Open Lattice Field Theories

non-Hermitian many-body Hamiltonians

Jay Hubisz
Syracuse University

With: Bharath Sambasivam (S.U.) and Judah Unmuth-Yockey (S.U. → FNAL)

Based on e-Print: 2012.05257 [hep-lat]

LATTICE 21

July 26, 2021

Also ongoing work with Sambasivam, Unmuth-Yockey, Gustafson, and Hite

The Power of EFT’s

The effective field theory paradigm dominates how we think about and do physics

We are most familiar with using it to integrate out physics at short distances when relevant length scales are large

However, it has applicability whenever there is an “environment” that is inaccessible to the experimenter.

(Non)unitarity and EFT’s

In the case of Wilsonian EFT, energy and momentum conservation ensures low energy EFTs are unitary

BUT: our “environment” could consist of:
light unobservable particles, stuff behind a horizon, or just something very complicated

Integrating out such environments will generically yield a non-unitary effective theory

Open Quantum Systems

Information flow

In many scenarios, it is an excellent approximation that information flow is predominantly “one-way” from system to environment

E.g. A massive particle that decays to light modes
(Phase space)

Or:
A system coupled to a bath of charge
(Scrambling)

Non-Hermitian H?

A large class of interesting effective field theories are well described by this particular kind of non-unitary dynamics:

$$i\frac{d}{dt}|\Psi\rangle = \hat{H}|\Psi\rangle \qquad \hat{H} = \hat{G} + i\hat{K}$$

Will “make sense” of this in a moment

- Systems at finite chemical potential
- QFT at complex coupling
  - Fisher/Lee-Yang zeros probe phase structure
  - Novel non-unitary critical points - out of equilibrium dynamics
- Topological terms
- Etc.

The sign problem

Would usually go to a thermal Euclidean counterpart and compute the partition function

$$Z = \sum e^{-\beta H}$$

Complex H leads to highly oscillatory numerical integrand

Poor convergence…

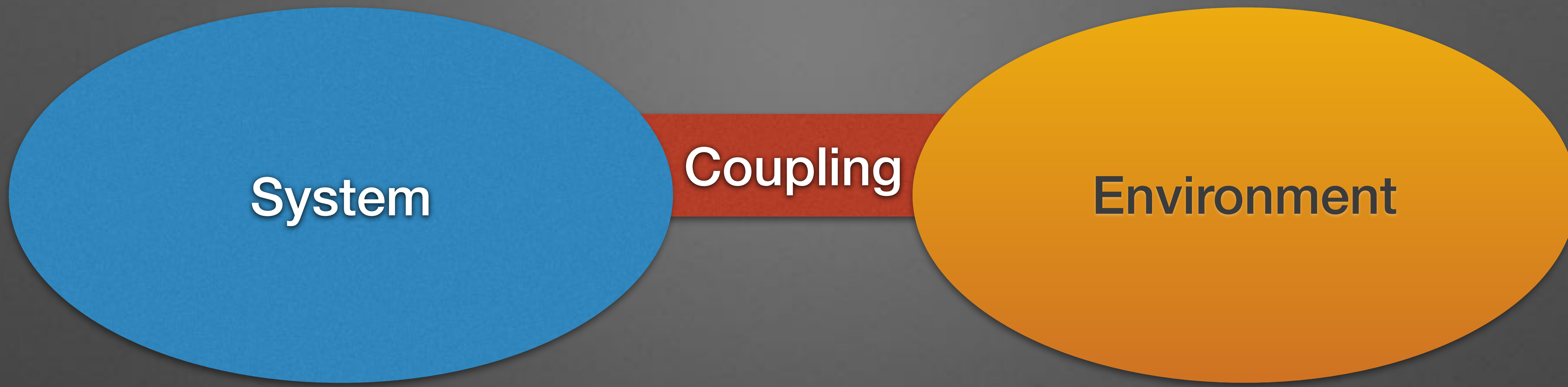
Does not yield to standard Monte Carlo techniques

Does quantum computing offer an opportunity to solve this problem?

