The static potential in $2+1+1$-flavor QCD

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For the TUMQCD-collaboration

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QCD static energy of a quark-antiquark pair

- Conceptually an observable $O(\nu)$ is computed on the lattice at a sufficiently high scale ν in order to allow for a weak-coupling perturbative approach. This gives rise to the window problem:

$$\Lambda_{\mathrm{QCD}} \ll \nu \ll 1/a\,,$$

which is hard to realize in practice.
- The QCD static energy of a quark-antiquark pair, $E(r)$, is an important physical observable in the continuum and on the lattice.
- It has been studied by the TUMQCD-collaboration since 2010 using quenched[1] and 2+1-flavor[2] simulations and in this talk is extended to 2+1+1-flavor simulations.
- For the static energy, the scale is set by the inverse distance $\nu = 1/r$ and further scales may be involved (ultra-soft $\mu_{\mathrm{us}} \sim \alpha_{\mathrm{s}}/r$).

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[1] N. Brambilla et al. In: *Phys. Rev. Lett.* 105 (2010), 212001.

[2] A. Bazavov et al. In: *Phys. Rev. D* 86 (2012), 114031; A. Bazavov et al. In: *Phys. Rev. D* 90.7 (2014), 074038; A. Bazavov et al. In: *Phys. Rev. D* 100.11 (2019), 114511.

Static quark-antiquark energy in perturbation theory

- Static energy is determined from the large-time behavior of Wilson loops

$$E(r) = \lim_{t\to\infty} \frac{\mathrm{i}}{t}\langle \ln W(t,r)\rangle\,, \quad W(t,r) = \exp\left[\mathrm{i}g\oint_{r,t}\mathrm{d}x^\mu A_\mu(x)\right] \tag{1}$$

- In perturbation theory known up to $\mathrm{N^3LL}$

$$E(r) = \Lambda_\mathrm{s} - \frac{C_\mathrm{F}\alpha_\mathrm{s}}{r}\left(1 + \#\alpha_\mathrm{s} + \#\alpha_\mathrm{s}^2 + \#\alpha_\mathrm{s}^3 + \#\alpha_\mathrm{s}^3\ln\alpha_\mathrm{s} + \dots\right)\,. \tag{2}$$

- US contributions can be understood in pNRQCD[3]

$$E(r) = \Lambda_\mathrm{s} + V_\mathrm{s}(r,\nu,\mu_\mathrm{us}) + \delta_\mathrm{us}(r,\nu,\mu_\mathrm{us})\,. \tag{3}$$

- The color singlet static potential, V_s, is affected by an r-independent renormalon that is dealt with by reintegrating the force[4] (formulae in[5])

$$F(r,1/r) = \left.\frac{\partial E(r,\nu)}{\partial r}\right|_{\nu=1/r} \Rightarrow \quad E(r) = \int_{r^*}^{r}\mathrm{d}r' F(r',1/r') + \mathrm{const}\,. \tag{4}$$

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3 N. Brambilla et al. In: *Nucl. Phys. B* 566 (2000), 275.

4 S. Necco and R. Sommer. In: *Nucl. Phys. B* 622 (2002), 328–346.

5 X. G. Tormo I. In: *Mod. Phys. Lett. A* 28 (2013), 1330028.

Finite charm quark effects

- Including a massive quark of mass m to the static potential, $V^{(N_f)}$, with N_f massless flavors yields the potential[6], known at two loop accuracy

$$V_m^{(N_f)}(r,\nu) = V^{(N_f+1)}(r,\nu) + \delta V^{(N_f+1)}(r,\nu)\,, \tag{5}$$

  where the *rhs* can be rewritten as a correction to the potential with N_f massless flavors (typo-corrected formulae to be found in[7]).
- Satisfies the limits at two loop accuracy

$$V_m^{(N_f)}(r,\nu) \underset{m\to\infty}{\longrightarrow} V^{(N_f)}(r,\nu) + \mathcal{O}((\alpha_s)^4)\,, \tag{6}$$

$$V_m^{(N_f)}(r,\nu) \underset{m\to 0}{\longrightarrow} V^{(N_f+1)}(r,\nu) + \mathcal{O}((\alpha_s)^4)\,. \tag{7}$$

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[6] D. Eiras and J. Soto. In: *Phys. Rev. D* 61 (2000), 114027; M. Melles. In: *Phys. Rev. D* 62 (2000), 074019; D. Eiras and J. Soto. In: *Phys. Lett. B* 491 (2000), 101–110; M. Melles. In: *Nucl. Phys. B Proc. Suppl.* 96 (2001), 472–476; A. H. Hoang. In: *hep-ph/0008102* (2000).

[7] S. Recksiegel and Y. Sumino. In: *Phys. Rev. D* 65 (2002), 054018.

Static energy on the lattice

- We extract the static energy from the correlator of two Wilson lines in Coulomb gauge and extract the ground state.
- We use the (rooted) Highly Improved Staggered Quark (HISQ)[8] action and a one-loop Symanzik-improved (Lüscher–Weisz) gauge action.
- Lattices are MILC ones[9] used in many state of the art calculations.
- We employ tree-level improvement for the distance to correct for artifacts.
- We also additionally simulate one step of hyper-cubic (HYP) smearing[10].
- Ensembles (focus on the physical $m_l/m_s = 1/27$ case in the following):

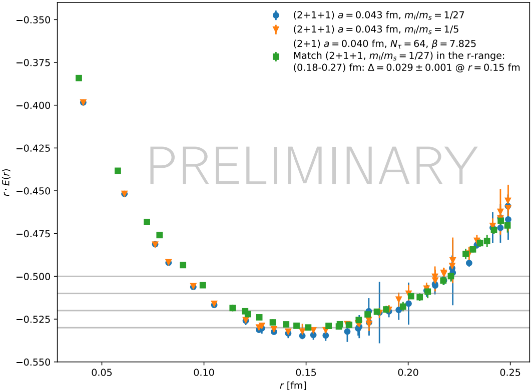
β	$N_\sigma^3 \times N_\tau$	a [fm]	m_l/m_s	r_{max} [fm]
7.0	$64^3 \times 192$	0.043	1/5	0.852
7.0	$144^3 \times 288$	0.043	1/27	0.852
7.28	$96^3 \times 288$	0.032	1/5	0.708

8 E. Follana et al. In: *Phys. Rev. D* 75 (2007), 054502.

9 A. Bazavov et al. In: *Phys. Rev. D* 98.7 (2018), 074512.

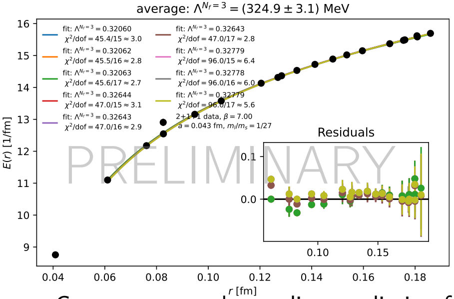
10 A. Hasenfratz and F. Knechtli. In: *Phys. Rev. D* 64 (2001), 034504.

