From tensors to qubits

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- Y. M., R. Sakai, J. Unmuth-Yockey, Tensor LFT with applications to quantum computing, arXiv:2010.06539; in revision for RMP.
- Y. M., QFT: a quantum computation approach, IoP book, 2021.
- Y. M., PRD 100, 014506 (2019) and PRD 102 014506 (2020).
- E. Gustafson, Y. M., J. Unmuth-Yockey, PRD 99 094503 (2019).
- E. Gustafson, Y. Zhu, P. Dreher, N. M. Linke, YM: Phase shifts, arXiv:2103.06848, rev. PRD, P. Dreher Monday 10PMET (Alg.).
- Jin Zhang, Shan-Wen Tsai and YM; arXiv:2104.06342 , Phys. Rev. B 103, 245137 (2021). Talks Th 9:00 PM and after (Alg.)
- Leon Hostetler, Jin Zhang , Ryo Sakai , Judah Unmuth-Yockey , A. Bazavov, YM; arXiv:2105.10450. rev. PRD, Th 9:45 PM (Alg.)
- YM, scalar QED with Rydberg arrays, arxiv:2107.11366 (today!)

Tensor Lattice Field Theory for Quantum Computing

Quantum computing (QC) requires a complete discretization

- Discretization of space: lattice gauge theory formulation
- Discretization of field integration: tensor methods for compact fields (as in Wilson lattice gauge theory)

Important ideas of the tensor reformulation:

- Character expansions (such as Fourier series): partition function and averages become discrete sums of contracted tensors.
- The "hard" integrals are done exactly and field integrations provide Kronecker deltas that encode the symmetries.
- For continuous field variables, the sums are infinite, but truncations to finite sums do not break symmetries.
- Examples show that the correct large distance physics can be obtained with "a few qubits per unit cells". Comparison with other methods based on field discretization or quantum links are highly desirable.
- The standard boundary conditions can be implemented

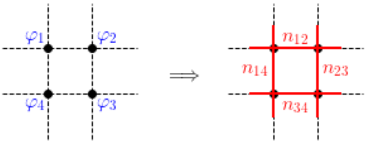
TLFT: From compact to discrete (O(2) example)

$$Z_{O(2)} = \prod_x \int_{-\pi}^{\pi} \frac{d\varphi_x}{2\pi} e^{\beta \sum\limits_{x,\mu} \cos(\varphi_{x+\hat{\mu}} - \varphi_x)} = \mathrm{Tr} \prod_x T^{(x)}_{n_{x-\hat{1},1}, n_{x,1}, \ldots, n_{x,D}}.$$

$$e^{\beta \cos(\varphi_{x+\hat{\mu}} - \varphi_x)} = \sum_{n_{x,\mu}=-\infty}^{\infty} e^{i n_{x,\mu} \varphi_{x+\hat{\mu}}} I_{n_{x,\mu}}(\beta) e^{-i n_{x,\mu} \varphi_x}.$$

$$T^{(x)}_{n_{x-\hat{1},1}, n_{x,1}, \ldots, n_{x-\hat{D},D}, n_{x,D}} = \sqrt{I_{n_{x-\hat{1},1}} I_{n_{x,1}}, \ldots, I_{n_{x-\hat{D},D}} I_{n_{x,D}}} \times \delta_{n_{x,\mathrm{out}}, n_{x,\mathrm{in}}},$$

$$\prod_x \int_{-\pi}^{\pi} d\varphi_x \Longrightarrow \sum_{\{n\}}$$



From tensor networks to quantum circuits (Ising)

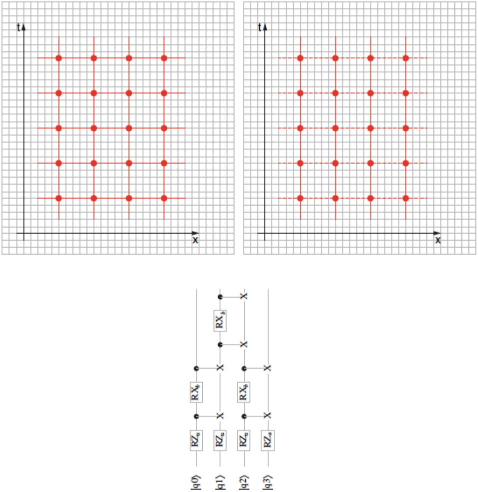
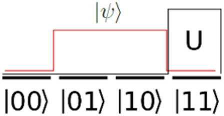


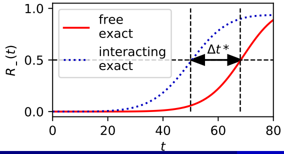
Figure 9.7. Graphical representation of the partition function for a spin model like the $O(2)$ model (left) and in the isolated rotors approximation (right). The dash lines stand for a single index 0 rather than a sum over all the states represented by straight lines. Circuit for 4 qubits with open boundary conditions used in reference

