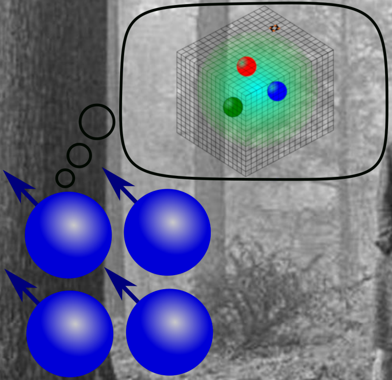
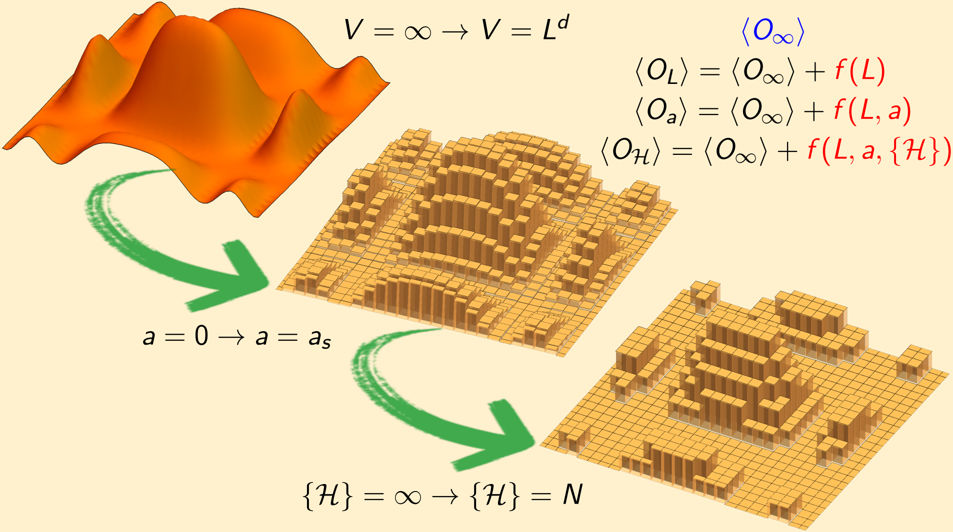
Toward Quantum Simulations using Discrete Subgroup Approximations

Hank Lamm



QFT is about infinities and how to regulate them



$$V = \infty \to V = L^d$$

$$a = 0 \to a = a_s$$

$$\{\mathcal{H}\} = \infty \to \{\mathcal{H}\} = N$$

$$\color{blue}{\langle O_\infty \rangle}$$
$$\langle O_L \rangle = \langle O_\infty \rangle + \color{red}{f(L)}$$
$$\langle O_a \rangle = \langle O_\infty \rangle + \color{red}{f(L, a)}$$
$$\langle O_{\mathcal{H}} \rangle = \langle O_\infty \rangle + \color{red}{f(L, a, \{\mathcal{H}\})}$$

How do I digitize a gluon

Exploring Digitizations of Quantum Fields for Quantum Devices

Erik Gustafson,[1] Hiroki Kawai,[2,*] Henry Lamm,[3,†] Indrakshi Raychowdhury,[4,‡]
Hersh Singh,[5,6,§] Jesse Stryker,[4,6,¶] and Judah Unmuth-Yockey[7]

[1] *University of Iowa, Iowa City, Iowa, 52242[**]*
[2] *Department of Physics, Boston University, 590 Commonwealth Avenue, Boston, MA 02215, USA*
[3] *Fermi National Accelerator Laboratory, Batavia, Illinois, 60510, USA*
[4] *Maryland Center for Fundamental Physics and Department of Physics, University of Maryland, College Park, MD 20742, USA*
[5] *Department of Physics, Box 90305, Duke University, Durham, North Carolina 27708, USA*
[6] *Institute for Nuclear Theory, University of Washington, Seattle, WA 98195, USA*
[7] *Syracuse University, Syracuse NY[††]*

In this LOI we undertake to enumerate promising digitization schemes for quantum fields that could allow near-term calculations on quantum devices. Further we discuss the outstanding questions that must be resolved in evaluating their potential, providing potential benchmarking on the way to practical quantum advantage in high energy physics.

