Quantum Optimization of Interpolators for Classical Lattice Calculations

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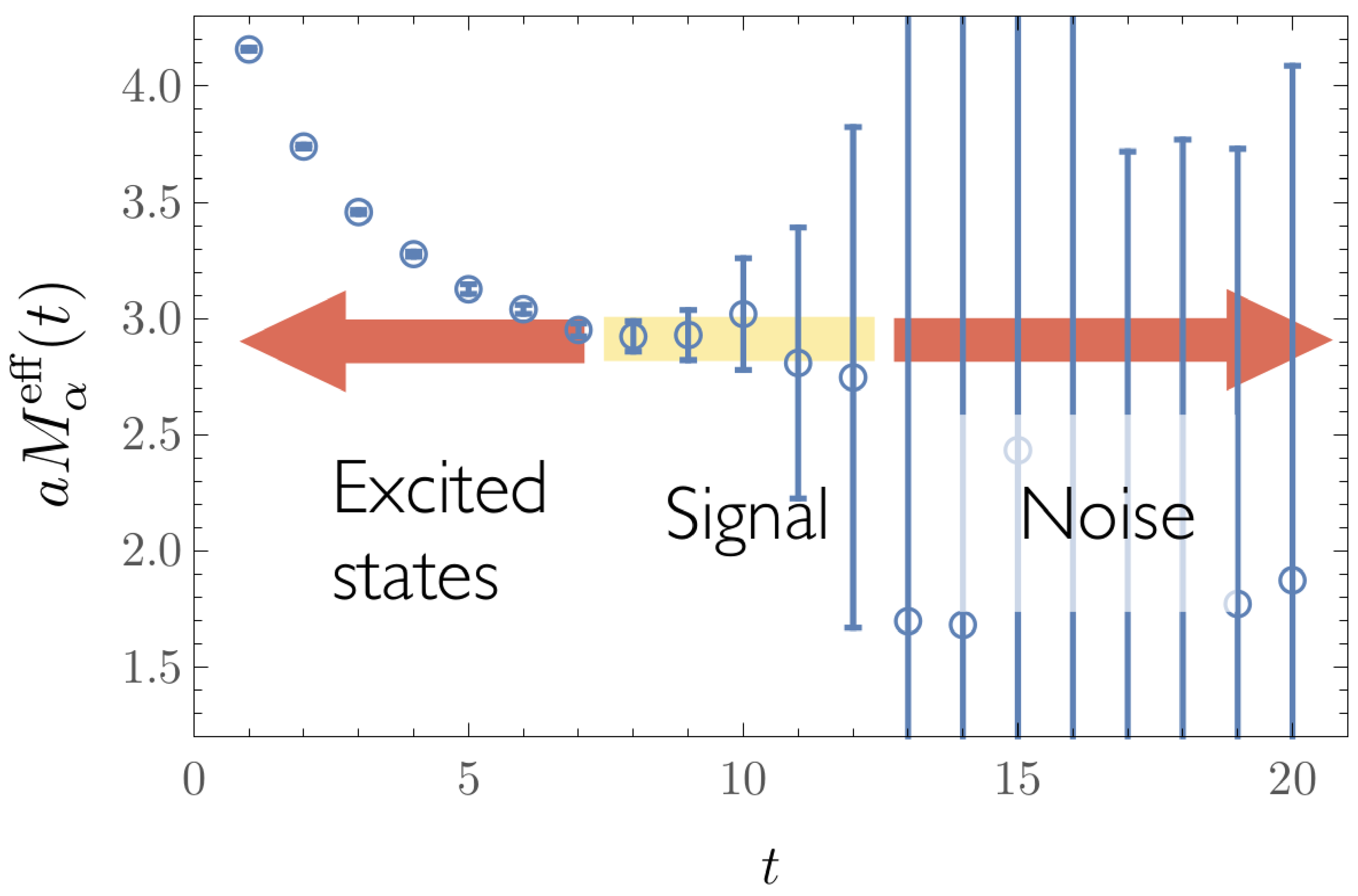
Interpolating Operators

Interpolating operators are used in lattice QFTs to prepare initial and final states in matrix elements of interest. Good interpolators result in ground-state dominance at earlier time, increasing the signal region. A typical effective mass plot [1] motivates optimized interpolator constructions:



A good interpolated state has a large overlap with the ground state and little overlap with low-lying excited states. This is evident from a two-point correlator:

$$C_{ij}(t) = \langle\Omega|\,\hat{O}_i(t)\,\hat{O}_j^{\dagger}(0)\,|\Omega\rangle = \sum_{E,\boldsymbol{\sigma}} \langle\Omega|\,\hat{O}_i\,|E,\boldsymbol{\sigma}\rangle\langle E,\boldsymbol{\sigma}|\,\hat{O}_j^{\dagger}\,|\Omega\rangle\, e^{-(E-E_{\Omega})t}.$$

Given a target state, we may construct interpolators with the same symmetries, but do not know beforehand which combination of them is optimal.

