The truncated $U(1)$ Abelian Higgs model and implication for its quantum simulation

Jin Zhang

University of Iowa

With: **Shan-Wen Tsai (UC-Riverside)**, **Yannick Meurice (UIowa)**
J. Unmuth-Yockey (Fermilab), **J. Zeiher (Max Planck Institute)** , **A. Bazavov (MSU)**

LATTICE 2021

10.1103/PhysRevB.103.245137.
10.1103/PhysRevLett.121.223201.

Motivation

- Truncations are needed in quantum simulation of models with continuous symmetry.
- For example: a proposal for quantum simulating the Abelian Higgs model in the field representation uses a multi-leg ladder (J. Zhang et al. 2018).
- Question: What are the truncation effects in the charge representation?

Outline

- The charge and field representations with a finite spin truncation.
- The truncated field representation of O(2) model is gapped.
- The dependence of the phase transition point on the spin truncation in the charge representation of O(2) model.
- Compare different methods to detect BKT transitions (next talk).
- Data collapse for the energy gap at nonzero gauge coupling in the charge representation.

Abelian Higgs model (compact sQED)

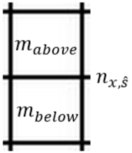
On the $L \times L_\tau$ Euclidean space-time lattice, the action of the compact sQED is

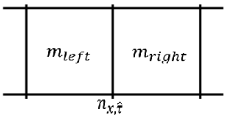
$$S = -\beta_{pl} \sum_x \cos\left(A_{x,\hat{s}} + A_{x+\hat{s},\hat{\tau}} - A_{x+\hat{\tau},\hat{s}} - A_{x,\hat{\tau}}\right) - \beta_\tau \cos\left(\theta_{x+\hat{\tau}} - \theta_x + A_{x,\hat{\tau}}\right) - \beta_s \cos\left(\theta_{x+\hat{s}} - \theta_x + A_{x,\hat{s}}\right)$$

By expanding the weights with modified Bessel functions, the path integral formulation is written as

$$e^{-S_{eff}} = \sum_{\{m_\Box\}} \left\{ \prod_\Box t_{m_\Box}(\beta_{pl}) \prod_x \left[t_{n_{x,\hat{s}}}(\beta_s) \times t_{n_{x,\hat{\tau}}}(\beta_\tau) \right] \right\},$$

Where $n_{x,\hat{s}} = m_{below} - m_{above}$, $n_{x,\hat{\tau}} = m_{right} - m_{left}$ (Gauss's Law).
$t_n(x) = I_n(x)/I_0(x)$, $I_n(x)$ is the nth order modified Bessel function of the 1st kind.





Hamiltonians in charge and field representations

- Using ns and taking $a_\tau \to 0$, we obtain the Hamiltonian in the charge representation,

$$\hat{H}_c = \frac{W}{2} \sum_{1 \le j,k < L} c_{jk} \hat{S}^z_j \hat{S}^z_k + \frac{Y}{2} \sum_{l=1}^{L+1} (\hat{S}^z_l)^2 - \frac{X}{2} \sum_{i=1}^{L} (\hat{U}^+_l \hat{U}^-_{l+1} + \hat{U}^-_l \hat{U}^+_{l+1}) \tag{1}$$

where $W = 1/\beta_{pl} a_\tau$, $c_{jk} = L + 1 - \max\{j, k\}$, $Y = 1/\beta_\tau a_\tau$, $X = \beta_s / a_\tau$, $\hat{U}^\pm |n\rangle = |n \pm 1\rangle$.

- Replacing ns by ms, or using $\hat{S}^z_l = \hat{\hat{S}}^z_l - \hat{\hat{S}}^z_{l-1}$ and $\hat{U}^+_l = \hat{U}^-_{l+1} \hat{U}^+_l$, we go to the field representation,

$$\hat{H}_f = \frac{W}{2} \sum_{p=1}^{L} (\hat{\hat{S}}^z_p)^2 + \frac{Y}{2} \sum_{p=1}^{L+1} (\hat{\hat{S}}^z_p - \hat{\hat{S}}^z_{p-1})^2 - X \sum_{p=1}^{L} \hat{\hat{U}}^x_p \,, \tag{2}$$

$S^z_0 = S^z_{L+1} = 0$ for OBC in the charge-0 sector. The charge-1 sector: $(\hat{\hat{S}}^z_{L/2+1} - \hat{\hat{S}}^z_{L/2})^2 \to (\hat{\hat{S}}^z_{L/2+1} - \hat{\hat{S}}^z_{L/2} - 1)^2$.