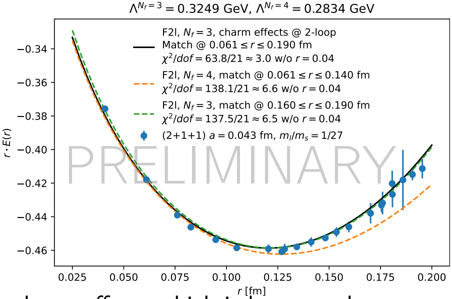
Static energy on the lattice - comparison



- At large $r \gg 1/m_c \sim 0.15$ fm, 2+1 and 2+1+1-flavor agree up to a constant because the charm quark decouples $\Rightarrow$ three-flavor running of α_s.
- At short distances $r \ll 1/m_c$ four-flavor running of α_s.
- The effects due to the different light-quark masses are smaller than the clearly visible effect due to the additional charm-quark at these distances.

Charm-quark effects in the data





- Compare to weak-coupling prediction for charm effects which is known only at two-loop accuracy.
- Determine sensible $\Lambda_{\overline{\mathrm{MS}}}$ using $m_c = 1.28$ GeV and three-loop running of α_s. Turns out $\sim 5\%$ higher than the 2019 2+1-flavor determination[11], but compatible within perturbative truncation errors.
- Compare weak-coupling to lattice: $N_f = 4$ matched to data at smaller r range, $N_f = 3$ + massive charm matched to data at small to larger r range, $N_f = 3$ matched to data at larger r range.
- $N_f = 3$ + massive charm gives best description of the data and interpolates between limiting cases.

11 A. Bazavov et al. In: *Phys. Rev. D* 100.11 (2019), 114511.

Lattice scales and the string tension

- The static energy based lattice scales r_i/a, $i = 0, 1, 2$, have not been determined in 2+1+1-flavor QCD yet. They are defined via

$$(r/a)^2 F(r/a)\Big|_{r/a=r_i/a} = \begin{cases} 1.65\,, & i = 0 \\ 1.0\,, & i = 1 \\ 0.5\,, & i = 2 \end{cases}\,, \tag{8}$$

  which have the continuum values

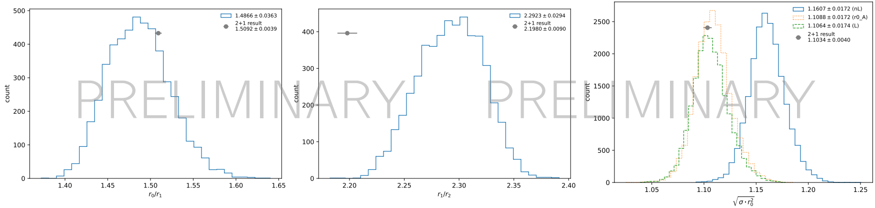
$$r_0 \sim 0.475 \text{ fm}\,, \quad r_1 \sim 0.3106 \text{ fm}\,, \quad r_2 \sim 0.145 \text{ fm} \sim 1/m_c\,.$$

- A general fit ansatz is given by the Cornell parametrization ($r \equiv r/a$)

$$aE(r) = -A/r + B + \sigma \cdot r\,, \quad \Rightarrow a^2 F(r) = A/r^2 + \sigma\,, \tag{9}$$

  where σ would correspond to the string tension at sufficiently large distances.
- Generate pseudo-random resamples of the data.
- For each r_i choose a range $\pm 15\%$ around the continuum value.
- For the σ determination, vary r_{min} around 0.6 fm and use different values for the A-parameter (0, A_{r_0}, $\pi/12$).
- Thin out the data in order to invert the correlation matrix (i) equidistant or (ii) randomly (10 points for r_i, 15 points for σ, keeping end points).

Determination results ($\beta = 7.0$, $m_l/m_s = 1/27$)



- We get consistent results for the scales r_i/a using the two procedures.
- The ratio r_0/r_1 should not be affected by the massive charm-quark. The result shown here is in good agreement with a previous 2+1-flavor determination[12]. This shows the decoupling at large distances.
- The ratio r_1/r_2 should be affected by the massive charm-quark. The result shown here deviates from the 2+1-flavor determination[13].
- Our value for $\sqrt{\sigma \cdot r_0^2}$ from the smeared ensemble agrees well with a determination[14] using up to 1 fm and the Lüscher term.

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12 A. Bazavov et al. In: *Phys. Rev. D* 90 (2014), 094503.

13 A. Bazavov, P. Petreczky, and J. H. Weber. In: *Phys. Rev. D* 97.1 (2018), 014510.

14 M. Cheng et al. In: *Phys. Rev. D* 77 (2008), 014511.

Summary

- We can see effects of a non-zero charm-quark mass in the data that is qualitatively described by perturbation theory at two-loop accuracy.
- $N_f = 3$ + massive charm interpolates between $N_f = 4$ and $N_f = 3$, similar in weak coupling and on the lattice.
- We can see decoupling of the charm-quark for the first time:
  - Short distance data ($r \ll 1/m_c$) is well described by $N_f = 3$ + massive charm and $N_f = 4$.
  - Long distance data ($r \gg 1/m_c$) is well described by $N_f = 3$.
- We get overall consistent estimates for the scales r_i/a and of $\sqrt{\sigma \cdot r_i^2}$ for the first time in 2+1+1-flavor simulations.
- We obtain consistent results within errors of the lattice scales r_0/a and r_1/a, or r_0/r_1, as well as the string tension $\sqrt{\sigma \cdot r_0^2}$ for different number of flavors.
- r_1/r_2 deviates due to the effects from the massive charm-quark on r_2/a.
- For the string tension we can see some dependence on the choices of A and $r_{\min}$ and effects due to $r_{\max} < 1$ fm.

- All results are still preliminary at this point and we do not yet have the final error budget.

