# Quark–gluon vertex with 2 flavours of $\mathcal{O}(a)$-improved Wilson fermions

Jon-Ivar Skullerud

with Ayşe Kızılersü, Orlando Oliveira, Paulo Silva, André Sternbeck

Maynooth University

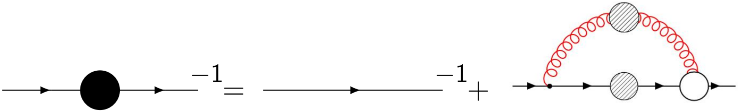
Lattice 21, 29 July 2021

Based on PRD103(2021)114515 [arXiv:2102.02945]

# Outline

# Preliminaries

## Dyson–Schwinger (gap) equation



- Inserting lattice gluon propagator in gap equation with a bare vertex gives insufficient $\chi$SB
- Abelian (Ball–Chiu) vertex also gives insufficient enhancement
- Vertex related to ghost self-energy via Slavnov–Taylor identity
- Nontrivial tensor structure may be crucial
- Effective charge most naturally defined from quark–gluon vertex?

# Tensor structure

12 independent form factors:
$\Gamma_\mu(p,q,k) = \sum_{i=1}^{4} \lambda_i(p^2,k^2,q^2) L_{i,\mu} + \sum_{i=5}^{12} \tau_i(p^2,k^2,q^2) T_{i,\mu}$

$$
\begin{aligned}
L_{1,\mu} &= \gamma_\mu & L_{2,\mu} &= -\not{P} P_\mu \\
L_{3,\mu} &= -iP_\mu & L_{4,\mu} &= -i\sigma_{\mu\nu} P_\nu \\
T_{1,\mu} &= -i\ell_\mu & T_{2,\mu} &= -\not{P}\ell_\mu \\
T_{3,\mu} &= \not{q} q_\mu - q^2\gamma_\mu & T_{4,\mu} &= -i\left[q^2\sigma_{\mu\nu}P_\nu + 2q_\mu\sigma_{\nu\lambda}p_\nu k_\lambda\right] \\
T_{5,\mu} &= -i\sigma_{\mu\nu}q_\nu & T_{6,\mu} &= (qP)\gamma_\mu - \not{q}P_\mu \\
T_{7,\mu} &= -\frac{i}{2}(qP)\sigma_{\mu\nu}P_\nu - iP_\mu\sigma_{\nu\lambda}p_\nu k_\lambda \\
T_{8,\mu} &= -\gamma_\mu\sigma_{\nu\lambda}p_\nu k_\lambda - \not{p}k_\mu + \not{k}p_\mu
\end{aligned}
$$

with $P_\mu \equiv p_\mu + k_\mu$, $\ell_\mu \equiv (pq)k_\mu - (kq)p_\mu$
$q$ = gluon momentum; $p, k$ = quark momenta

# Extracting form factors

Soft gluon kinematics ($q = 0$): 3 form factors

$$\Gamma_\mu(p) = \lambda_1(p^2)\gamma_\mu + 4\lambda_2(p^2)\not{p}p_\mu + 2i\lambda_3(p^2)p_\mu\,.$$

## Non-covariant

$$\lambda_1 = \mathrm{Tr}_4\,\gamma_\mu\Gamma_\mu\Big|_{p_\mu=0}$$

$$\lambda_2 = \frac{p_\mu p_\nu}{4p^2}\,\mathrm{Tr}_4\,\gamma_\nu\Gamma_\mu\Big|_{\nu\neq\mu}$$

$$\lambda_3 = \frac{p_\mu}{2ip^2}\,\mathrm{Tr}_4\,\Gamma_\mu$$

## Covariant

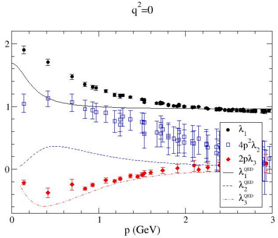
$$\lambda_1 = \frac{1}{3}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\mathrm{Tr}_4\,\gamma_\nu\Gamma_\mu$$

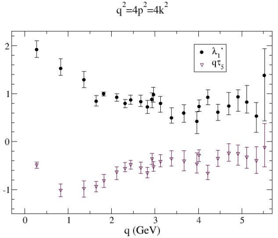
$$\lambda_2 = \frac{1}{12p^2}\left(\delta_{\mu\nu} - 4\frac{p_\mu p_\nu}{p^2}\right)\mathrm{Tr}_4\,\gamma_\nu\Gamma_\mu$$

$$\lambda_3 = \frac{p_\mu}{2ip^2}\,\mathrm{Tr}_4\,\Gamma_\mu$$

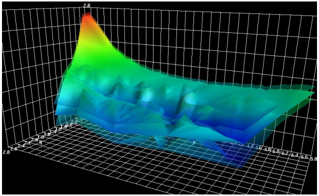
Lattice momenta: $p_\mu \to K_\mu(p) = \frac{1}{a}\sin(ap_\mu)$
where $p_\mu =$ Fourier mode.