Lots of choices for digitizing gauge bosons[1]:

- Some combination of: Hamiltonian, basis, and **truncation**
- I am going to focus on **discrete subgroups**

What qualities make a GOOD scheme?

- What **quantum resources** are required to get physical point?
- What symmetries are being **broken** in digitization?
- Can the scheme be simulated **classically**?

[1] Gustafson, E. et al. In: *Snowmass 2021 LOI* TF10-97 (2020).

This is not a triviality!

$$H_{\text{trunc}} = \frac{c}{a_s}\left[\frac{g_H^2}{2}\sum_l E_l^2 + \frac{1}{g_H^2}\sum_p \text{Tr}\, U_p\right] + \mathcal{O}_{\text{trunc}}$$

- This **defines** your EFT and what **continuum theory** you approach
- Qubit costs as a **function of renormalized parameters** is only meaningful metric
- $\mathcal{O}_{\text{trunc}}$ is **not** always obtained from $U \to U + \delta$ but can be **lowest** dim $\mathcal{O}$ which breaks symmetry

I'm going to talk about lattice actions

$$\langle x|e^{-iHt}|y\rangle = \int \mathcal{D}\phi e^{iS}$$

The **anisotropic Wilson** action is

$$S_{\rm W} = \frac{1}{g_t^2}\xi\sum_t \mathrm{Tr}\, U_t + \frac{1}{g_s^2}\frac{1}{\xi}\sum_s \mathrm{Tr}\, U_s \tag{1}$$

thru **transfer matrix,** $\langle i|e^{-a_0 H}|j\rangle$ derives the H_{KS}

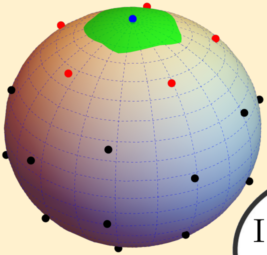
$$H_{KS} = \frac{c}{a_s}\left[\frac{g_H^2}{2}\sum_l E_l^2 + \frac{1}{g_H^2}\sum_p \mathrm{Tr}\, U_p\right] \tag{2}$$

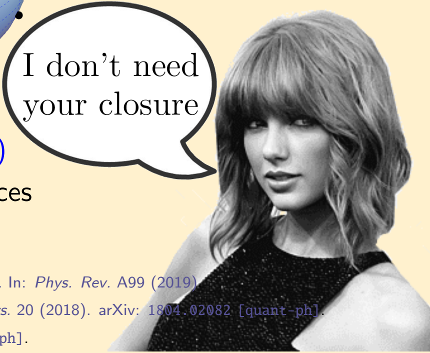
At finite a_0, $H \neq H_{KS} \implies$ consequences for trotterization[2]

[2] Carena, M., **H. Lamm**, Y.-Y. Li, and W. Liu. In: (July 2021). arXiv: 2107.01166 [hep-lat].

Discrete subgroups allow plug-and-play[3][4][5]

Replace $G \rightarrow H$ in $e^{-S}, e^{-i\mathcal{H}}$





- $SU(3) \rightarrow \mathbb{V}$ reduces qubits by $O(10^2)$
- I **believe** endgame will be **3x3** matrices

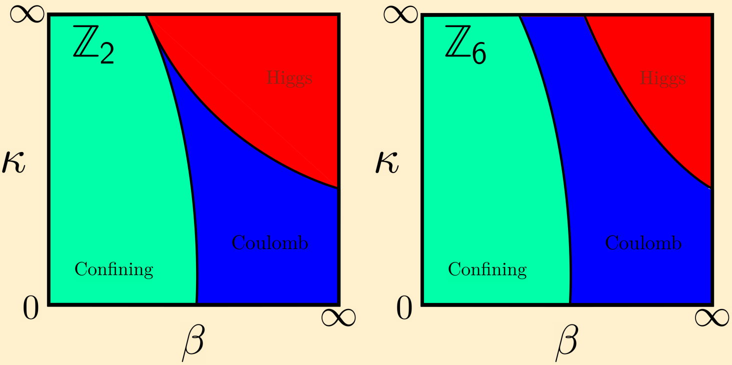
[3] Bhanot, G. In: *Phys. Lett.* 108B (1982), Hackett, D. C. et al. In: *Phys. Rev.* A99 (2019)