Backup slides

Static energy on the lattice

- Extract the static energy from the Wilson line correlator (in Coulomb gauge)

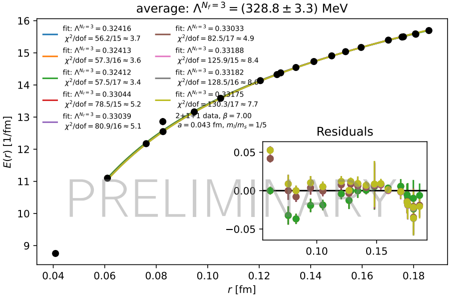
$$W(\vec{r}/a, t/a) = \prod_{u=0}^{t/a} U_4(\vec{r}/a, u)\,, \tag{10}$$

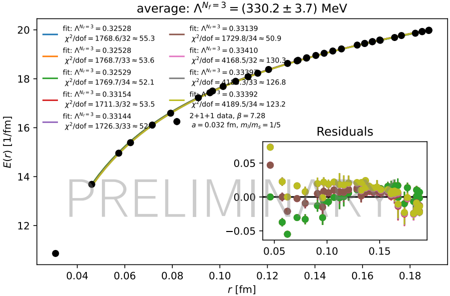
$$C(\vec{r}/a, t/a) = \left\langle \sum_{\vec{y}=R(\vec{r})} \frac{1}{N_c\, N_{\vec{r}}} \mathrm{Tr}\left[W^\dagger(\vec{y}/a, t/a) W(\vec{0}, t/a) \right] \right\rangle\,, \tag{11}$$

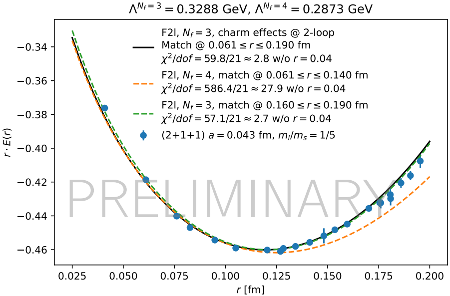
$$= \sum_{n=0}^{\infty} C_n(\vec{r}/a) \left(\mathrm{e}^{-tE_n(\vec{r}/a)} + \mathrm{e}^{-(aN_\tau - t)E_n(\vec{r}/a)} \right)\,, \tag{12}$$

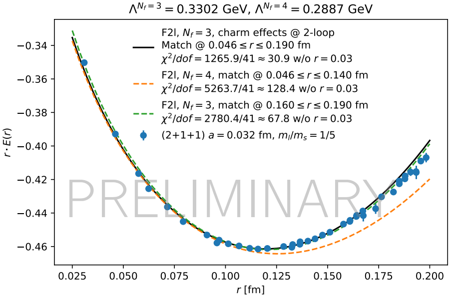
  where U_4 is a temporal link, $N_c = 3$, the sum is over all distances $\vec{y}$ that are a cubic rotation-reflection R of $\vec{r}$, $N_{\vec{r}}$ is the number of such operations, and N_τ is the temporal extent of the lattice.
- We are only interested in the lowest-lying state, $aE_0(\vec{r}/a)$ which we get using multi-exponential fits, i.e., truncating the above sum to a finite number of states over a time interval $[t_{\mathrm{min}}, t_{\mathrm{max}}]$.
- We do not have the backwards-propagating state with the static $Q\bar{Q}$.