O(2) limit: field representation with $|m|_{max} = S$

- The energy gap converges exponentially with S.
- Field representation has no quantum phase transitions for finite S.

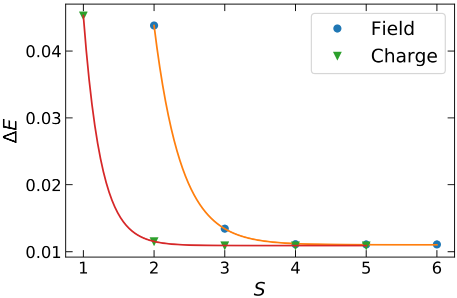


Figure: $L = 64$, $Y = 1.126$. The solid lines are fit to $A\exp(-\alpha S) + c$.

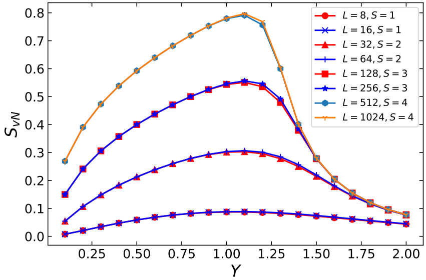


Figure: Entanglement entropy S_{vN} $v.s.$ Y for $S = 1, 2, 3, 4$, saturates at $L_0 \approx 2^3, 2^5, 2^7, 2^9$.

O(2) limit: charge representation with $|n|_{max} = 1$

Spin-1 truncation: BKT critical line to large-D phase. Level crossing between ground states in sectors $(M, P) = (0, 1), (0, -1)$.

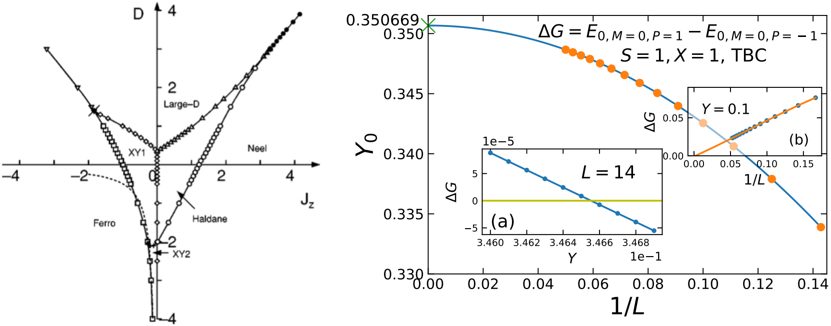


Figure: Left: Phase diagram of spin-1 truncation with $J_z S_l^z S_{l+1}^z$ term, $D = Y$ (Chen, Hida and Sanctuary, 2003). Right: level crossing between ground states in two parity sectors.

O(2) limit: charge representation with $|n|_{max} = S$

Level crossing between excitations $(M, k, P) = (4, 0, 1), (0, 0, 1)$ (Nomura & Kitazawa, 1998).

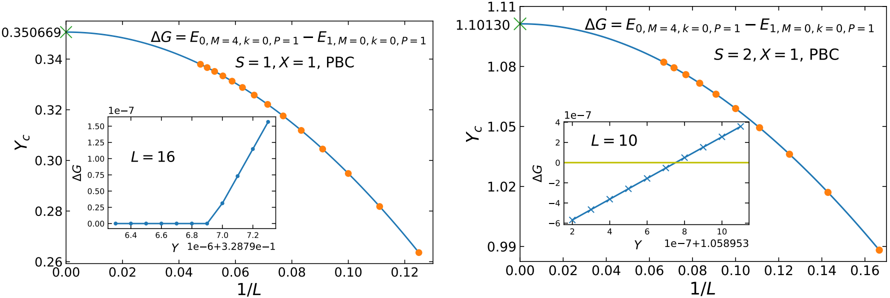


Figure: The extrapolation procedure of finite size Y_c for $S = 1$ (left) and $S = 2$ (right). The degeneracy signals a BKT point with SU(2) symmetry.