[4] Bender, J., E. Zohar, A. Farace, and J. I. Cirac. In: *New J. Phys.* 20 (2018). arXiv: 1804.02082 [quant-ph].

[5] Haase, J. F. et al. In: (June 2020). arXiv: 2006.14160 [quant-ph].

Discrete groups can't reach continuum[6][7][8]

$$S = \beta \sum \mathrm{Re}\,\mathrm{Tr}\, U_P + \kappa \sum \phi \cdot D\{U_P\} \cdot \phi^\dagger + c.c.$$



Integrating over ϕ leads to S_{eff} with new irreps of G

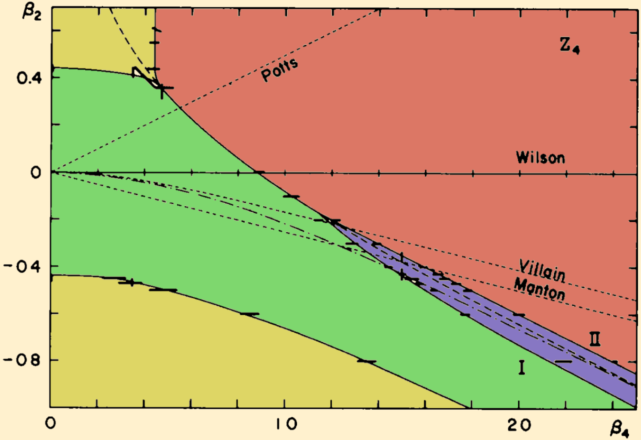
[6] Fradkin, E. H. and S. H. Shenker. In: *Phys. Rev. D* 19 (1979).

[7] Horn, D., M. Weinstein, and S. Yankielowicz. In: *Phys. Rev. D* 19 (1979).

[8] Labastida, J. M. F., E. Sanchez-Velasco, R. E. Shrock, and P. Wills. In: *Phys. Rev. D* 34 (1986).

Modifed actions can lower truncation needed[9]

$$f(z) = \beta_0 + \tfrac{1}{2}\beta_4(z + z^{-1}) + \beta_2 z^2 .$$



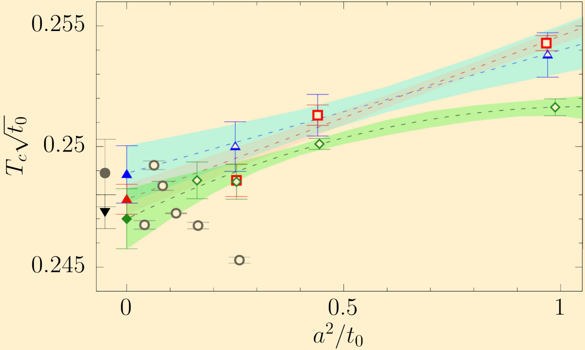
Fewer qubits at cost of circuit depth

[9] Fukugita, M., T. Kaneko, and M. Kobayashi. In: *Nucl. Phys. B* 215 (1983).

$T_c\sqrt{t_0}$ suggests $a \approx 0.07$ fm ≈ 2 GeV^{-1} possible[11]

$$S = \sum \frac{\beta_0}{3} \text{Re Tr } U + \beta_1 f(U) \text{ with } f(U) = \{\text{Tr}^2\, U + \text{Tr } U^2, |TrU|^2\}$$

Compare to SU(3)[10]



On-going work to extract quenched spectroscopy

[10] Francis, A., O. Kaczmarek, M. Laine, T. Neuhaus, and H. Ohno. In: *Phys. Rev. D* 91 (2015).