Charm-quark effects in the data

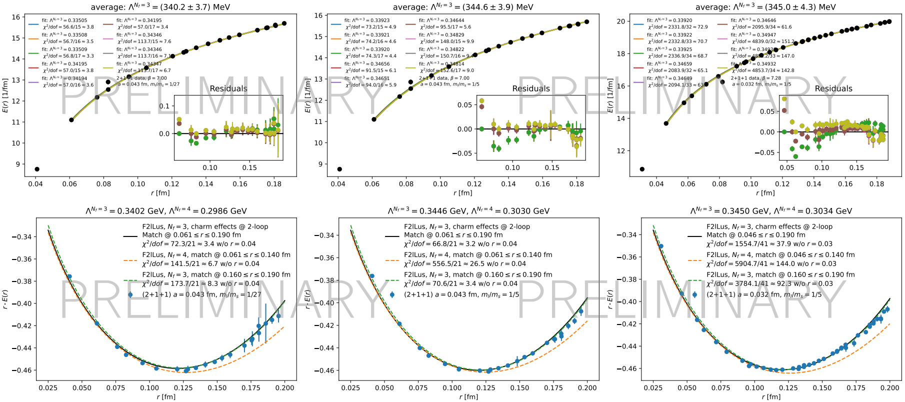






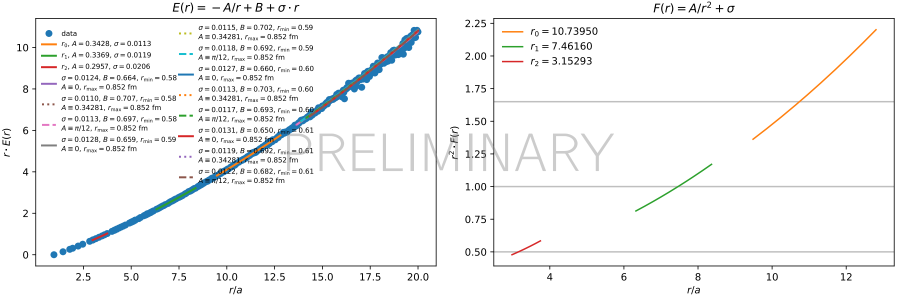


F2lLus



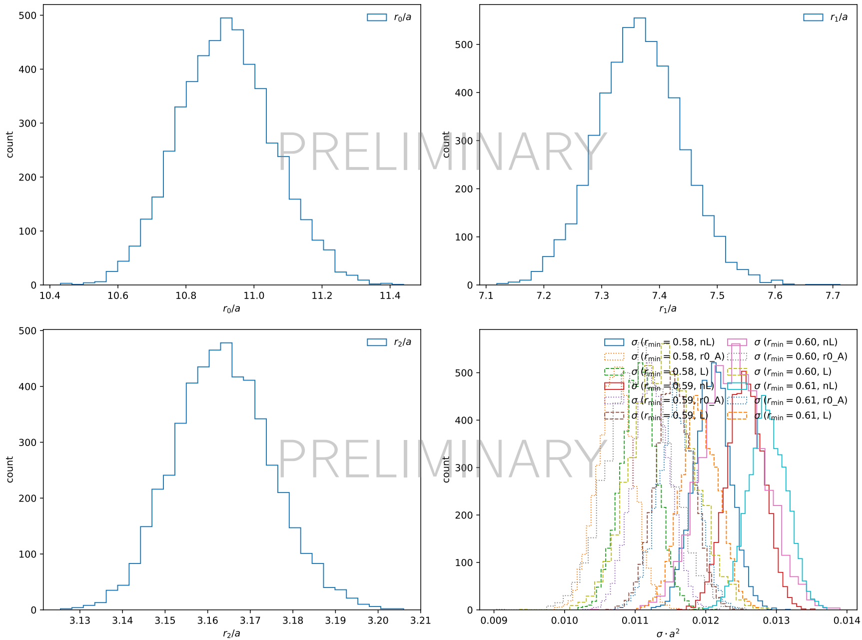
average: $\Lambda^{N_f=3} = (340.2 \pm 3.7)$ MeV
average: $\Lambda^{N_f=3} = (344.6 \pm 3.9)$ MeV
average: $\Lambda^{N_f=3} = (345.0 \pm 4.3)$ MeV
Residuals
PRELIMINARY
$E(r)$ [1/fm]
r [fm]
$\Lambda^{N_f=3} = 0.3402$ GeV, $\Lambda^{N_f=4} = 0.2986$ GeV
F2lLus, $N_f = 3$, charm effects @ 2-loop
Match @ $0.061 \le r \le 0.190$ fm
$\chi^2/dof = 72.3/21 \approx 3.4$ w/o $r = 0.04$
F2lLus, $N_f = 4$, match @ $0.061 \le r \le 0.140$ fm
$\chi^2/dof = 141.5/21 \approx 6.7$ w/o $r = 0.04$
F2lLus, $N_f = 3$, match @ $0.160 \le r \le 0.190$ fm
$\chi^2/dof = 173.7/21 \approx 8.3$ w/o $r = 0.04$
(2+1+1) $a = 0.043$ fm, $m_l/m_s = 1/27$
$\Lambda^{N_f=3} = 0.3446$ GeV, $\Lambda^{N_f=4} = 0.3030$ GeV
F2lLus, $N_f = 3$, charm effects @ 2-loop
Match @ $0.061 \le r \le 0.190$ fm
$\chi^2/dof = 66.8/21 \approx 3.2$ w/o $r = 0.04$
F2lLus, $N_f = 4$, match @ $0.061 \le r \le 0.140$ fm
$\chi^2/dof = 556.5/21 \approx 26.5$ w/o $r = 0.04$
F2lLus, $N_f = 3$, match @ $0.160 \le r \le 0.190$ fm
$\chi^2/dof = 70.6/21 \approx 3.4$ w/o $r = 0.04$
(2+1+1) $a = 0.043$ fm, $m_l/m_s = 1/5$
$\Lambda^{N_f=3} = 0.3450$ GeV, $\Lambda^{N_f=4} = 0.3034$ GeV
F2lLus, $N_f = 3$, charm effects @ 2-loop
Match @ $0.046 \le r \le 0.190$ fm
$\chi^2/dof = 1554.7/41 \approx 37.9$ w/o $r = 0.03$
F2lLus, $N_f = 4$, match @ $0.046 \le r \le 0.140$ fm
$\chi^2/dof = 5904.7/41 \approx 144.0$ w/o $r = 0.03$
F2lLus, $N_f = 3$, match @ $0.160 \le r \le 0.190$ fm
$\chi^2/dof = 3784.1/41 \approx 92.3$ w/o $r = 0.03$
(2+1+1) $a = 0.032$ fm, $m_l/m_s = 1/5$
$r \cdot E(r)$

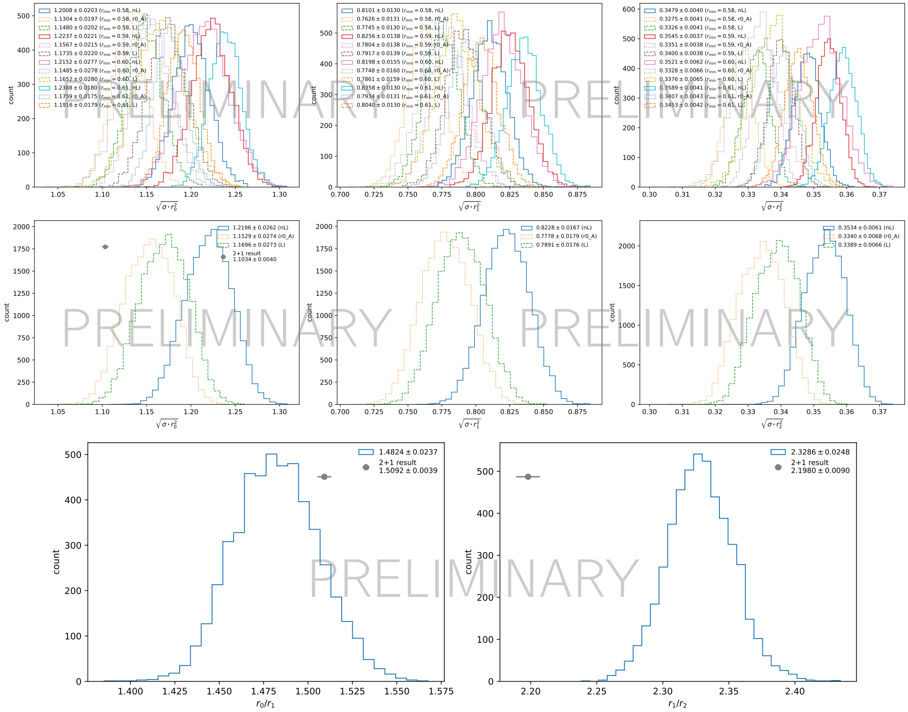
Determination procedure



- Generate pseudo-random resamples of the data that are distributed normally, taking into account the full correlation matrix.
- For each r_i choose a range $\pm 15\%$ around the continuum value and fit a Cornell. Eliminate parameter B by normalizing $aE(1,0,0) \equiv 0$. Read off r_i from the $r^2 F$ plot.
- For the determination of σ, vary r_{min} around 0.6 fm and use different constraints for the A-parameter: setting it to zero, using the value from the r_0-determination, and setting it to $\pi/12$ (Lüscher term).
- Thin out the data in order to invert the correlation matrix (i) equidistant or (ii) randomly (10 points for r_i, 15 points for σ, keeping end points).