Quantum computers don't do this:

$$i\frac{d}{dt}|\Psi\rangle = \hat{H}|\Psi\rangle \qquad \hat{H} = \hat{G} + i\hat{K}$$

Quantum operations



$$\rho_{\mathrm{All}} \rightarrow U \rho_{\mathrm{All}} U^{\dagger}$$

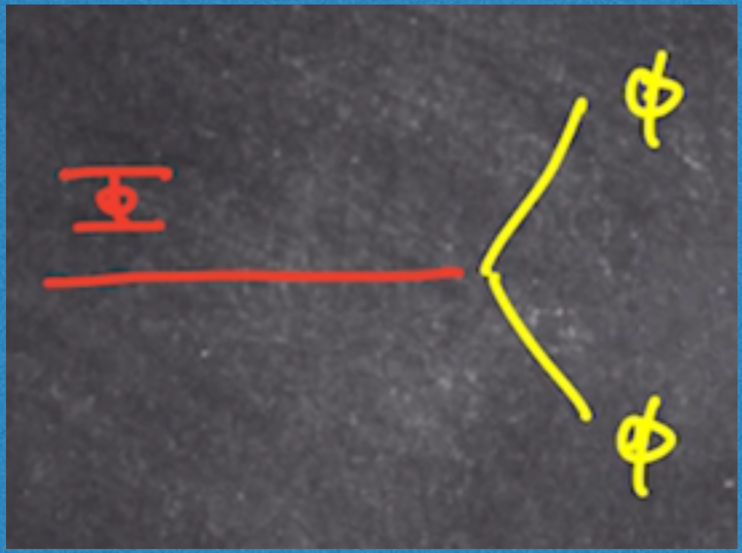
$$\rho_{\mathrm{sys}} \rightarrow \mathrm{Tr}_{\mathrm{env}}[U \rho_{\mathrm{All}} U^{\dagger}] = \sum_{m}^{d_{\mathrm{env}}} E_m \rho_{\mathrm{sys}} E_m{}^{\dagger}$$

The E’s are the Kraus operators for non-unitary evolution of the non-isolated system

Orchestrating non-hermitian evolution

Design a system + environment model

Simplest non-Hermitian system: particle decay



ϕ particles hard to detect

Φ is system, ϕ environment

System measurements: Φ is there/not there (mixed state)

$$\rho = \begin{pmatrix} e^{-\Gamma t} & 0 \\ 0 & (1 - e^{-\Gamma t}) \end{pmatrix}$$

$$\rho_{\text{sys}} \rightarrow e^{-\Gamma t/2} \rho_{\text{sys}} e^{-\Gamma t/2}$$

$$\hat{H} = -i\Gamma/2\hat{I}$$

A quantum operation:

$$\rho \rightarrow E_0 \rho E_0{}^\dagger + E_1 \rho E_1{}^\dagger$$

$$E_0 = \begin{pmatrix} e^{-\Gamma t/2} & 0 \\ 0 & 0 \end{pmatrix}, \quad E_1 = \begin{pmatrix} 0 & 0 \\ \sqrt{1 - e^{-\Gamma t}} & 0 \end{pmatrix}$$

Non-trivial system

$K_0 - \bar{K}_0$ oscillation and decay:

Non-hermitian evolution via $\hat{H} = \hat{G} + i\hat{K}$

Oscillation matrix $\hat{G}$, decay matrix $\hat{K} = -\hat{\Gamma}/2$

$$\rho = \begin{pmatrix} \rho_{K\bar{K}} & 0 \\ 0 & 1 - \mathrm{Tr}\rho_{K\bar{K}} \end{pmatrix} \rightarrow \begin{pmatrix} e^{-i\hat{H}t}\rho_{K\bar{K}}e^{i\hat{H}^\dagger t} & 0 \\ 0 & 1 - \mathrm{Tr}\rho'_{K\bar{K}} \end{pmatrix}$$

