

Another look at the three-gluon vertex in the minimal Landau gauge

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Outline

- 1 Introduction and Motivation
- 2 Results
 - Three gluon vertex
- 3 Conclusions and outlook

Green's functions

- Green's functions summarize the dynamics of the theory
 - QCD: information on confinement and chiral symmetry breaking
- n -point complete Green's functions

$$G^{(n)}(x_1, \dots, x_n) = \langle 0 | T (\phi(x_1) \cdots \phi(x_n)) | 0 \rangle,$$

- decomposition in terms of one particle irreducible (1PI) functions $\Gamma^{(n)}$
 - access to form factors that define $\Gamma^{(n)}$
- lattice approach allows for first principles determination of the complete Green's functions of QCD

Three gluon vertex

- allow to measure e.g. strong coupling constant
- fundamental role in the structure of Dyson-Schwinger equations
- infrared behaviour predicted by DSE equations
 - infrared suppression — form factors (FF) decrease as $p \rightarrow 0$
 - zero crossing — FF change sign in IR region
 - logarithmic divergence at origin — $FF \rightarrow -\infty$ for $p \rightarrow 0$
- infrared behaviour corroborated by lattice simulations
 - quenched and dynamical simulations

Aguilar, Soto, Ferreira, Papavassiliou, Rodríguez-Quintero, PLB 818(2021)136352

Aguilar, Soto, Ferreira, Papavassiliou, Rodríguez-Quintero, Zafeiropoulos, EPJC 80(2020)154

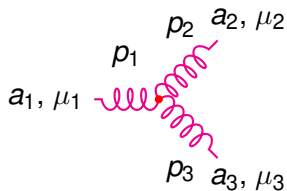
Aguilar, Ferreira, Papavassiliou, EPJC 80(2020)887

Maas, Vujanović, arXiv:2006.08248 [hep-lat]

Three point complete Green's function

$$\langle A_{\mu_1}^{a_1}(p_1) A_{\mu_2}^{a_2}(p_2) A_{\mu_3}^{a_3}(p_3) \rangle =$$

$$V \delta(p_1 + p_2 + p_3) G_{\mu_1 \mu_2 \mu_3}^{a_1 a_2 a_3}(p_1, p_2, p_3)$$



$$G_{\mu_1 \mu_2 \mu_3}^{a_1 a_2 a_3}(p_1, p_2, p_3) =$$

$$D_{\mu_1 \nu_1}^{a_1 b_1}(p_1) D_{\mu_2 \nu_2}^{a_2 b_2}(p_2) D_{\mu_3 \nu_3}^{a_3 b_3}(p_3) \Gamma_{\nu_1 \nu_2 \nu_3}^{b_1 b_2 b_3}(p_1, p_2, p_3)$$

- color structure:

$$\Gamma_{\mu_1 \mu_2 \mu_3}^{a_1 a_2 a_3}(p_1, p_2, p_3) = f_{a_1 a_2 a_3} \Gamma_{\mu_1 \mu_2 \mu_3}(p_1, p_2, p_3)$$

Three gluon vertex

- Bose symmetry requires vertex to be symmetric under interchange of any pair (p_i, a_i, μ_i)
 - $\Gamma_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3)$ must be antisymmetric
- $\Gamma_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3)$ in the continuum: requires six Lorentz invariant form factors
 - two associated to the transverse component $\Gamma^{(t)}$
 - remaining associated to the longitudinal $\Gamma^{(l)}$.

J. S. Ball, T.-W. Chiu, Phys. Rev. **D22**, 2550 (1980)

Lattice setup

- Wilson gauge action, $\beta = 6.0$
 - 64^4 , 2000 configurations
 - 80^4 , 1800 configurations
- rotation to the Landau gauge: FFT-SD method
- gluon field

Duarte, Oliveira, PJS, PRD 94(2016)074502

$$ag_0 A_\mu(x + a\hat{e}_\mu) = \frac{U_\mu(x) - U^\dagger(x)}{2ig_0} - \frac{\text{Tr} [U_\mu(x) - U^\dagger(x)]}{6ig_0}$$

in momentum space:

$$A_\mu(\hat{p}) = \sum_x e^{-i\hat{p}(x+a\hat{e}_\mu)} A_\mu(x + a\hat{e}_\mu), \quad \hat{p}_\mu = \frac{2\pi n_\mu}{aL_\mu}$$



