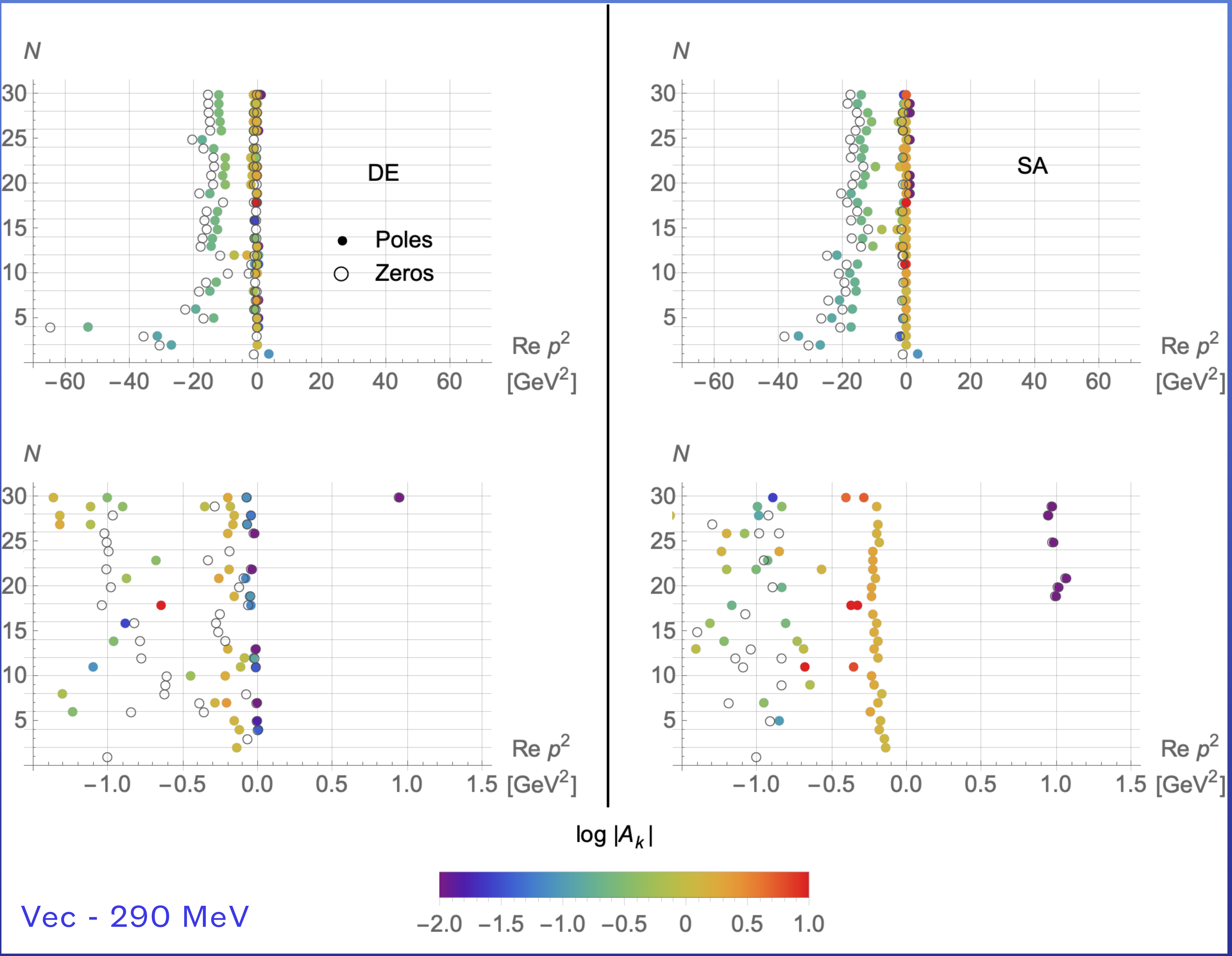
Vec - 290 MeV



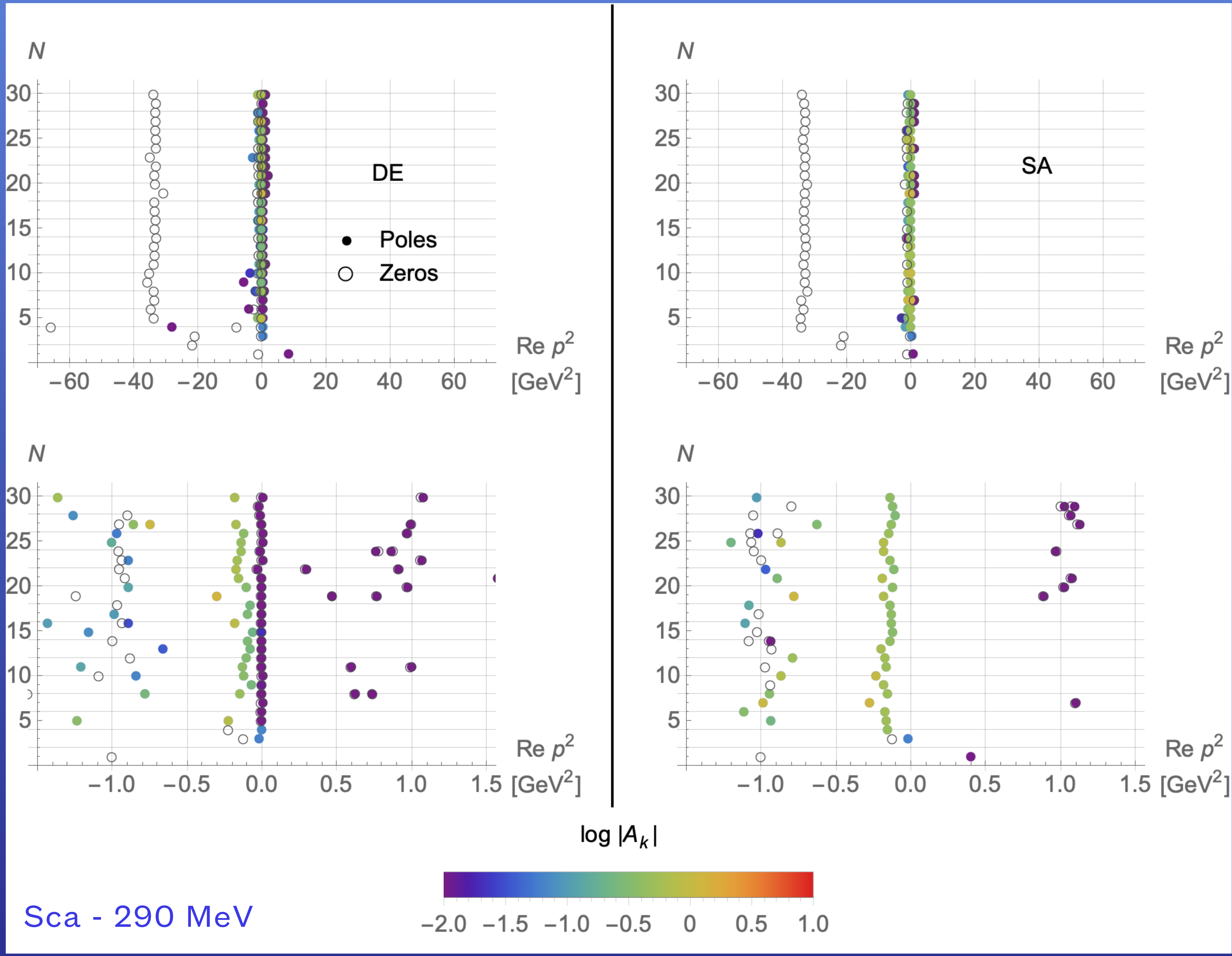
Sca - 290 MeV

No clear sign of complex poles

Quark Propagator



Vec - 290 MeV



Sca - 290 MeV

Quark Propagator

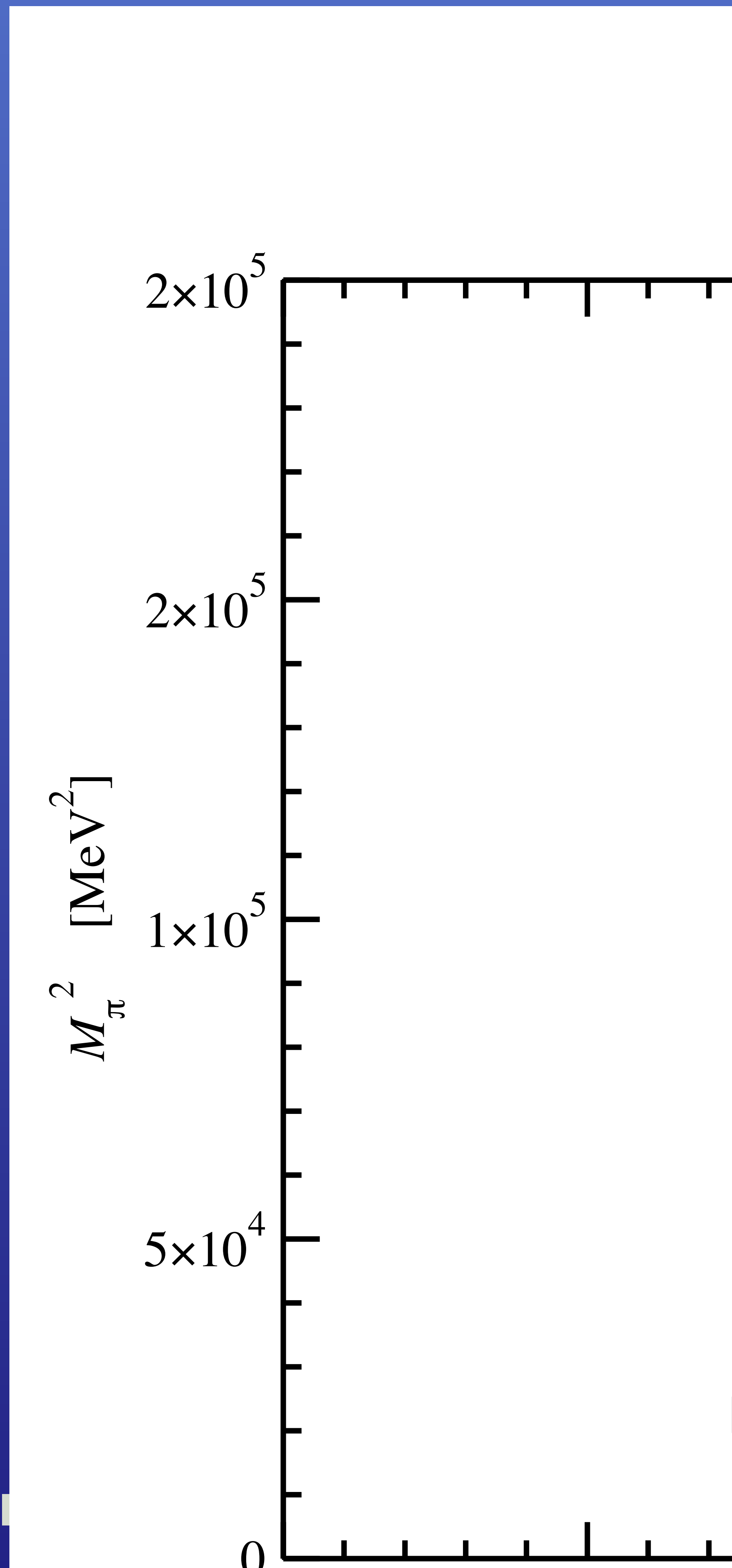
Vec - 290 MeV

M_π	150 MeV	290 MeV	422 MeV
Vec	0.19(6)	0.22(3)	0.26(3)
Sca	0.16(3)	0.19(6)	0.21(6)

Quark Propagator

Vec - 290 MeV

M_π	150 MeV	290 MeV	422 MeV
Vec	0.19(6)	0.22(3)	0.26(3)
Sca	0.16(3)	0.19(6)	0.21(6)



Conclusions

Padé Approximants allow to access the analytic structure from Landau gauge lattice propagators

Landau gauge gluon propagator: pair of complex conjugate poles + branch cut

Landau gauge ghost propagator: pole at zero momentum + branch cut with precise location of the branch point

Landau gauge quark propagator: pole at Minkowski momenta that grows (linearly) with the pion mass squared; not clear if there is a branch cut for $p < 1$ GeV

Problems and Future work:

Precise location of branch cut (gluon propagator)

Investigate for the presence of multiple poles

Acknowledgments

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