Quantum operation:

$$E_0 = \begin{pmatrix} e^{-i\hat{H}t} & & 0 \\ & & 0 \\ 0 & 0 & 0 \end{pmatrix}, E_1 = \begin{pmatrix} & 0_{2x2} & 0 \\ & & 0 \\ \sqrt{1 - e^{-\Gamma_K t}} & 0 & 0 \end{pmatrix}, E_2 = \begin{pmatrix} & 0_{2x2} & 0 \\ & & 0 \\ 0 & \sqrt{1 - e^{-\Gamma_{\bar{K}} t}} & 0 \end{pmatrix}$$

$K\bar{K}$ system lives in 2x2 block, evolves according to $\hat{H}$

$\rho_{K\bar{K}}$ not strictly a density matrix - trace depleted,

$E_{1,2}$ maintain unitarity - $\mathrm{Tr}\rho = 1$

The General Case:

Arbitrary $\hat{H} = \hat{G} + i\hat{K}$

Want $\rho = \begin{pmatrix} \rho_{\text{sys}} & \vec{0} \\ \vec{0}^T & 1 - \text{Tr}\rho_{\text{sys}} \end{pmatrix} \rightarrow \begin{pmatrix} e^{-i\hat{H}t}\rho_{\text{sys}}e^{i\hat{H}^\dagger t} & \vec{0} \\ \vec{0}^T & 1 - \text{Tr}\rho'_{\text{sys}} \end{pmatrix}$

Issue: if $\hat{K}$ has any positive eigenvalues, $\text{Tr}\rho_{\text{sys}}$ might exceed one

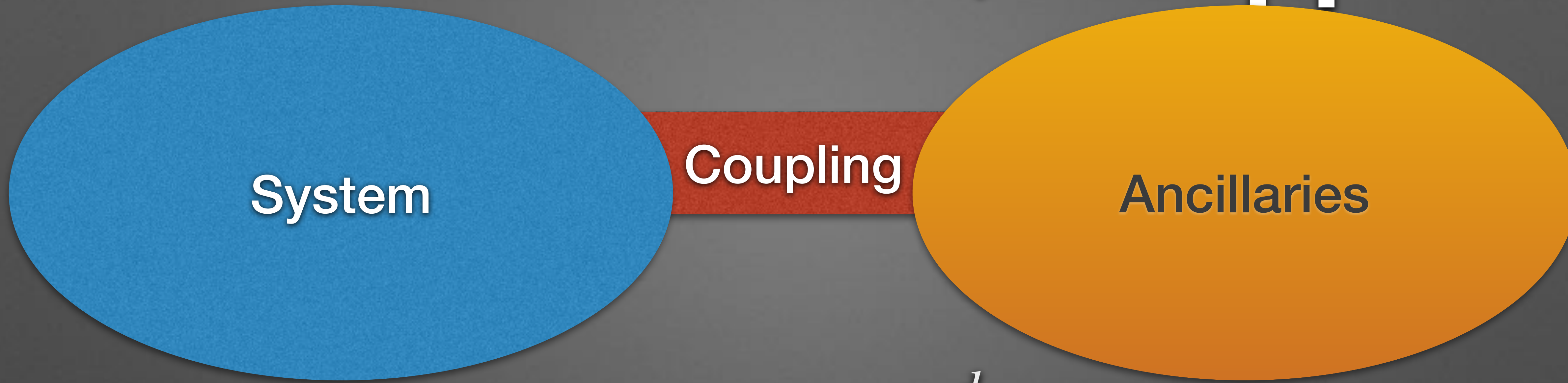
Shift $\hat{H}$ by an imaginary constant such that $-\hat{K}$ is a positive operator

$$\hat{K} \rightarrow \hat{K} - \Lambda\hat{I}$$

(only affects rate at which $\text{Tr}\rho_{\text{sys}}$ is depleted)

$$E_0 = \begin{pmatrix} e^{-i\hat{H}t} & \vec{0} \\ \vec{0}^T & 0 \end{pmatrix}, E_i = \begin{pmatrix} 0_{NxN} & \vec{0} \\ \ldots\sqrt{1 - e^{-\Gamma_i t}}\ldots & 0 \end{pmatrix}$$