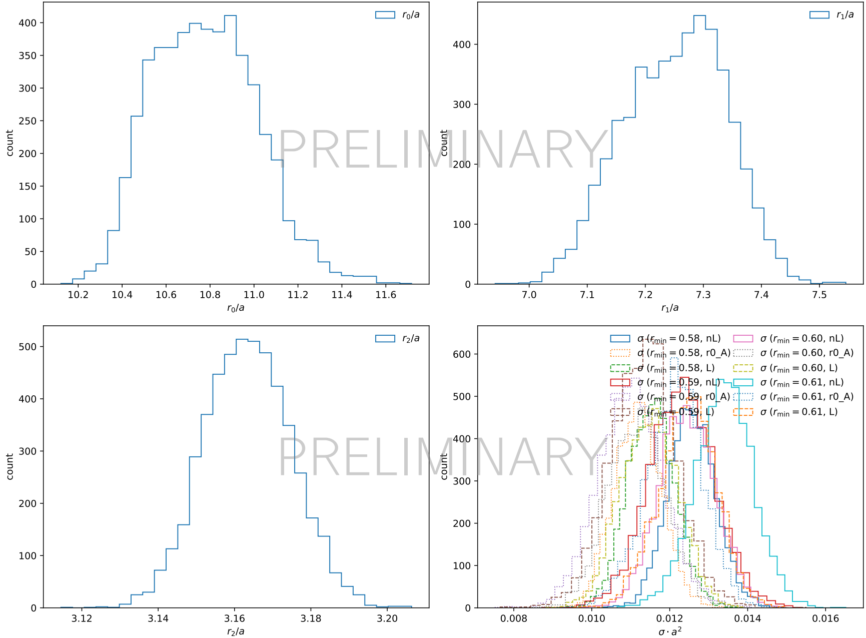
Method 1 ($\beta = 7.0$, $m_l/m_s = 1/27$)

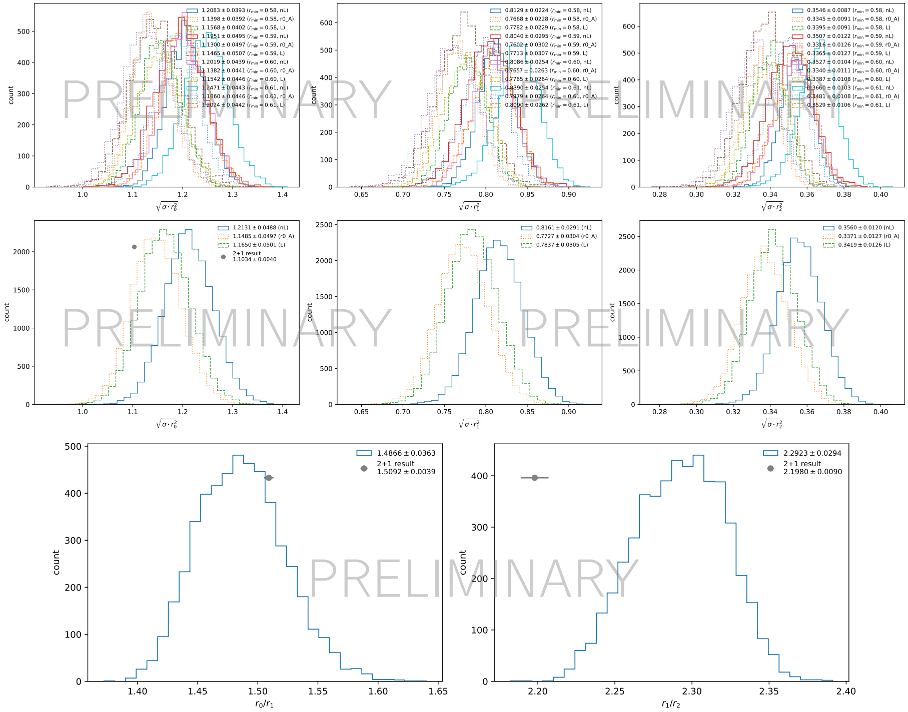




1.2008 ± 0.0203 (r_{min} = 0.58, nL)
1.1304 ± 0.0197 (r_{min} = 0.58, r0_A)
1.1480 ± 0.0202 (r_{min} = 0.58, L)
1.2237 ± 0.0221 (r_{min} = 0.59, nL)
1.1567 ± 0.0215 (r_{min} = 0.59, r0_A)
1.1735 ± 0.0220 (r_{min} = 0.59, L)
1.2152 ± 0.0277 (r_{min} = 0.60, nL)
1.1485 ± 0.0278 (r_{min} = 0.60, r0_A)
1.1652 ± 0.0280 (r_{min} = 0.60, L)
1.2388 ± 0.0180 (r_{min} = 0.61, nL)
1.1759 ± 0.0175 (r_{min} = 0.61, r0_A)
1.1916 ± 0.0179 (r_{min} = 0.61, L)
0.8101 ± 0.0130 (r_{min} = 0.58, nL)
0.7626 ± 0.0131 (r_{min} = 0.58, r0_A)
0.7745 ± 0.0130 (r_{min} = 0.58, L)
0.8256 ± 0.0138 (r_{min} = 0.59, nL)
0.7804 ± 0.0138 (r_{min} = 0.59, r0_A)
0.7917 ± 0.0139 (r_{min} = 0.59, L)
0.8198 ± 0.0155 (r_{min} = 0.60, nL)
0.7748 ± 0.0160 (r_{min} = 0.60, r0_A)
0.7861 ± 0.0159 (r_{min} = 0.60, L)
0.8358 ± 0.0130 (r_{min} = 0.61, nL)
0.7934 ± 0.0131 (r_{min} = 0.61, r0_A)
0.8040 ± 0.0130 (r_{min} = 0.61, L)
0.3479 ± 0.0040 (r_{min} = 0.58, nL)
0.3275 ± 0.0041 (r_{min} = 0.58, r0_A)
0.3326 ± 0.0041 (r_{min} = 0.58, L)
0.3545 ± 0.0037 (r_{min} = 0.59, nL)
0.3351 ± 0.0038 (r_{min} = 0.59, r0_A)
0.3400 ± 0.0038 (r_{min} = 0.59, L)
0.3521 ± 0.0062 (r_{min} = 0.60, nL)
0.3328 ± 0.0066 (r_{min} = 0.60, r0_A)
0.3376 ± 0.0065 (r_{min} = 0.60, L)
0.3589 ± 0.0041 (r_{min} = 0.61, nL)
0.3407 ± 0.0043 (r_{min} = 0.61, r0_A)
0.3453 ± 0.0042 (r_{min} = 0.61, L)
1.2196 ± 0.0262 (nL)
1.1529 ± 0.0274 (r0_A)
1.1696 ± 0.0273 (L)
2+1 result
1.1034 ± 0.0040
0.8228 ± 0.0167 (nL)
0.7778 ± 0.0179 (r0_A)
0.7891 ± 0.0176 (L)
0.3534 ± 0.0061 (nL)
0.3340 ± 0.0068 (r0_A)
0.3389 ± 0.0066 (L)
1.4824 ± 0.0237
2+1 result
1.5092 ± 0.0039
2.3286 ± 0.0248
2+1 result
2.1980 ± 0.0090
count
$\sqrt{\sigma \cdot r_0^2}$
$\sqrt{\sigma \cdot r_1^2}$
$\sqrt{\sigma \cdot r_2^2}$
r_0/r_1
r_1/r_2
PRELIMINARY

