Efficiency Study of Overrelaxation and Stochastic Overrelaxation Algorithms for SU(3) Landau Gauge-Fixing

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Introduction

To obtain gauge dependent quantities on the lattice, like the propagators, need to fix the gauge of configurations $U = \{U_\mu(n)\}$. For Landau gauge ($\partial_\mu A_\mu = 0$) done by maximizing functional

$$\mathcal{E}[g] \equiv \frac{1}{dN_c|\Lambda|} \sum_{\mu=1}^{d} \sum_{n\in\Lambda} \frac{1}{2} \mathrm{tr}\left[U_\mu^{(g)}(n) + U_\mu^{(g)\dagger}(n)\right], \quad (1)$$

where $U_\mu^{(g)}(n) \equiv g(n)U_\mu(n)g^\dagger(n+\hat{\mu})$, with respect to $\mathcal{G} = \{g(n)\}$. Since the algorithms considered are local, we define a local version of this functional

$$\mathcal{E}[g(n)] = \frac{1}{dN_c|\Lambda|} \mathrm{tr}[g(n)h(n)], \quad (2)$$

where $h(n) \equiv \sum_{\mu=1}^{d} \left(U_\mu(n)g^\dagger(n+\hat{\mu}) + U_\mu^\dagger(n-\hat{\mu})g^\dagger(n-\hat{\mu})\right)$.
We consider a config gauge-fixed if

$$\frac{1}{|\Lambda|} \sum_{n\in\Lambda} \sum_{b=1}^{N_c^2-1} \left(\nabla_\mu A_\mu^b(n)\right)^2 < 10^{-12}. \quad (3)$$

For SU(2), $g(n) = h^\dagger(n)/\sqrt{\det h(n)}$ maximizes local functional.
For SU(3) we take $g^{(\mathrm{new})}(n) = A(n)g^{(\mathrm{old})}(n)$ with $A = T.S.R.$ R, S e T are SU(2) matrices embedded in SU(3) matrix, calculated following the SU(2) method. $g(n)$ maximizes local functional asymptotically and iteratively. Number of iterations is free parameter of optimization.
To speed up the gauge-fixing procedure, we can also use modified versions of the algorithm (see below).

Overrelaxation (OR)

Instead of A(n) use $A^\omega(n)$ with $1 < \omega < 2$

$$A^{\mathrm{OR}}(n) = \mathrm{Proj}_{\mathrm{SU(3)}} \sum_{m=0}^{\infty} \frac{\Gamma(\omega+1)}{\Gamma(\omega+1-m)m!}(A - \mathbb{1})^m, \quad (4)$$

and ω is another parameter of optimization.

Stochastic Overrelaxation (SOR)

Instead of A(n) use stochastic process

$$A^{\mathrm{SRE}}(n) \equiv \begin{cases} A(n) & \text{with probability } 1-p, \\ (A(n))^2 & \text{with probability } p, \end{cases} \quad (5)$$

and p is another parameter of optimization.

Optimization

Optimization of the number of iterations shows that 2 hits is best if extra time to do extra hit is taken into account.

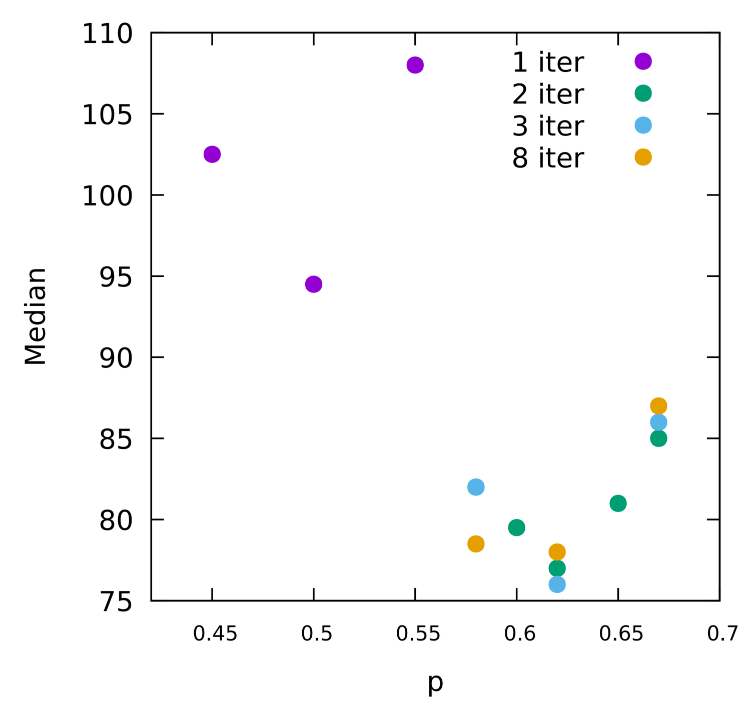

Figure 1: Optimization of # of iterations for SOR on a 4^4 lattice. Similar results for OR and larger lattices.