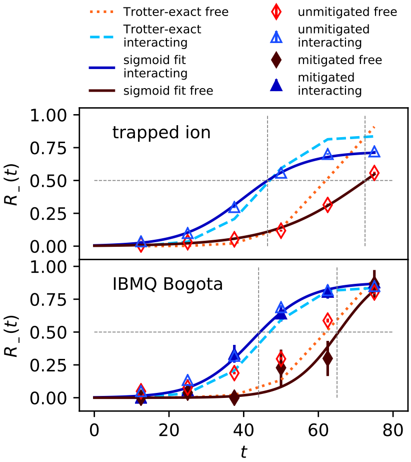
Erik Gustafson, Yingyue Zhu, Patrick Dreher, Norbert M. Linke, YM: see arXiv:2103.06848; P. Dreher, Monday 10:00 PM (Alg.)

Quantum devices: IBMQ and Trapped Ions (N. Linke facilities).



- 4-site Quantum Ising model.
- $R_{-}(t)$: renormalized reflection probability to be in $|-k\rangle$. (FT!)
- $\Delta t^{\star} = \delta'(k)/(\partial E/\partial k)$ due to interaction (Wigner)





Truncations (PRD 100 014506, PRD 102 014506)

- Tensorial truncations are compatible with the general identities derived from global and local symmetries.
- Symmetries are encoded in tensor selection rules, not in the numerical values of the tensor elements.
- Universal properties of these models can be reproduced with highly simplified formulations desirable for implementations with quantum computers or for quantum simulations experiments ("few qubits per unit cells"?).
- Noether theorem: for each symmetry there is a corresponding tensor redundancy (and we can "fix" the corresponding integration variable).
- Noise robust enforcement of Gauss's law (with bosons):
  - Abelian Higgs model: discrete version of $\partial_j E^j = \rho$
  - Pure gauge: discrete version of $E^k = \partial_j C^{jk}$
- See also: J. Bender and E. Zohar arXiv:2008.01349; L. Tagliacozzo, A. Celi, and M. Lewenstein, Phys. Rev. X 4, 041024; K. Jansen's talk

Transfer matrix and Gauss's law for Abelian gauge th.

PHYSICAL REVIEW D **102,** 014506 (2020)

Discrete aspects of continuous symmetries in the tensorial formulation of Abelian gauge theories

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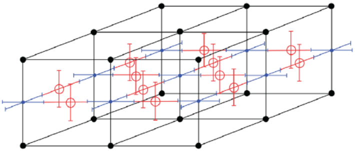
We show that standard identities and theorems for lattice models with $U(1)$ symmetry get reexpressed discretely in the tensorial formulation of these models. We also explain the geometrical analogy between the continuous lattice equations of motion and the discrete selection rules of the tensors. We further construct a gauge-invariant transfer matrix in arbitrary dimensions, show the equivalence with its gauge-fixed version in a maximal temporal gauge, and explain how a discrete Gauss's law is always enforced. Moreover, we propose a noise-robust way to implement Gauss's law in arbitrary dimensions, and we reformulate Noether's theorem for global, local, continuous, or discrete Abelian symmetries: for each given symmetry, there is one corresponding tensor redundancy. We discuss semiclassical approximations for classical solutions with periodic boundary conditions in two solvable cases, and we show the correspondence of their weak coupling limit with the tensor formulation after Poisson summation. Finally, we briefly discuss connections with other approaches and implications for quantum computing.



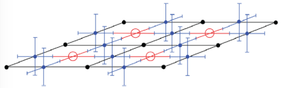


FIG. 3. Magnetic layer of the transfer matrix for $D = 3$ on a time slice. Small circles (blue) are used for the A tensors and large circles (red) for the B tensors.

Optical lattice implementation of the Abelian Higgs model with a ladder (1+1); spin-2 truncation

After taking the time continuum limit:

$$\bar{H} = \frac{\tilde{U}_g}{2}\sum_i \left(\bar{L}^z_{(i)}\right)^2 + \frac{\tilde{Y}}{2}\sum_i (\bar{L}^z_{(i)} - \bar{L}^z_{(i+1)})^2 - \tilde{X}\sum_i \bar{U}^x_{(i)}$$

5 states ladder with 9 rungs

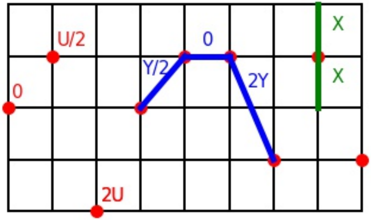


Figure: Ladder with one atom per rung: tunneling along the vertical direction ($\bar{L}^z = \pm 2,\ \pm 1,\ 0$), no tunneling in the the horizontal direction but short range attractive interactions. A parabolic potential is applied in the spin (vertical) direction. See PRL 121 223201.