How to make a QO happen:



$$\rho \to \mathrm{Tr}_{\mathrm{env}}[U\rho_{\mathrm{All}}U^{\dagger}] = \sum_{m}^{d_{\mathrm{anc}}} E_m \rho_{\mathrm{sys}} {E_m}^{\dagger}$$

Ancillary system of at most dimension N+1, where N is dimension of system undergoing NH evolution

Ancillary measurement: outcomes 0,…,N

Outcome 0: $\rho \to \dfrac{E_0 \rho {E_0}^{\dagger}}{\mathrm{Tr} E_0 \rho {E_0}^{\dagger}}$ Otherwise, system decayed

Process can be Trotterized - one QO per time step

Features in non-hermitian quantum systems

Exceptional points - eigenvectors are not orthogonal

As hamiltonian parameters are varied, eigensystems can collapse

Hamiltonian diagonalizable only up to Jordan block form

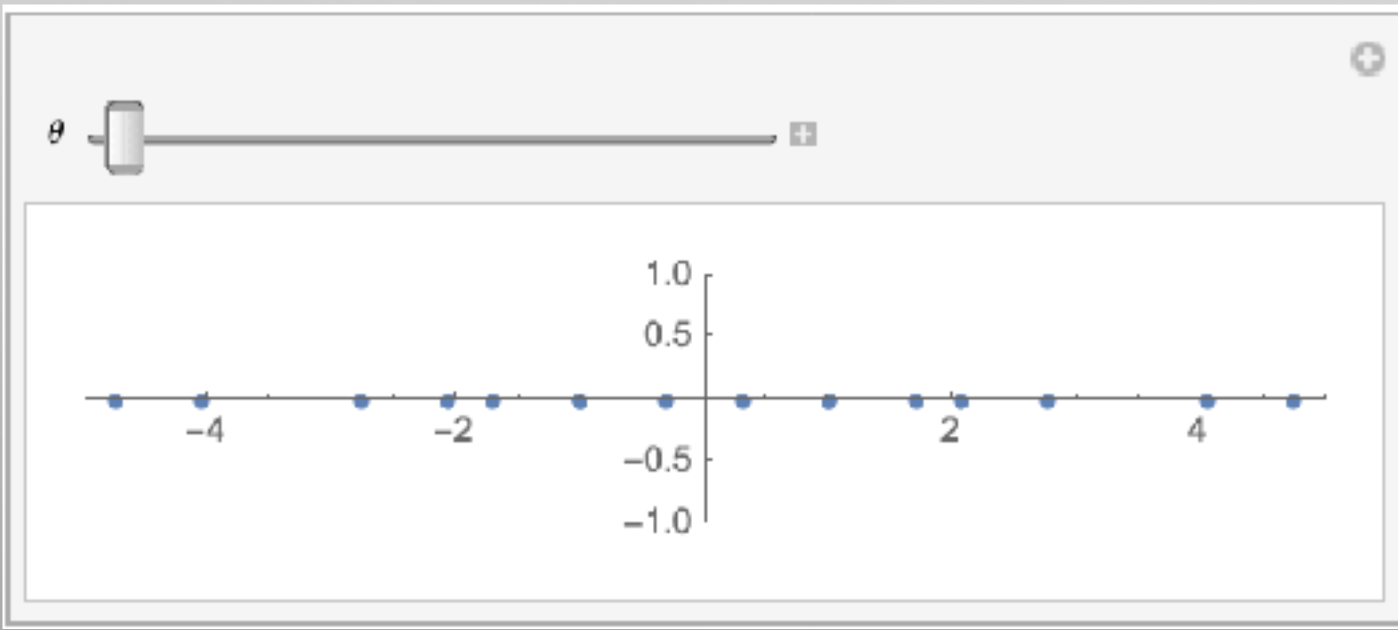
Quantum 1D Ising model with imaginary longitudinal field