Proof of Concept Using the Schwinger Model

In this work, we aim to show that an optimal combination of pre-selected interpolators may be obtained as

$$\boldsymbol{\alpha} = \arg\min_{\boldsymbol{\alpha}} \frac{\langle\Omega|\hat{O}(\boldsymbol{\alpha})\hat{H}\,\hat{O}^{\dagger}(\boldsymbol{\alpha})|\Omega\rangle}{\langle\Omega|\hat{O}(\boldsymbol{\alpha})\,\hat{O}^{\dagger}(\boldsymbol{\alpha})|\Omega\rangle},\quad \hat{O}(\boldsymbol{\alpha}) = \sum_i \alpha_i \hat{O}_i,$$

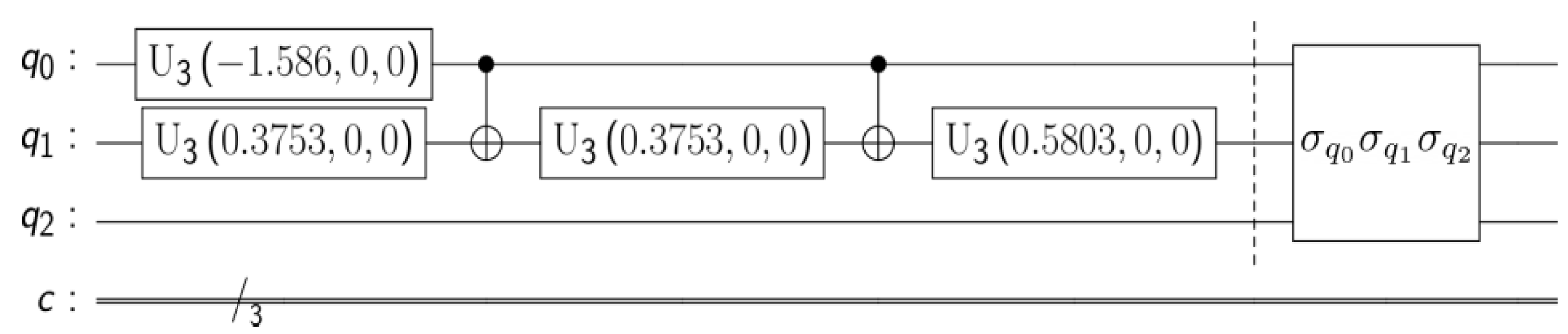
where the vacuum state is approximated on a small-scale quantum device. Once found, the optimal construction can be used in standard lattice QFT calculations. For a proof-of-concept, we choose a set of 2 interpolators to compute the pseudoscalar meson mass in a single-flavor massive Schwinger model.

Quantum Simulation

We approximate the vacuum state of the Schwinger model with a variational quantum eigensolver (VQE). The target Hamiltonian is

$$\hat{H} = \sum_n \beta\left[\left(\hat{\sigma}_n^{+}\,\hat{U}(n,\,n+1)\,\hat{\sigma}_{n+1}^{-} + \text{h.c.}\right) + m\,(-)^n\,\hat{\sigma}_n^{z}\right] + \sum_n \hat{\mathcal{E}}_n^2.$$

We follow the encoding outlined by Klco *et. al* [2] and measure the requisite matrix elements with a quantum circuit:



Here, the 2-qubit variational layer approximates the vacuum state, and the 3-qubit Pauli strings are used to decompose the operators.