Nearest neighbor Rydberg interactions

PHYSICAL REVIEW LETTERS **121,** 223201 (2018)

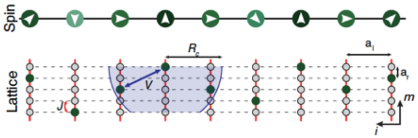


FIG. 3. Multileg ladder implementation for spin-2. The upper part shows the possible m_z projections. Below, we show the corresponding realization in a ladder within an optical lattice. The atoms (green disks) are allowed to hop within a rung with a strength J, while no hopping is allowed along the legs. The lattice constants along rungs and legs are a_r and a_l, respectively. Coupling between atoms in different rungs is implemented via an isotropic Rydberg-dressed interaction V with a cutoff distance R_c (marked by blue shading).

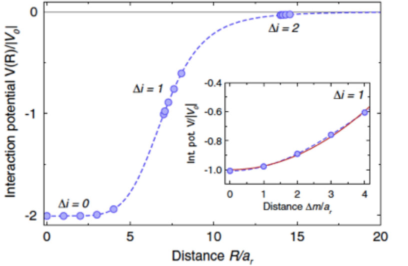


FIG. 4. Quadratic interactions on an asymmetric ladder for $s = 2$. The isotropic Rydberg-dressed potential (dashed blue line)

Quantum Simulation of the Universal Features of the Polyakov Loop

Jin Zhang,[1] J. Unmuth-Yockey,[2] J. Zeiher,[3] A. Bazavov,[4] S.-W. Tsai,[1] and Y. Meurice[5]

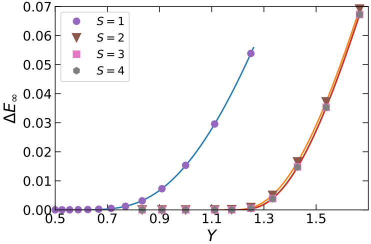
Proposal to quantum simulate the 1+1 Abelian Higgs model using a ladder of Rydberg atoms. The horizontal dimension is space, the vertical direction is the electric field degree of freedom.

Critical behavior of Spin truncations in the $O(2)$ limit

Jin Zhang, Shan-Wen Tsai and YM; arXiv:2104.06342 , Phys. Rev. B 103, 245137 (2021). Thursday 9:00 and 9:30 PM (Alg.)

$$\hat{H}_{charge} = \frac{Y}{2}\sum_{l=1}^{L+1}(\hat{S}_l^z)^2 - \frac{X}{2}\sum_{i=1}^{L}(\hat{U}_l^+\hat{U}_{l+1}^- + \hat{U}_l^-\hat{U}_{l+1}^+)$$

Energy gaps $\Delta E_{V=\infty}$ for spin truncations $S = 1, 2, 3, 4$ (by J. Zhang).



Fits:

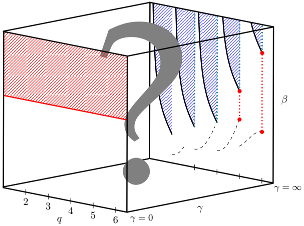
- $S \geq 2$: $A\exp(-b/\sqrt{Y - Y_c})$ (regular KT)
- $S = 1$: $A\sqrt{Y - Y_c}\exp\left[-b/(Y - Y_c)\right]$ ($SU(2)$ symmetry; KT separatrix, CFT: WZW model)

- $Y_c = 0.350666928(2)$ for $S = 1$; 1.101304(6) for $S = 2$; ... using level crossing spectroscopy.
- The dual field representation is gapped at finite S.

Interpolation among Z_q clock models

Leon Hostetler, Jin Zhang , Ryo Sakai , Judah Unmuth-Yockey , Alexei Bazavov , and YM; arXiv:2105.10450. L.H.: Thursday 9:45 PM (Alg.)

- $O(2)$ model with Symm. breaking : $\Delta S = \gamma \sum_x \cos(q\varphi_x)$
- $\gamma \to \infty$: $\varphi = \frac{2\pi k}{q}$ $k = 0, 1, .., \lfloor q \rfloor$
- Integer q: Z_q symmetry
- Non-integer q: Z_2 symmetry
- Phase diagram: see right panel



Implementation with Rydberg arrays?

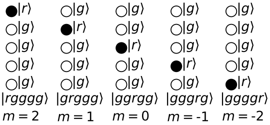
A. Keesling, .., M. Lukin et al. Nature 568: 1D array of ^{87}Rb atoms evenly separated by a controllable distance, homogeneously coupled to the excited Rydberg state $|r\rangle$ with detuning Δ.