Lattice setup

- tree level improved momentum

$$p_\mu = \frac{2}{a} \sin\left(\frac{a\hat{p}_\mu}{2}\right)$$

- accessing the 1PI three gluon vertex from the lattice

$$\begin{aligned} G_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3) &= \text{Tr} \langle A_{\mu_1}(p_1) A_{\mu_2}(p_2) A_{\mu_3}(p_3) \rangle = \\ &= V\delta(p_1 + p_2 + p_3) \frac{N_c(N_c^2 - 1)}{4} D(p_1^2) D(p_2^2) D(p_3^2) \\ &P_{\mu_1\nu_1}(p_1) P_{\mu_2\nu_2}(p_2) P_{\mu_3\nu_3}(p_3) \Gamma_{\nu_1\nu_2\nu_3}(p_1, p_2, p_3) \end{aligned}$$

Momentum configuration

- focus on the asymmetric momentum configuration: $p_2 = 0$

B. Allés et al, Nucl. Phys. B**502**, 325 (1997)

$$G_{\mu_1\mu_2\mu_3}(p, 0, -p) = V \frac{N_c(N_c^2 - 1)}{4} [D(p^2)]^2 D(0) \frac{\Gamma(p^2)}{3} p_{\mu_2} T_{\mu_1\mu_3}(p)$$

$$\Gamma(p^2) = 2 \left[A(p^2, p^2; 0) + p^2 C(p^2, p^2; 0) \right]$$

$$G_{\mu\alpha\mu}(p, 0, -p) p_\alpha = V \frac{N_c(N_c^2 - 1)}{4} [D(p^2)]^2 D(0) \Gamma(p^2) p^2$$

Handling of noise, lattice artefacts

- binning in momentum
- $H(4)$ extrapolation of the lattice data
 - lattice momentum p_μ
 - invariants of the remnant $H(4)$ symmetry group associated with a hypercubic lattice

$$p^2 = p^{[2]} = \sum_{\mu} p_{\mu}^2, \quad p^{[4]} = \sum_{\mu} p_{\mu}^4, \quad p^{[6]} = \sum_{\mu} p_{\mu}^6, \quad p^{[8]} = \sum_{\mu} p_{\mu}^8$$

- on the lattice, a scalar quantity F is a function of all $H(4)$ invariants, i.e. $F_{Lat} = F(p^{[2]}, p^{[4]}, p^{[6]}, p^{[8]})$
- continuum limit given by $F(p^{[2]}, 0, 0, 0)$

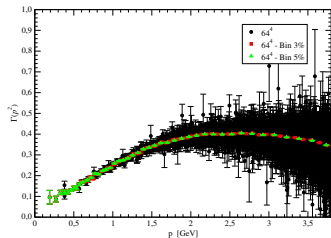
Becirevic et al, PRD 60(1999)094509
Soto, Roiesnel, JHEP 09(2009)007

Outline

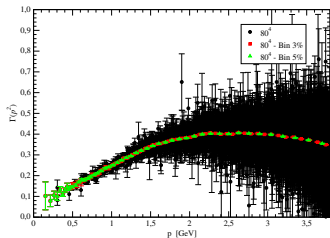
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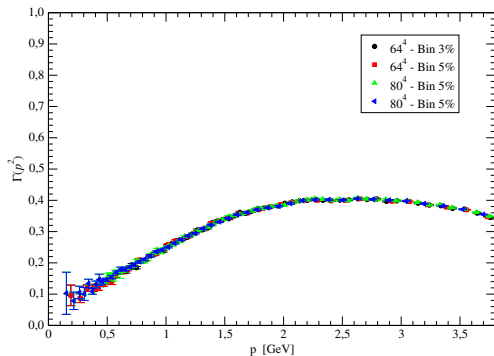
Original and binned $\Gamma(p^2)$