# History: quenched quark–gluon vertex




- Enhancement of vector part over and above abelian
- No enhancement of scalar part?
- Substantial contribution from "chromomagnetic" term



$p^2 = k^2$

# Simulation parameters

Using Regensburg QCD collaboration ensembles ($N_f = 2$ clover)

| Name | $\beta$ | $\kappa$ | $a$ [fm] | $V$ | $m_\pi$ [MeV] | $N_{\text{cfg}}$ | $N_{\text{src}}$ |
|---|---|---|---|---|---|---|---|
| L08 | 5.20 | 0.13596 | 0.081 | $32^3 \times 64$ | 280 | 900 | 4 |
| H07 | 5.29 | 0.13620 | 0.071 | $32^3 \times 64$ | 422 | 900 | 4 |
| L07 | 5.29 | 0.13632 | 0.071 | $32^3 \times 64$ | 295 | 908 | 4 |
| L07-64 | 5.29 | 0.13632 | 0.071 | $64^3 \times 64$ | 290 | 750 | 2 |
| H06 | 5.40 | 0.13647 | 0.060 | $32^3 \times 64$ | 426 | 900 | 4 |
| Q07 | 6.16 | 0.1340 | 0.071 | $32^3 \times 64$ | 1000 | 998 | 4 |

Off-shell improved quark propagator

$$S_R(x,y) = \left\langle\left\langle (1 - c_q a \overrightarrow{\not{D}}(x)) S_0(x,y;U)(1 + c_q a \overleftarrow{\not{D}}(y)) \right\rangle\right\rangle$$

Fixed to Landau gauge with overrelaxation algorithm

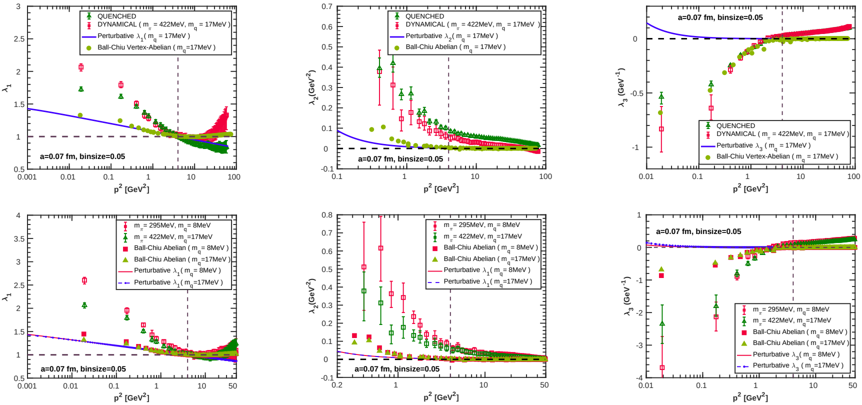
# Tree-level correction

$$\Gamma^{(0)}_\mu(p,0,p) = \frac{1}{\left(1+c_q^2a^2K^2(p)\right)^4}\times$$
$$\times\Bigg\{\gamma_{\boldsymbol{\mu}}\left[\left(1+c_q^2a^2K^2(p)\right)^2C_\mu(p)\right]$$
$$-4a^2K_\mu \not{K}(p)\left[2c_q^2C_\mu(p)-c_q\left(1-c_q^2a^2K^2(p)\right)\right]$$
$$-2iaK_\mu\Big[-2c_q^2a^2K^2(p)+\frac{1}{2}\left(1-c_q^2a^2K^2(p)\right)$$
$$-2c_q\left(1-c_q^2a^2K^2(p)\right)C_\mu(p)\Big]\Bigg\}$$