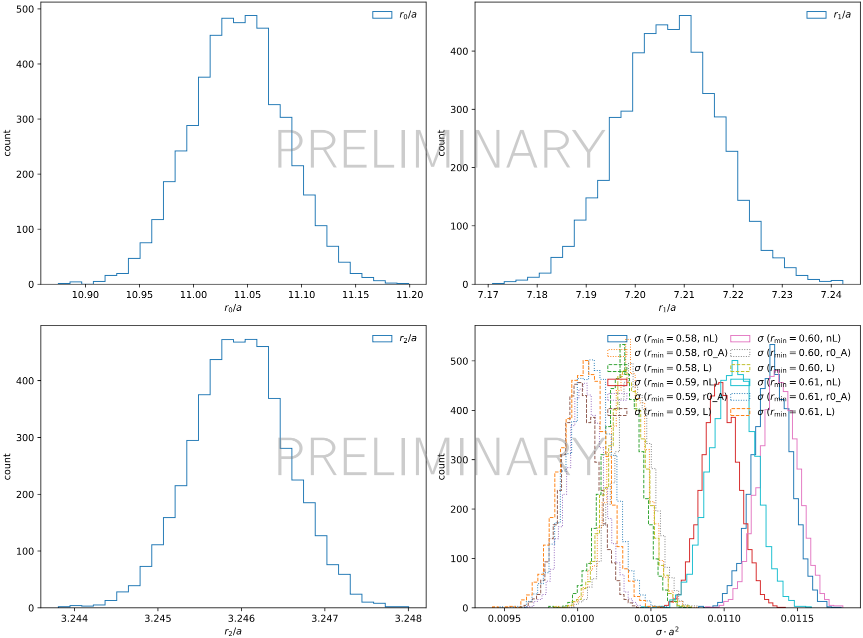
Method 2 ($\beta = 7.0$, $m_l/m_s = 1/27$)

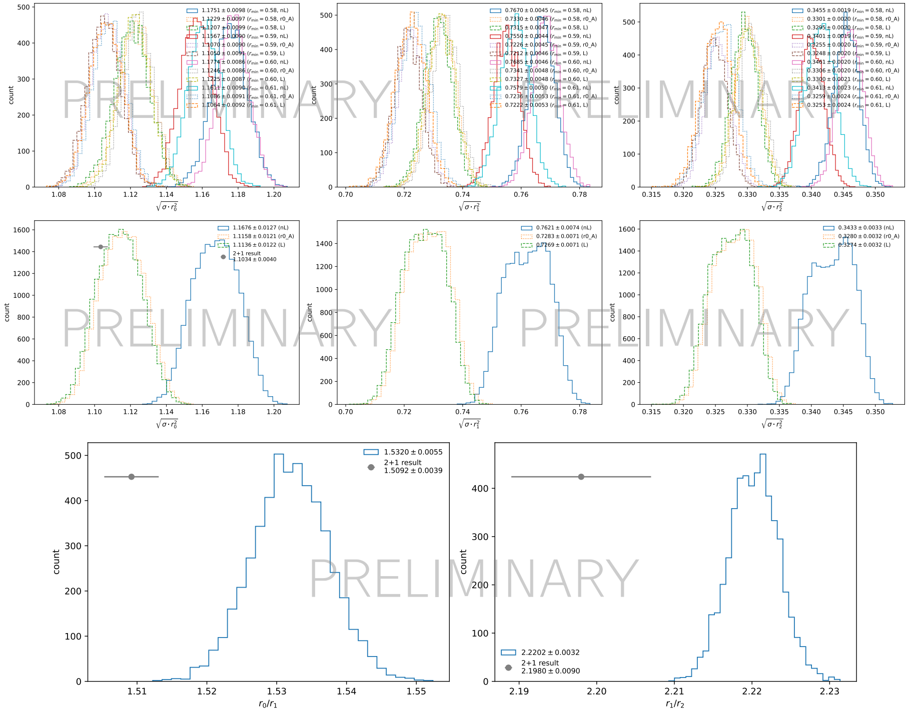




1.2083 ± 0.0393 (r_{min} = 0.58, nL)
1.1398 ± 0.0392 (r_{min} = 0.58, r0_A)
1.1568 ± 0.0402 (r_{min} = 0.58, L)
1.1951 ± 0.0495 (r_{min} = 0.59, nL)
1.1300 ± 0.0497 (r_{min} = 0.59, r0_A)
1.1465 ± 0.0507 (r_{min} = 0.59, L)
1.2019 ± 0.0439 (r_{min} = 0.60, nL)
1.1382 ± 0.0441 (r_{min} = 0.60, r0_A)
1.1542 ± 0.0446 (r_{min} = 0.60, L)
1.2471 ± 0.0443 (r_{min} = 0.61, nL)
1.1860 ± 0.0446 (r_{min} = 0.61, r0_A)
1.2024 ± 0.0442 (r_{min} = 0.61, L)
count
$\sqrt{\sigma \cdot r_0^2}$
0.8129 ± 0.0224 (r_{min} = 0.58, nL)
0.7668 ± 0.0228 (r_{min} = 0.58, r0_A)
0.7782 ± 0.0229 (r_{min} = 0.58, L)
0.8040 ± 0.0295 (r_{min} = 0.59, nL)
0.7602 ± 0.0302 (r_{min} = 0.59, r0_A)
0.7713 ± 0.0307 (r_{min} = 0.59, L)
0.8086 ± 0.0254 (r_{min} = 0.60, nL)
0.7657 ± 0.0263 (r_{min} = 0.60, r0_A)
0.7765 ± 0.0264 (r_{min} = 0.60, L)
0.8390 ± 0.0254 (r_{min} = 0.61, nL)
0.7979 ± 0.0264 (r_{min} = 0.61, r0_A)
0.8090 ± 0.0262 (r_{min} = 0.61, L)
count
$\sqrt{\sigma \cdot r_1^2}$
0.3546 ± 0.0087 (r_{min} = 0.58, nL)
0.3345 ± 0.0091 (r_{min} = 0.58, r0_A)
0.3395 ± 0.0091 (r_{min} = 0.58, L)
0.3507 ± 0.0122 (r_{min} = 0.59, nL)
0.3316 ± 0.0126 (r_{min} = 0.59, r0_A)
0.3365 ± 0.0127 (r_{min} = 0.59, L)
0.3527 ± 0.0104 (r_{min} = 0.60, nL)
0.3340 ± 0.0111 (r_{min} = 0.60, r0_A)
0.3387 ± 0.0108 (r_{min} = 0.60, L)
0.3660 ± 0.0103 (r_{min} = 0.61, nL)
0.3481 ± 0.0108 (r_{min} = 0.61, r0_A)
0.3529 ± 0.0106 (r_{min} = 0.61, L)
count
$\sqrt{\sigma \cdot r_2^2}$
1.2131 ± 0.0488 (nL)
1.1485 ± 0.0497 (r0_A)
1.1650 ± 0.0501 (L)
2+1 result
1.1034 ± 0.0040
count
$\sqrt{\sigma \cdot r_0^2}$
0.8161 ± 0.0291 (nL)
0.7727 ± 0.0304 (r0_A)
0.7837 ± 0.0305 (L)
count
$\sqrt{\sigma \cdot r_1^2}$
0.3560 ± 0.0120 (nL)
0.3371 ± 0.0127 (r0_A)
0.3419 ± 0.0126 (L)
count
$\sqrt{\sigma \cdot r_2^2}$
1.4866 ± 0.0363
2+1 result
1.5092 ± 0.0039
count
r_0/r_1
2.2923 ± 0.0294
2+1 result
2.1980 ± 0.0090
count
r_1/r_2
PRELIMINARY