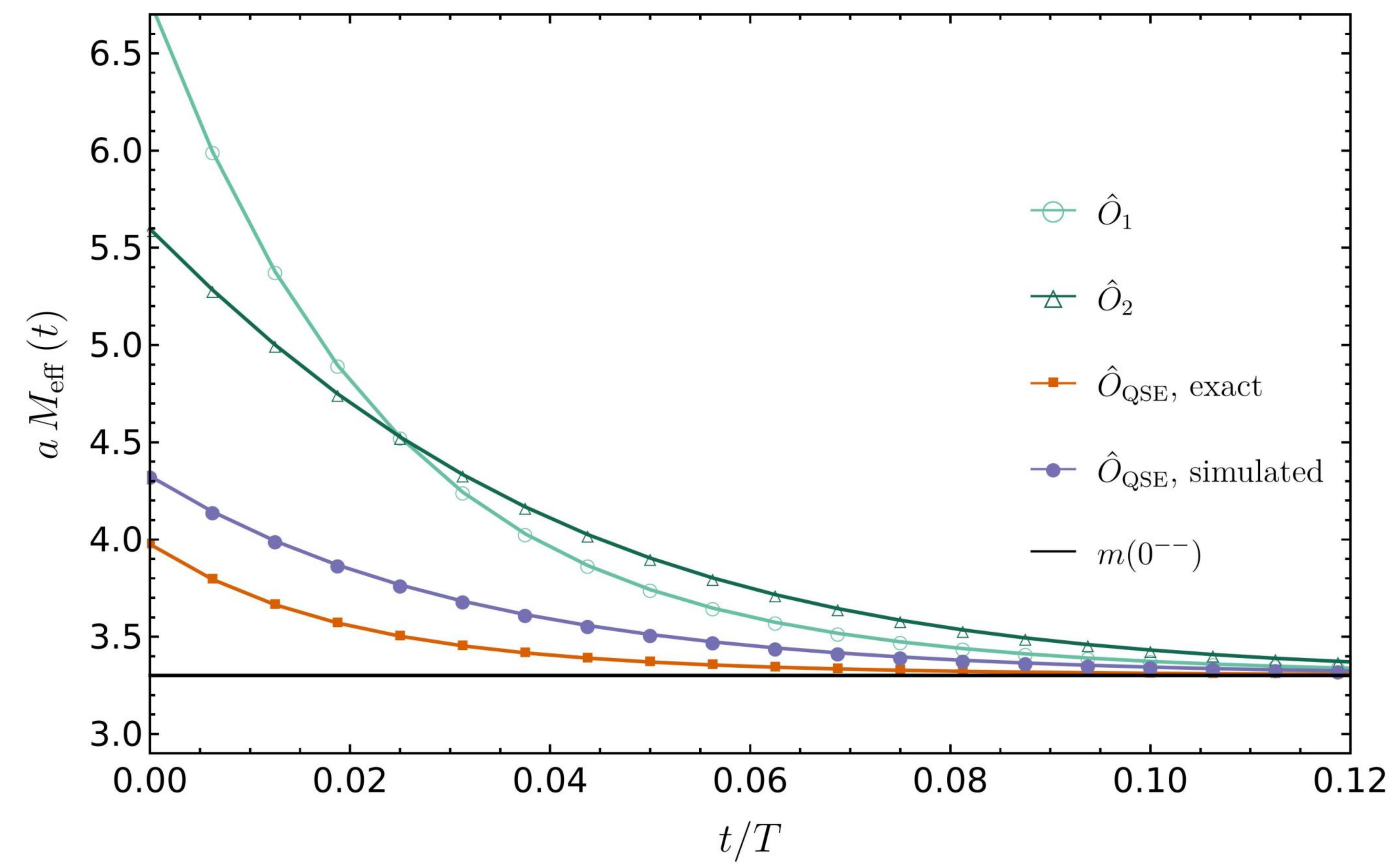
Classical Simulation

The optimal interpolator combination will be used on a standard gauge field ensemble for the Schwinger model, generated according to the staggered action

$$S = \sum_{n,\,m\,\in\,\Lambda} \bar{\chi}_n D(n|m)\chi_m + \sum_{n\,\in\,\Lambda} \beta\,\mathrm{Re}[1 - U_{12}(n)].$$

Results

Interpolator optimization from the VQE for this study is complete. Based on exact diagonalization, we expect to see the following improvement in the effective mass curve for a pseudoscalar meson:



Above, the orange curve is the best possible combination for the chosen interpolator set. Additional excited-state contamination in the purple curve is due to the variational approximation of the vaccuum state and simulated noise on the quantum device. Results from Monte-Carlo data are forthcoming.

References:
[1] The effective mass plot is taken from the slides by P. E. Shanahan, "Machine Learning for Lattice Quantum Field Theory Calculations" (conference presentation, KITP, Santa Barbara, CA, February 15, 2019).
[2] The encoding for the Schwinger model VQE is given in N. Klco, E. F. Dumitrescu, A. J. McCaskey, T. D. Morris, R. C. Pooser, M. Sanz, E. Solano, P. Lougovski, and M. J. Savage, Phys. Rev. A 98, 032331 (2018).
[3] The proposal for quantum-accelerated interpolator constructions is made in A. Avkhadiev, P. E. Shanahan and R. D. Young, Phys. Rev. Letters 124, 080501 (2020).

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