Locating BKT transitions: level spectroscopy

- Y_c converges exponentially with S for $\hat{U}^{\pm}$
- Y_c converges polynomially with S for $\hat{S}^{\pm}$

	$\hat{U}^{\pm}$	$\hat{S}^{\pm}$
$S=1$	0.35066928(2)	0.35066928(2)
$S=2$	1.101304(6)	0.932201(4)
$S=3$	1.125614(17)	1.03308(3)
$S=4$	1.125898(19)	1.07103(2)
$S=5$		1.08952(3)
$S=\infty$	1.126188(13)	1.12614(8)

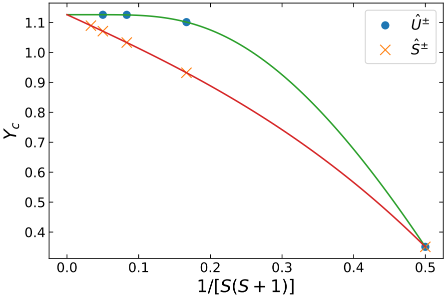


Figure: Phase transition points Y_c for different S. The $\hat{U}^{\pm}$ operators are replaced by $\hat{S}^{\pm}/\sqrt{S(S+1)}$ in the second column.

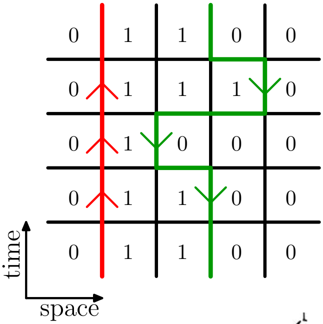
Nonzero β_{pl}: the observable, field representation

PRL 121, 223201 (2018)

Polyakov loop: a Wilson loop wrapped around the temporal direction of the lattice. This operator:

- is a product of gauge fields in the time direction.
- is an order parameter for confinement in gauge theories.
- has a continuous-time limit which adds a term to the Hamiltonian

$$\hat{P} = \prod_{n=1}^{L_\tau} \exp(i\hat{A}_{x^*+n\hat{\tau},\tau}).$$

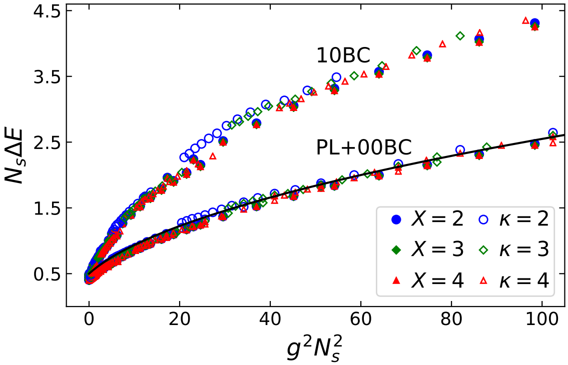


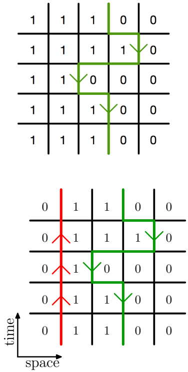
$$\hat{H}_f \to \hat{H}'_f = \frac{W}{2}\sum_{p=1}^{L}(\hat{\hat{S}}^z_p)^2 + \frac{Y}{2}\sum_{p\neq\frac{L}{2}}^{\prime}(\hat{\hat{S}}^z_{p+1} - \hat{\hat{S}}^z_p)^2$$

$$+\frac{Y}{2}(\hat{\hat{S}}^z_{\frac{L}{2}+1} - \hat{\hat{S}}^z_{\frac{L}{2}} - 1)^2 - X\sum_{p=1}^{L}\hat{\hat{U}}^x_p$$

P-loop: universal function + collapse across limits

PRL 121, 223201 (2018)