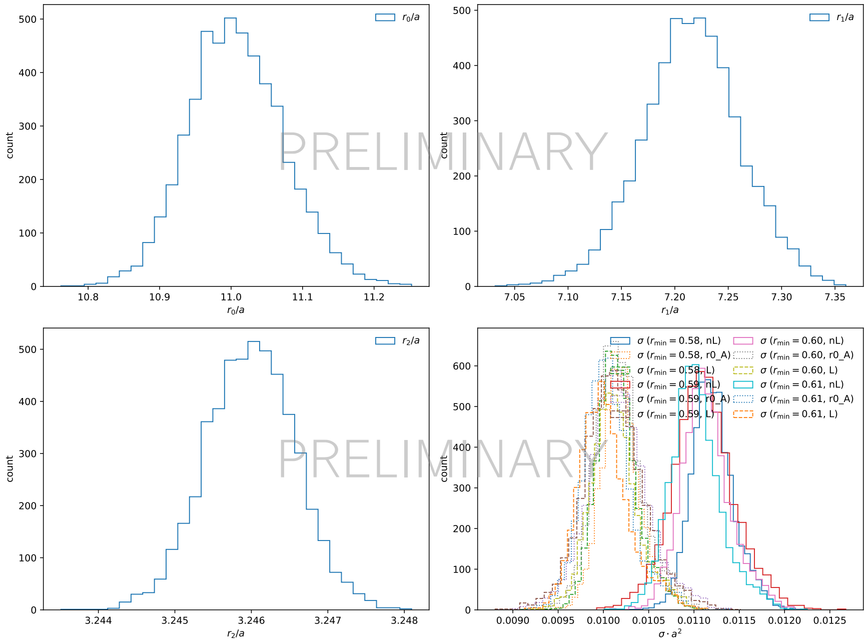
Smeared method 1 ($\beta = 7.0$, $m_l/m_s = 1/27$)