64^4

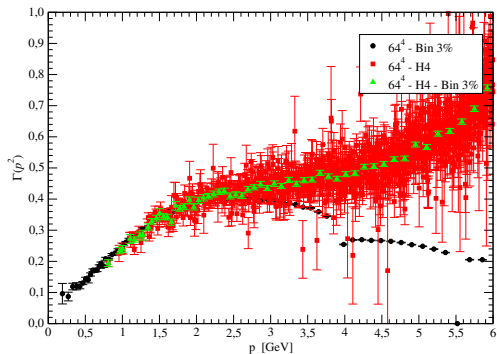


80^4

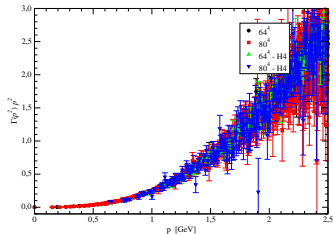
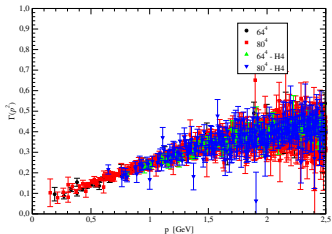


Binned $\Gamma(p^2)$ 

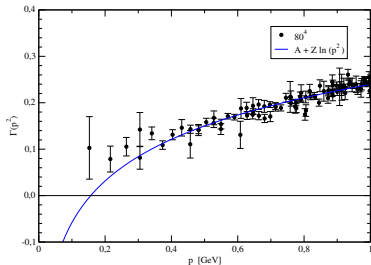
$H(4)$ extrapolation of $\Gamma(p^2)$



H(4) extrapolation — infrared region

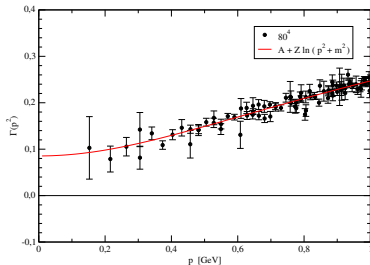
 $p^2 \Gamma(p^2)$

 $\Gamma(p^2)$


$\Gamma(p^2)$ — Infrared behaviour



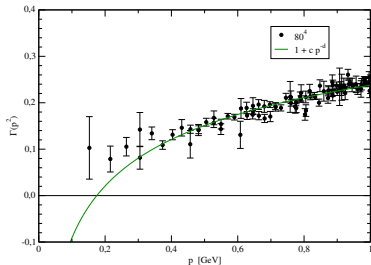
- $\Gamma(p^2) = A + Z \ln(p^2)$
- 80^4 lattice, $p < 1\text{GeV}$
- $A = 0.2395(16)$
- $Z = 0.0646(21)$
- $\chi^2/dof = 1.23$
- $p_0 = 157\text{MeV}$

$\Gamma(p^2)$ — Infrared behaviour



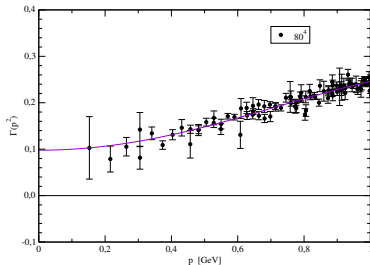
- $\Gamma(p^2) = A + Z \ln(p^2 + m^2)$
- 80^4 lattice, $p < 1\text{GeV}$
- $A = 0.208(24)$
- $Z = 0.124(27)$
- $m = 0.61(15)$
- $\chi^2/\text{dof} = 0.95$

$\Gamma(p^2)$ — Infrared behaviour



- $\Gamma(p^2) = 1 + c p^{-d}$
- 80^4 lattice, $p < 1\text{GeV}$
- $c = -0.7621(15)$
- $d = 0.1558(49)$
- $\chi^2/dof = 1.35$
- $p_0 = 175\text{MeV}$

$\Gamma(p^2)$ — Infrared behaviour



- $\Gamma(p^2) = a + bp^2 + cp^4$
- 80^4 lattice, $p < 1\text{GeV}$
- $a = 0.0978(60)$
- $b = 0.218(22)$
- $c = -0.070(18)$
- $\chi^2/\text{dof} = 0.98$

Conclusions

- Improved calculation of the three gluon vertex on the lattice
 - particular kinematical configuration $p_2 = 0$
 - two different lattice volumes: $(6.5 \text{ fm})^4$ and $(8.2 \text{ fm})^4$
 - lattice spacing $a = 0.102 \text{ fm}$
- $H(4)$ extrapolation pushes the vertex to higher values in UV regime
- functional study in the infrared region
 - some functional forms compatible with zero crossing and IR divergence

Outlook

- Three gluon vertex
 - explore other momentum configurations
- Four gluon vertex



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