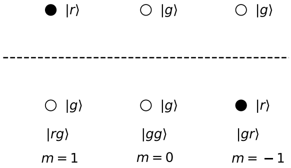
$$H = \frac{\Omega}{2} \sum_i (|g_i\rangle\langle r_i| + |r_i\rangle\langle g_i|) - \Delta \sum_i n_i + \sum_{i<j} V_{ij} n_i n_j$$

For $R_b/a \simeq q$ integer and Δ large enough: Z_q ordering.

Rydberg atom simulators

- 1 atom has 2 states: $|g\rangle$: ground state, $|r\rangle$: Rydberg state.
- n: occupation of $|r\rangle$; “qubit" picture: $|g\rangle \rightarrow |0\rangle$ and $|r\rangle \rightarrow |1\rangle$.
- Blockade: $|rr\rangle$ suppressed if two atoms are too close.
- $$H = \frac{\Omega}{2}\sum_i (|g_i\rangle\langle r_i| + |r_i\rangle\langle g_i|) - \Delta \sum_i n_i + \sum_{i<j} \Omega R_b^6/R_{ij}^6 n_i n_j,$$
- i and j run over atoms in configurable arrays.
- programming=assembling atoms with tweezers (M. Lukin, Harvard; A. Keesling, QuEra, ...).
- Proposals for spin 2 and spin 1 (YM arxiv:2107.11366)





One spin system: target model

$$H = \frac{U}{2}(L^z)^2 - XU^x.$$

H is invariant under charge conjugation: we use $\mathcal{C}$ eigenstates

$$|\pm\rangle \equiv \frac{1}{\sqrt{2}}(|1\rangle \pm |-1\rangle).$$

$$\mathcal{C}|\pm\rangle = \pm|\pm\rangle.$$

They are also eigenstates of $(L^z)^2$ with eigenvalue 1, and $\mathcal{C}|0\rangle = |0\rangle$. There is only one $\mathcal{C}$-odd state which is $|-\rangle$.

$$U^x|-\rangle = 0;\ H|-\rangle = \frac{U}{2}|-\rangle.$$

This one of the two building blocks of scalar QED; the other is the spin-spin interaction (see arxiv:2107.11366)

Two Rydberg atom implementation

$$H^{2R} = -\Delta(n_{+1} + n_{-1}) + V_0 n_{+1} n_{-1} + \frac{\Omega}{2} \sum_{\pm 1} (|g_{\pm 1}\rangle\langle r_{\pm 1}| + |r_{\pm 1}\rangle\langle g_{\pm 1}|).$$

state	ket	short	energy ($\Omega = 0$)
● ○	$\|rg\rangle$	$\|1\rangle$	$-\Delta$
○ ○	$\|gg\rangle$	$\|0\rangle$	0
○ ●	$\|gr\rangle$	$\|-1\rangle$	$-\Delta$
● ●	$\|rr\rangle$	$\|2\rangle$	$-2\Delta + V_0$

$\Delta = -\frac{U}{2}$ and $\Omega = -X$.
Except for possible transitions to $|rr\rangle$, the correspondence is exact and the linear formula applies for arbitrary values of X. Example:

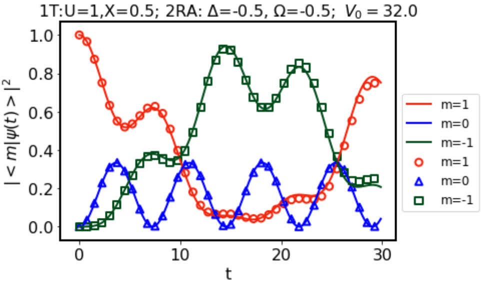


Figure: $|\langle m|U(t)|m=1\rangle|^2$, one site with exact Hamiltonian $U = 1,\ X = 0.5$ (solid lines) and Rydberg Hamiltonian with $\Omega = -0.5$, $\Delta = -0.5$ and $V_0 = 64|\Omega| = 32$ (empty symbols). See YM arxiv:2107.11366 for details and setups with 3 (another spin-1 proposal), and 4 or 6 atoms for spin-spin interactions.

Conclusions

- Tensor Lattice Field Theory is a generic tool to discretize path integral formulations of lattice models.
- TRG: exact blocking, a friendly competitor to QC.
- Microscopic truncations respect symmetries and have interesting critical properties
- TLFT: gauge-invariant approach.
- Noether theorem: for each symmetry, there is a corresponding tensor redundancy.
- Noise-robust economical implementation of Gauss's law for pure gauge models.
- Rydberg atom implementations of scalar QED in the near future!
- Thanks for listening!