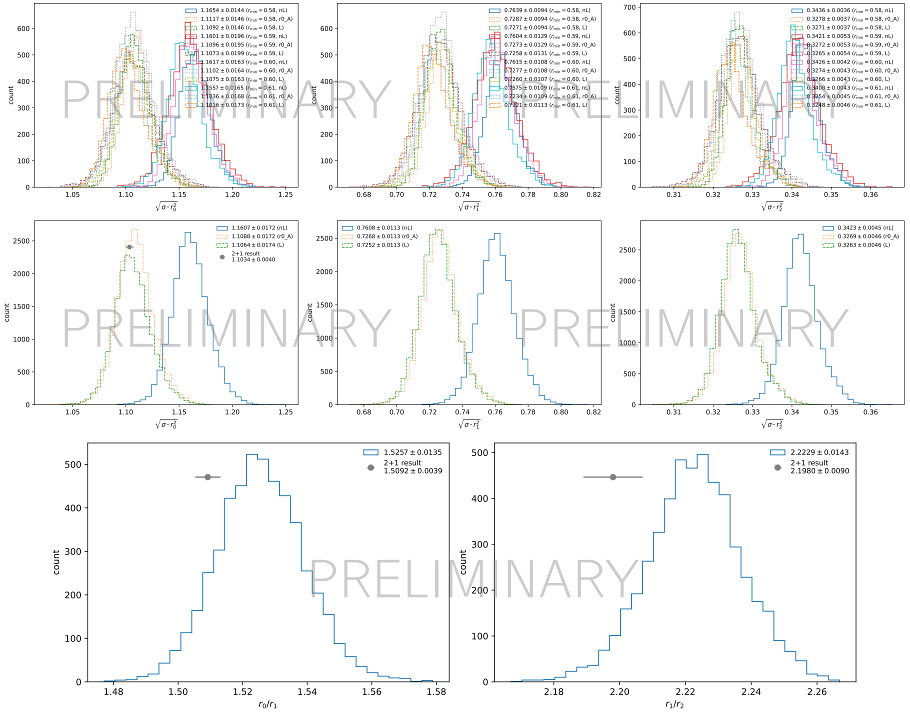




1.1751 ± 0.0098 ($r_{\min}$ = 0.58, nL)
1.1229 ± 0.0097 ($r_{\min}$ = 0.58, r0_A)
1.1207 ± 0.0099 ($r_{\min}$ = 0.58, L)
1.1567 ± 0.0090 ($r_{\min}$ = 0.59, nL)
1.1070 ± 0.0090 ($r_{\min}$ = 0.59, r0_A)
1.1050 ± 0.0091 ($r_{\min}$ = 0.59, L)
1.1774 ± 0.0086 ($r_{\min}$ = 0.60, nL)
1.1246 ± 0.0086 ($r_{\min}$ = 0.60, r0_A)
1.1225 ± 0.0087 ($r_{\min}$ = 0.60, L)
1.1611 ± 0.0090 ($r_{\min}$ = 0.61, nL)
1.1086 ± 0.0091 ($r_{\min}$ = 0.61, r0_A)
1.1064 ± 0.0092 ($r_{\min}$ = 0.61, L)
count
$\sqrt{\sigma \cdot r_0^2}$
0.7670 ± 0.0045 ($r_{\min}$ = 0.58, nL)
0.7330 ± 0.0046 ($r_{\min}$ = 0.58, r0_A)
0.7315 ± 0.0047 ($r_{\min}$ = 0.58, L)
0.7550 ± 0.0044 ($r_{\min}$ = 0.59, nL)
0.7226 ± 0.0045 ($r_{\min}$ = 0.59, r0_A)
0.7212 ± 0.0046 ($r_{\min}$ = 0.59, L)
0.7685 ± 0.0046 ($r_{\min}$ = 0.60, nL)
0.7341 ± 0.0048 ($r_{\min}$ = 0.60, r0_A)
0.7327 ± 0.0048 ($r_{\min}$ = 0.60, L)
0.7579 ± 0.0050 ($r_{\min}$ = 0.61, nL)
0.7236 ± 0.0053 ($r_{\min}$ = 0.61, r0_A)
0.7222 ± 0.0053 ($r_{\min}$ = 0.61, L)
count
$\sqrt{\sigma \cdot r_1^2}$
0.3455 ± 0.0019 ($r_{\min}$ = 0.58, nL)
0.3301 ± 0.0020 ($r_{\min}$ = 0.58, r0_A)
0.3295 ± 0.0020 ($r_{\min}$ = 0.58, L)
0.3401 ± 0.0019 ($r_{\min}$ = 0.59, nL)
0.3255 ± 0.0020 ($r_{\min}$ = 0.59, r0_A)
0.3248 ± 0.0020 ($r_{\min}$ = 0.59, L)
0.3461 ± 0.0020 ($r_{\min}$ = 0.60, nL)
0.3306 ± 0.0020 ($r_{\min}$ = 0.60, r0_A)
0.3300 ± 0.0021 ($r_{\min}$ = 0.60, L)
0.3413 ± 0.0023 ($r_{\min}$ = 0.61, nL)
0.3259 ± 0.0024 ($r_{\min}$ = 0.61, r0_A)
0.3253 ± 0.0024 ($r_{\min}$ = 0.61, L)
count
$\sqrt{\sigma \cdot r_2^2}$
1.1676 ± 0.0127 (nL)
1.1158 ± 0.0121 (r0_A)
1.1136 ± 0.0122 (L)
2+1 result
1.1034 ± 0.0040
count
$\sqrt{\sigma \cdot r_0^2}$
0.7621 ± 0.0074 (nL)
0.7283 ± 0.0071 (r0_A)
0.7269 ± 0.0071 (L)
count
$\sqrt{\sigma \cdot r_1^2}$
0.3433 ± 0.0033 (nL)
0.3280 ± 0.0032 (r0_A)
0.3274 ± 0.0032 (L)
count
$\sqrt{\sigma \cdot r_2^2}$
1.5320 ± 0.0055
2+1 result
1.5092 ± 0.0039
count
r_0/r_1
2.2202 ± 0.0032
2+1 result
2.1980 ± 0.0090
count
r_1/r_2
PRELIMINARY

Smeared method 2 ($\beta = 7.0$, $m_l/m_s = 1/27$)





1.1654 ± 0.0144 (r_{min} = 0.58, nL)
1.1117 ± 0.0146 (r_{min} = 0.58, r0_A)
1.1092 ± 0.0146 (r_{min} = 0.58, L)
1.1601 ± 0.0196 (r_{min} = 0.59, nL)
1.1096 ± 0.0195 (r_{min} = 0.59, r0_A)
1.1073 ± 0.0199 (r_{min} = 0.59, L)
1.1617 ± 0.0163 (r_{min} = 0.60, nL)
1.1102 ± 0.0164 (r_{min} = 0.60, r0_A)
1.1075 ± 0.0163 (r_{min} = 0.60, L)
1.1557 ± 0.0165 (r_{min} = 0.61, nL)
1.1036 ± 0.0168 (r_{min} = 0.61, r0_A)
1.1016 ± 0.0173 (r_{min} = 0.61, L)
count
$\sqrt{\sigma \cdot r_0^2}$
0.7639 ± 0.0094 (r_{min} = 0.58, nL)
0.7287 ± 0.0094 (r_{min} = 0.58, r0_A)
0.7271 ± 0.0094 (r_{min} = 0.58, L)
0.7604 ± 0.0129 (r_{min} = 0.59, nL)
0.7273 ± 0.0129 (r_{min} = 0.59, r0_A)
0.7258 ± 0.0131 (r_{min} = 0.59, L)
0.7615 ± 0.0108 (r_{min} = 0.60, nL)
0.7277 ± 0.0108 (r_{min} = 0.60, r0_A)
0.7260 ± 0.0107 (r_{min} = 0.60, L)
0.7575 ± 0.0109 (r_{min} = 0.61, nL)
0.7234 ± 0.0109 (r_{min} = 0.61, r0_A)
0.7221 ± 0.0113 (r_{min} = 0.61, L)
$\sqrt{\sigma \cdot r_1^2}$
0.3436 ± 0.0036 (r_{min} = 0.58, nL)
0.3278 ± 0.0037 (r_{min} = 0.58, r0_A)
0.3271 ± 0.0037 (r_{min} = 0.58, L)
0.3421 ± 0.0053 (r_{min} = 0.59, nL)
0.3272 ± 0.0053 (r_{min} = 0.59, r0_A)
0.3265 ± 0.0054 (r_{min} = 0.59, L)
0.3426 ± 0.0042 (r_{min} = 0.60, nL)
0.3274 ± 0.0043 (r_{min} = 0.60, r0_A)
0.3266 ± 0.0043 (r_{min} = 0.60, L)
0.3408 ± 0.0043 (r_{min} = 0.61, nL)
0.3254 ± 0.0045 (r_{min} = 0.61, r0_A)
0.3248 ± 0.0046 (r_{min} = 0.61, L)
$\sqrt{\sigma \cdot r_2^2}$
1.1607 ± 0.0172 (nL)
1.1088 ± 0.0172 (r0_A)
1.1064 ± 0.0174 (L)
2+1 result
1.1034 ± 0.0040
0.7608 ± 0.0113 (nL)
0.7268 ± 0.0113 (r0_A)
0.7252 ± 0.0113 (L)
0.3423 ± 0.0045 (nL)
0.3269 ± 0.0046 (r0_A)
0.3263 ± 0.0046 (L)
1.5257 ± 0.0135
2+1 result
1.5092 ± 0.0039
r_0/r_1
2.2229 ± 0.0143
2+1 result
2.1980 ± 0.0090
r_1/r_2
PRELIMINARY