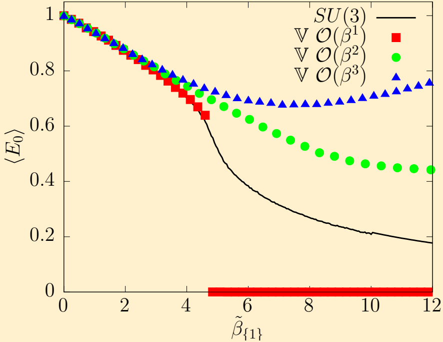
[11] Alexandru, A. et al. In: *Phys.Rev.D* 100 (2019). arXiv: 1906.11213 [hep-lat].

Systematics from Group Space Decimation[12]

Decimate via $U = u \cdot \epsilon$ in analogy to Wilsonian renormalization:

$$Z = \int_G DU\, e^{-S[U]} = \sum_{u\in H} \int_\Omega D\epsilon\, e^{-S[u,\epsilon]} = \sum_{u\in H} e^{-S[u]}\,,$$

$$S[u] = \sum_p \beta_{\{1\}} \frac{1}{3} \mathrm{Re}\, \chi_{\{1\}} + \beta_{\{2\}} \frac{1}{6} \mathrm{Re}\, \chi_{\{2\}} + \beta_{\{1,-1\}} \frac{1}{8} \chi_{\{1,-1\}} + \cdots$$



[12] Flyvbjerg, H. In: *Nucl. Phys.* B243 (1984), Ji, Y., **H. Lamm**, and S. Zhu. In: *Phys. Rev. D* 102 (2020). arXiv: 2005.14221 [hep-lat].

Systematics from Character Expansion

Any $SU(3)$-invariant LGT can be expanded[13]

$$e^{S(U)} = \sum_{\{\lambda,\mu\}} \beta_{\{\lambda,\mu\}} \chi_{\{\lambda,\mu\}}(U) = \sum_r \beta'_r \chi'_r(u) = e^{S(u)} = \exp\left(\sum_{i=1}^{17} \gamma_i \chi'_i(u)\right)$$

Where for the **Wilson** action, one finds[14]:

χ'_i	γ_i
$\mathbb{1}$	$-0.0555556\beta^2 - 0.00462963\beta^3 + 0.000771605\beta^4 + 0.000160751\beta^5 + \ldots$
$\mathrm{Re}\,\mathrm{Tr}\,u$	$0.166667\beta + 0.000964506\beta^4 - 0.000186471\beta^5 - 0.0000471536\beta^6 + \ldots$
$\lvert\mathrm{Tr}\,u\rvert^2 - 1$	$-0.0555556\beta^2 - 0.00925926\beta^3 + 0.000685871\beta^5 + 0.000101214\beta^6 + \ldots$
$\frac{1}{2}\left(\mathrm{Tr}^2(u) + \mathrm{Tr}(u^2)\right) + c.c.$	$0.00462963\beta^3 + 0.00147891\beta^4 - 0.000218621\beta^5 - 0.0000878772\beta^6 + \ldots$
$\frac{1}{6}\left(\mathrm{Tr}^3(u) + 2\,\mathrm{Tr}(u^3) + 3\,\mathrm{Tr}(u)\,\mathrm{Tr}(u^2)\right)$	$-0.00308642\beta^3 - 0.000771605\beta^4 + 0.000643004\beta^5 + 0.000136936\beta^6 + \ldots$
χ_j	$\mathcal{O}(\beta^4)$

[13] Flyvbjerg, H. In: *Journal of mathematical physics* 26 (1985).

[14] Ji, Y., **H. Lamm**, and S. Zhu. In: (2021). in prep: 21XX.XXXXX.

It's time to go

Digitization has many knobs, and their settings remain unclear

- Hamiltonians
  - H_{KS} will slowly approach continuum
  - Other H may distinguish differently between schemes
- Discrete Subgroups have advantages
- Theory errors matter!
  - e.g. Finite volume, finite a, a_t, decimation errors, fidelity to obtain **realisitic** resource estimates