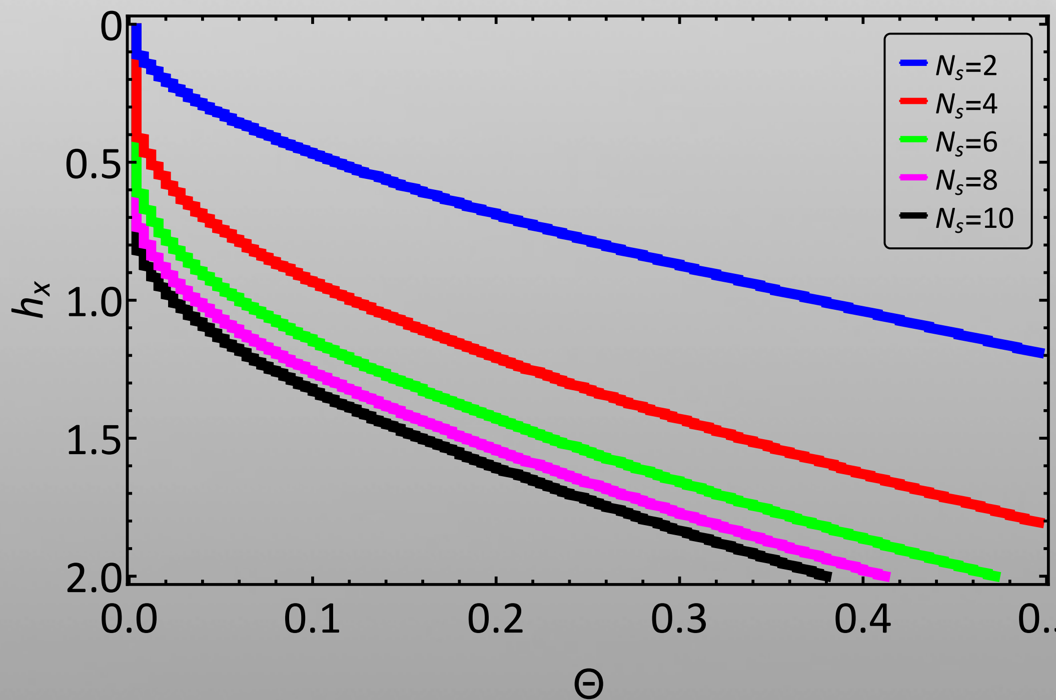
$$\hat{H}_{\text{Ising}} = \underbrace{-\sum_{\langle ij \rangle} \hat{\sigma}_i^z \hat{\sigma}_j^z - \frac{h_x}{\lambda} \sum_i \hat{\sigma}_i^x}_{\hat{G}} + \underbrace{i \frac{\Theta}{\lambda} \sum_i \hat{\sigma}_i^z}_{i\hat{K}}$$

If $\Theta = 0$, quantum phase transition at $h_x = 1$ as $N_{\text{site}} \to \infty$

The Exceptional Line

As Θ is varied, the two lowest states (by real part) collapse into Jordan block, then move off real axis





This is for N=4 and $h_x = 1$

In the large volume limit, the contour of exceptional points corresponds to a line of non-unitary CFTs. C = -22/5 (Cardy 1985)