A sample of 200 configs (100 configs for 16^4 lattice) generated by heat bath algorithm with pure-gauge Wilson action, allowed us to obtain optimal parameters for OR and SOR by looking at distribution of sweeps until gauge-fixed.

Constant physics analysis gives the dynamical critical exponent of the algorithms. The best fit of the form CN^z gives $C = 14(2)$ and $z = 1.06(5)$ for OR and $C = 14.7(8)$ and $z = 1.13(2)$ for SOR, much better than the $z = 2$ for the original method.

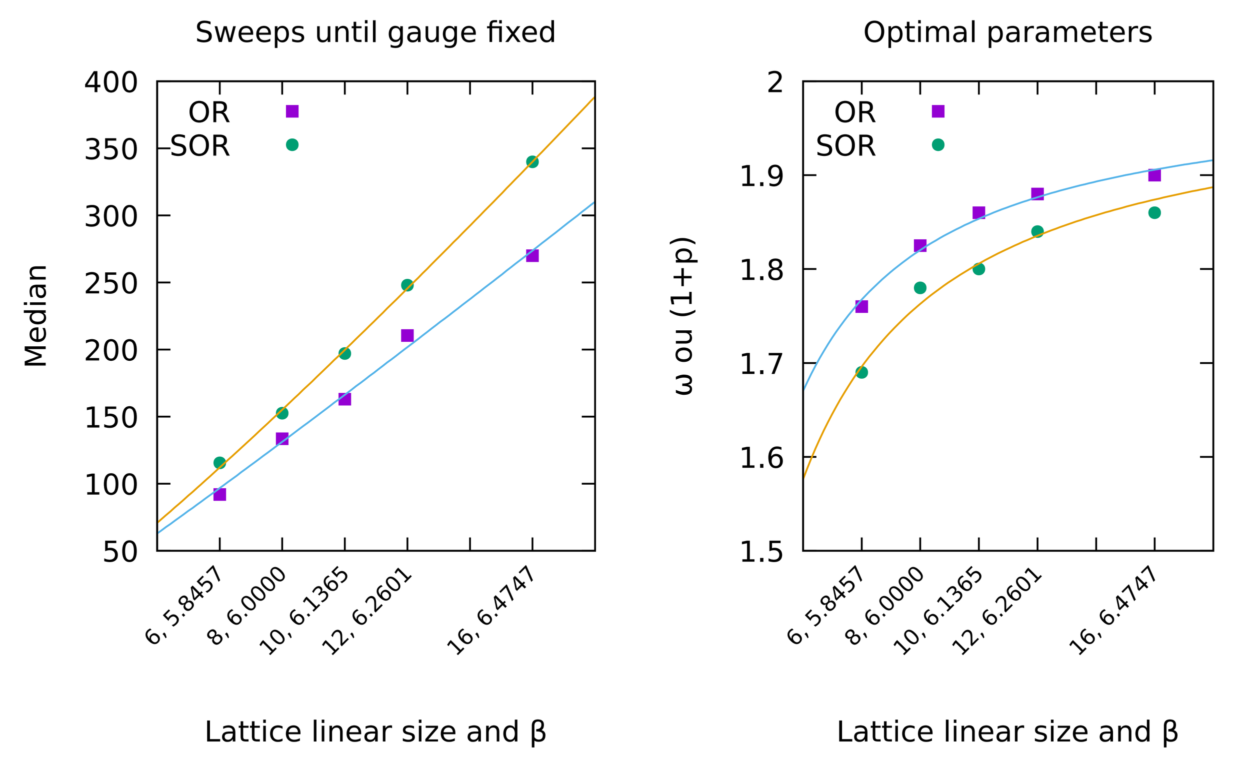

Figure 2: Gauge-fixing at constant physics

Conclusion

- OR and SOR reduce critical exponent from $z = 2$ of the original algorithm to $z \approx 1$ using optimal parameters
- 2 iterations is enough to reduce substantially # of sweeps needed to fix the gauge
- more than 2 iterations do not contribute to further reduce # of sweeps to fix the gauge
- OR performs betten than SOR for SU(3)

References

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Future Research

- Research on Landau-gauge propagators for gluons and quarks in the vacuum being conducted;
- Two-point functions at high-temperature;
- Project to calculate three-point functions in the vacuum and at high-temperature on the way.

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