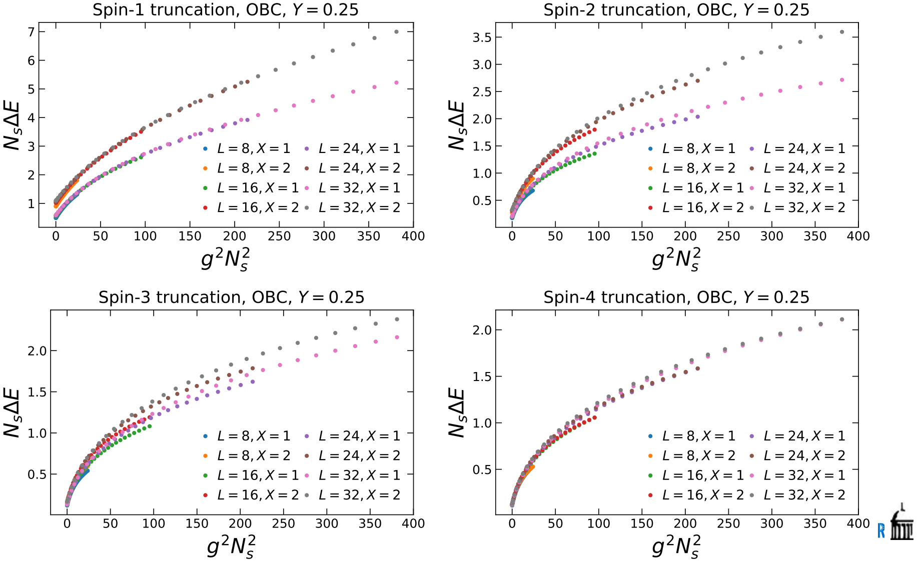




Data collapse of $N_s\Delta E$ defined from the insertion of the Polyakov loop (lower set) or with 1-0 boundary conditions (upper set) (collapse of 24 data sets each). The black solid line is $\sqrt{(gN_s/4)^2+0.25}$.

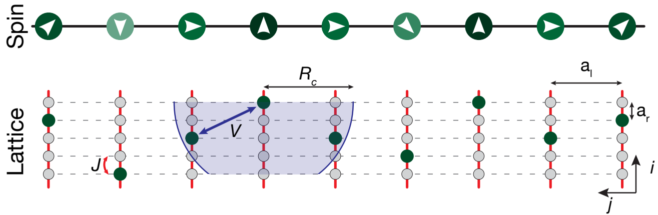
P-loop: data collapse in charge representation

The universal function depends on X for fixed Y for small S.



Experimental proposal

Cold atoms on multi-leg ladder with Rydberg-dressed interaction



$$\hat{H}_f = \frac{W}{2}\sum_{p=1}^{L}(\hat{\mathcal{S}}^z_p)^2 + \frac{Y}{2}\sum_{p=1}^{L+1}(\hat{\mathcal{S}}^z_p - \hat{\mathcal{S}}^z_{p-1})^2 - X\sum_{p=1}^{L}\hat{U}^x_p,$$

$$\hat{H}_{ladder} = -\frac{J}{2}\sum_{i=1}^{L}\sum_{m=-s}^{s-1}\left(\hat{a}^\dagger_{m,i}\hat{a}_{m+1,i} + h.c\right) - \sum_{i=1}^{L}\sum_{m=-s}^{s}\epsilon_{m,i}\hat{n}_{m,i} + \sum_{i,i'=1}^{L}\sum_{m,m'=-s}^{s} V_{m,m',i,i'}\,\hat{n}_{m,i}\hat{n}_{m',i'} \qquad (3)$$

Conclusions

- The field representation of the O(2) model is always gapped for finite S. But we will not need a big S to probe the finite size effects accurately.
- The charge representation has true quantum phase transitions. The phase transition is an infinite-order Gaussian transition for $S = 1$, and is a true BKT transition for $S \geq 2$.
- The phase transition point converges exponentially with S for $\hat{U}^{\pm}$ operators, while it converges algebraically for $\hat{S}^{\pm}$ operators.
- Calculations of the Polyakov loop at finite L and small gauge coupling show a universal behavior in both charge and field representations (collapse related to the BKT transition of the limiting $O(2)$ model).
- A ladder of cold atoms with L rungs, one atom per rung, and $2S + 1$ legs is a candidate system for experimental realization of the full sQED.