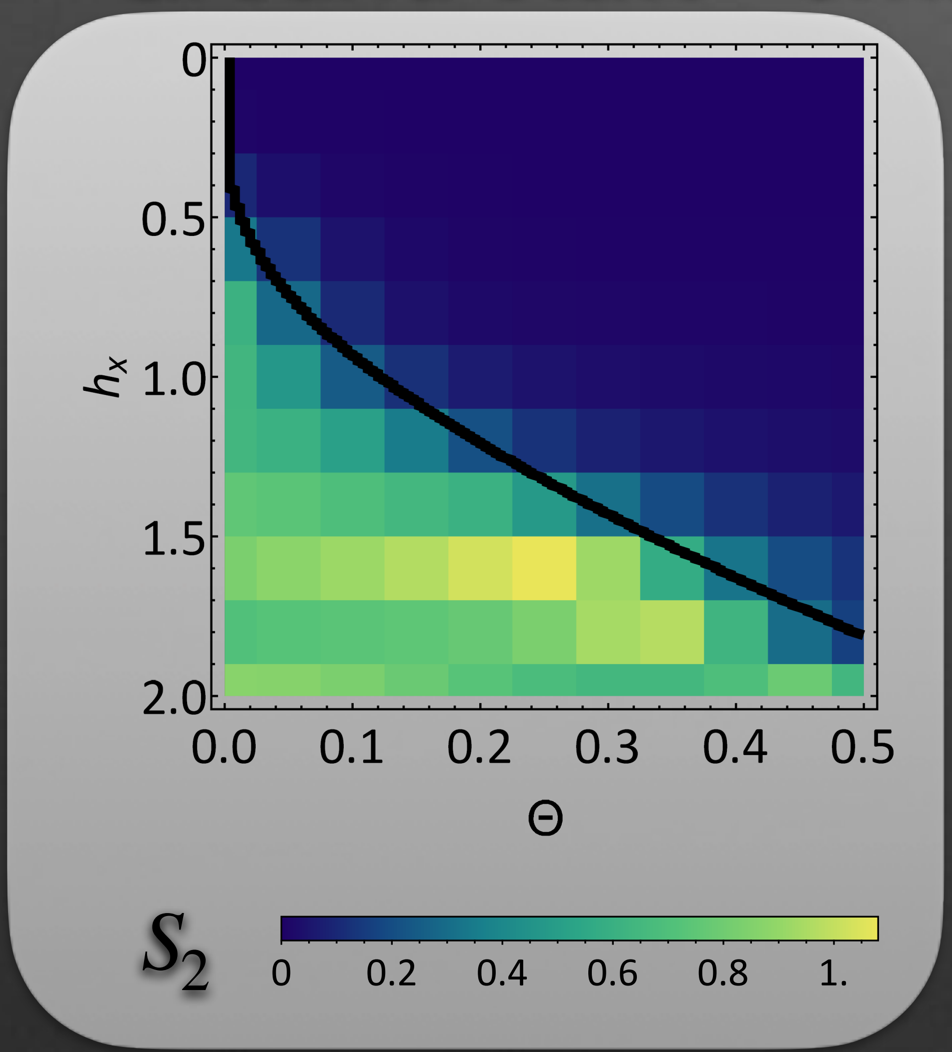
The $h_x = 1$ quantum critical point lives at the merger of critical lines at $\pm\Theta_c$

Lee-Yang edge singularity in a quantum system

Measuring non-unitary criticality

Example: The Rényi entropies measure entanglement

"Ground state" entanglement at critical line is maximal



Can be inferred from parity measurements
Islam et. al. 2015

When eigenvalues are complex, System is driven into state with largest complex part

In large Θ region, ground state is an attractor

Conclusions and outlook

- Any non-hermitian Hamiltonian can be accommodated in a unitary system + environment model
  - thus realized on quantum hardware without sign problem
- Constructed a simple quantum operator set that can be implemented with quantum algorithms
  - there are others - see Bharath Sambasivam's talk
- Opportunity to study novel features of open (complexities) lattice field theories - Hamiltonian exceptional points $\leftrightarrow$ singular features in partition function
- Future: spin/clock/O(N) models at finite μ, topological terms, hardware implementation (see Michael Hite's talk), QITE application (Motta et. al. 2019/Kamakari et. al. 2021)