$$K_\mu(p)=\frac{1}{a}\sin(p_\mu a)\,,\qquad C_\mu(p)=\cos(p_\mu a)\,.$$

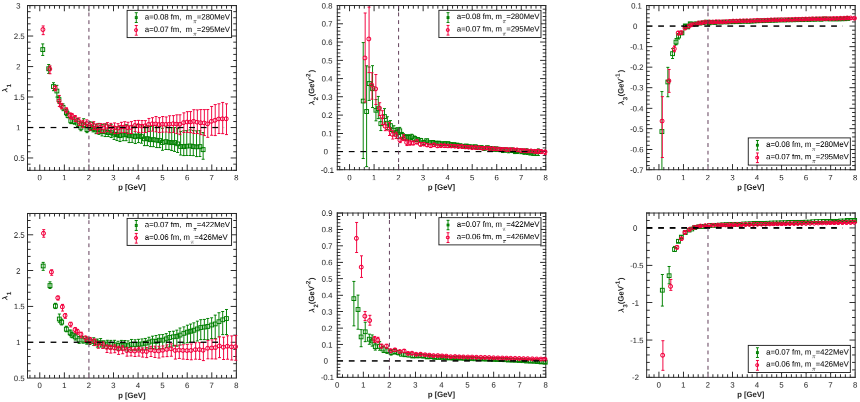
- Divide ($\lambda_1$) or subtract ($\lambda_{2,3}$) tree-level expression
- Two different lattice tensors for each of $L_{2\mu}$ and $L_{3\mu}$

# Quark mass and flavour dependence



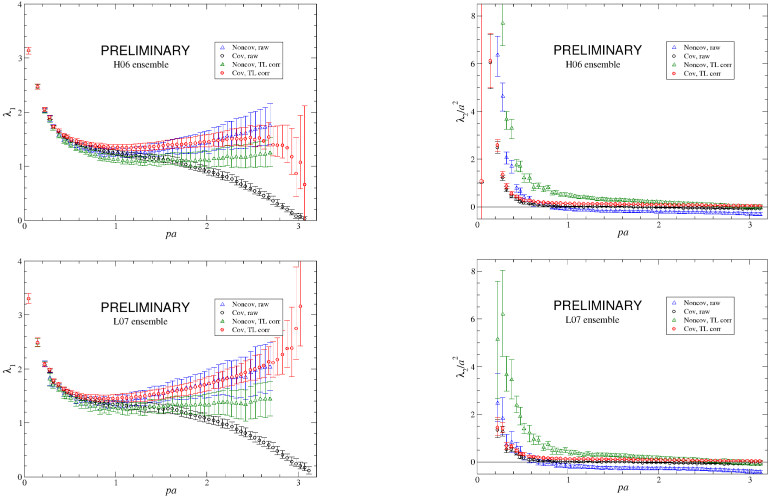

- Moderate quark mass and (un)quenching effects

# Lattice spacing dependence

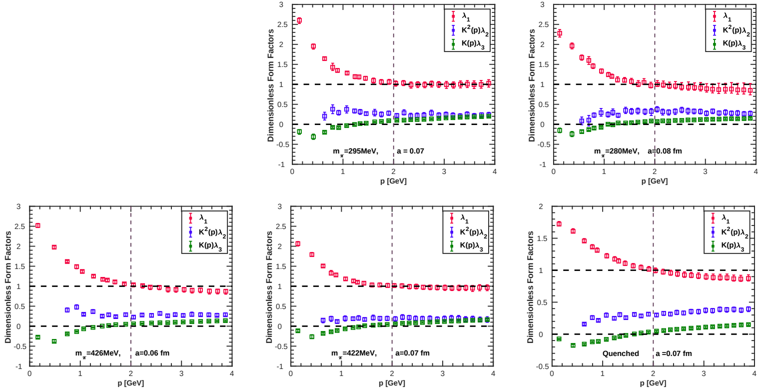



- Top: 290 MeV; Bottom: 420 MeV.
- Stronger IR enhancement with reduced lattice spacing

# Covariant vs non-covariant $\lambda_{1,2}$



# Contributions from $\lambda_{1,2,3}$




- Greater infrared enhancement with lighter quarks
- Moderate lattice spacing effects
- Significant contribution from $\chi$SB form factor $\lambda_3$

# Summary and outlook

- Enhancement in infrared — driver of chiral symmetry breaking
- Stronger enhancement
  - with dynamical quarks
  - for smaller quark masses
  - towards continuum limit
- Significant contribution from $\chi$SB form factor $\lambda_3$
- Covariant and non-covariant extraction

  $\lambda_1$: Good agreement
  $\lambda_2$: Discrepancy needs investigation
- Other kinematics, form factors in progress